Speaker

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Title

Semi-regular Polygons on Regular Tilings

Abstract

One of the well-known theorems of discrete geometry states: if a regular polygon is placed on an integer lattice such that its vertices lie at the lattice points, then the polygon is a square. The situation is more tricky when we switch to *semi-regular* (equiangular and equilateral) polygons. In the second half of the 20th century, D. Ball showed that an equiangular polygon with vertices at integer points is a quadrangle or an octagon. As for equilateral polygons, it turns out that one can construct a polygon with any given even number of sides such that all its vertices are points with integer coordinates.

The set of vertices of an integer lattice in the plane can also be viewed as the set of vertices of a square tiling. This tiling is one of the eleven existing *regular tilings*, that is, tilings of the plane with regular polygons without gaps or overlaps, in which any two polygons either do not intersect or intersect at a vertex or an edge, and for which all vertices of the tiling are equivalent. In a talk at the Kolmogorov Readings in 2001, tenth-graders I. Sedoshkin and E. Mychka (AESC, Moscow State University) presented the results of their research, which can be summarized as follows. On each of the regular tilings, only those regular polygons that are "visible to the naked eye" can be placed.

In this talk, we examine a generalization of the above problem, namely, the question of placing semi-regular polygons on regular tilings. The complete solution to this problem obtained by the speaker in early 2011 can be summarized as follows. If a regular polygon can be placed on a given regular tiling, then the number of its sides is 3, 4, 6, 8, 12, 16, or 24. An equilateral polygon with any given number of sides can be found for each of the regular tilings, except for the square and truncated square. For the two latter tilings, the additional condition that the number of sides of the polygon be even is required.