Growing binary trees

Khaydar Nurligareev
LIP6, Sorbonne University
Khaydar.Nurligareev@lip6.fr

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Abstract

In this talk, we study a growing process that generates the family of binary trees. The process follows the next steps: we start with an anchor (that is, an active leaf), and at each step, we replace every anchor either by a (dead) leaf or by an internal node with two children that both are anchors themselves. Our interest is focused on active trees obtained using the above process, that is, in trees with at least one anchor.

Let us denote by $t_{n,m}$ the number of binary trees with n internal nodes and m anchors. For a fixed value of n, the total number of such trees (when m varies) is the Catalan number C_n. We empirically observe that the proportion $t_{n,2}/C_n$ tends to a certain limit, as $n \to \infty$, and show that this proportion is greater than 5/8. On the other hand, for a fixed value of n, let us denote by a_n the number of nonzero elements $t_{n,m}$. We establish the behavior of the sequence (a_n) and show that it is known in the literature as a meta-Fibonacci sequence [1]. We also provide relations between the generating functions of growing binary trees and the so-called Mandelbrot polynomials [2].

Finally, we study the active binary trees with respect to height. Here, we establish the limit shapes of the nonzero domains

$$\{(n,k)\colon t_{n,2k,h} \neq 0\}, \qquad \text{as } h \to \infty,$$

and

$$\{(k,h)\colon t_{n,2k,h} \neq 0\}, \qquad \text{as } n \to \infty,$$

where $t_{n,m,h}$ the number of active trees of height h with n internal nodes and m anchors.

This talk is based on the ongoing work with Antoine Genitrini.

References

[1] Stephen M. Tanny, *A well-behaved cousin of the Hofstadter sequence*, Discrete Mathematics, 1992.

[2] Neil J. Calkin, Eunice Y. S. Chan, Robert M. Corless, *Some Facts and Conjectures about Mandelbrot Polynomials*, Maple Transactions, 2021.