Speaker

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Title

Brick wall excursions: combinatorial interpretation of Random Flight moments

Abstract

Consider a short uniform random walk in $\mathbb{R}^d$ that starts at the origin and consists of m independent unit steps in random directions (this walk is also known in the literature as random flights). Take the distance to the origin (after m such steps) and compute a sequence of its even moments. As was shown in 2015 by Borwein, Straub, and Vignat, in dimensions $d = 2$ and $d = 4$, this is an integer sequence. While for $d = 2$, the $2n$th moment is equal to the number of abelian squares of length $2n$ over an alphabet with m letters, for $d = 4$ no interpretation was known.

The aim of this talk is to provide such an interpretation, both for $d = 2$ and $d = 4$, in terms of $2n$-step lattice paths in dimension $m - 1$. Our construction relies on a bijection between Dyck paths with a prescribed number of peaks and words of a certain type. In addition, this bijection allows us to derive closed formulas for the number of lattice paths provided with certain statistics.

This talk is based on the ongoing work with Sergey Kirgizov and Michael Wallner.