Speaker

Khaydar Nurligareev

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Title

Regular plane multi-tilings

Abstract

In the early 1970s, at one of his math circles for high school students, Andrei Kolmogorov set a problem involving the classification of the regular tilings. By a *tiling*, he meant a covering of a plane with regular polygons without gaps or overlaps, in which any two polygons either share a side, share a vertex, or do not intersect at all. By a *regular tiling*, he meant a tiling that can be superimposed onto itself in such a way that a given vertex coincides with another vertex chosen beforehand. There is a finite number of regular tilings, and the young students were asked to find them all and verify that no other regular tilings exist.

Every regular tiling is determined by its *vertex type* (that is, the order in which the polygons appear when traversing around a vertex of the tiling counterclockwise). Therefore, to classify regular tilings, it is sufficient to identify all possible vertex types and then, for each type found, determine whether a regular tiling corresponding to that type exists. A regular tiling may not exist, since, when placing polygons next to each other so that all vertex types are the same, in some cases we cannot avoid overlaps. This raises the following question: what happens if we allow the tiles to overlap? Generally speaking, we obtain a multi-tiling. Is the number of layers in this tiling finite? And if so, what is the number of layers? This talk is devoted to answering all these questions.