Interaction solos

D. Mazza
(LIPN)

A. Dorman
(Roma Tre - LIPN)

PANDA

November 24th, 2011

1. Motivations

2. Interaction solos calculus

3. Subsystems and first translation

4. Translations

5. Non-determinisms

“In the beginning was the sign”... (Hilbert, 1922)

Then came the picture.

- **Proof-nets** (*MLL*)
  - Strongly confluent, normal “forest” form...
- **Lafont** abstracted.
  - Strongly confluent, normal form, natural observability...
  - Universal system (combinators), execution semantics, geometry of interaction...
- **Other people** (Alexiev, Yoshida, Mazza, Beffara-Maurel, Ehrhard-Regnier,...) generalized.
  - Not confluent, no nice form, no implicit observability...

... no structure.

Looking for the words again

Interaction Solos calculus

Let Λ be a labeling alphabet, with arities and co-arities.

Let N be a set of names.

$$
\begin{aligned}
S &::= \quad \alpha(x_1, \ldots, x_m; y_1, \ldots, y_n) \\
W &::= \quad \{x_1, \ldots, x_n\} \\
P, Q &::= \quad 0 \mid S \mid W \mid (P|Q) \mid \nu x\, P
\end{aligned}
$$

(Notice there is no constructor for replication.)

Structural congruence

... is the smallest congruence that satisfies

	$P_1 \mid (P_2 \mid P_3)$	$\equiv$	$(P_1 \mid P_2) \mid P_3$	
Parallel	$P_1 \mid P_2$	$\equiv$	$P_2 \mid P_1$	
	$P \mid 0$	$\equiv$	P	
	$\nu v\, \nu w\, P$	$\equiv$	$\nu w\, \nu v\, P$	
Restriction	$\nu z\, 0$	$\equiv$	0	
	$\nu z\, (P_1 \mid P_2)$	$\equiv$	$P_1 \mid \nu z\, P_2$	if $z \notin \mathtt{fn}(P_1)$
	Wires	are	sets.	
Wires	$\{x\}$	$\equiv$	0	
	$W \mid W'$	$\equiv$	$W \cup W'$	if $W \cap W' \neq \emptyset$

Interaction

Interactions are a generalisation of process calculi reductions. Where in solos, reductions identify names, interactions can evolve into any given process.

$$\alpha(\tilde{a};\tilde{x}) \mid \beta(\tilde{b};\tilde{y}) \mid a_i{=}b_j \xrightarrow{\ e\ } R \mid a_i{=}b_j$$

where $a_i \in \tilde{a}$, $b_j \in \tilde{b}$ and $\mathtt{fn}(R) \subseteq \{\tilde{a}, \tilde{x}, \tilde{b}, \tilde{y}\}$ (which are all different).

***Remark*:** unique restriction on R: symmetry when $\alpha_i = \beta_j$.

As usual...

$$\frac{P \xrightarrow{\quad e \quad} Q}{\sigma P \longrightarrow \sigma Q} \qquad \text{where} \begin{cases} (P \xrightarrow{e} Q) \in \mathcal{R} \\ \sigma \text{ is any name substitution.} \end{cases}$$

$$\frac{P \longrightarrow Q}{P \mid R \longrightarrow Q \mid R} \qquad\qquad \frac{P \longrightarrow Q}{\nu x\, P \longrightarrow \nu x\, Q}$$

$$\frac{P \equiv P' \longrightarrow Q' \equiv Q}{P \longrightarrow Q}$$

As expressive as

... for instance **Explicit Fusion** (without replication, because...)

$$P, Q ::= 0 \mid P|Q \mid \nu x\, P \mid \bar{x}.P \mid x.P \mid \langle x \rangle \mid x{=}y$$

$$z.P \mid \bar{z}.Q \searrow P@Q$$

$$\nu\tilde{a}_1\,(\langle\tilde{d}_1\rangle \mid P')\ @\ \nu\tilde{a}_2\,(\langle\tilde{d}_2\rangle \mid Q') \;=\; \nu\tilde{a}_1\tilde{a}_2\,(\tilde{d}_1{=}\tilde{d}_2 \mid P' \mid Q').$$

In solo calculus:

- $\Sigma = \{\alpha_{i,j} \mid i,j \in \mathbb{N}\} \cup \{\beta_{i,j} \mid i,j \in \mathbb{N}\} \cup \{\partial\}$.
- $\forall i,j$, let $\mathtt{ar}(\alpha_{i,j}) = \mathtt{ar}(\beta_{i,j}) = i + 2j$ and $\mathtt{coar}(\alpha_{i,j}) = \mathtt{coar}(\beta_{i,j}) = 1$

$$[\![x.P]\!] = \nu\tilde{a}\tilde{p}'\ \left(\alpha_{m,n}(x;\tilde{d},\tilde{p},\tilde{p}') \mid [\![P']\!]\right) \quad \textit{where} \begin{cases} P = \nu\tilde{a}\,(\langle\tilde{d}\rangle \mid P') \\ \tilde{p}' = \mathtt{fn}(P'), m = |\tilde{d}| \text{ and } n = |\tilde{p}| = |\tilde{p}'| \end{cases}$$

$$\alpha(x;a_1,\ldots,a_i,u_1,\ldots,u_n,v_1,\ldots,v_n) \mid \beta(y;b_1,\ldots,b_i,s_1,\ldots,s_m,t_1,\ldots,t_m) \mid x{=}y$$

$$\xrightarrow{e} \quad a_1{=}b_1 \mid \ldots \mid a_i{=}b_i \mid u_1{=}v_1 \mid \ldots \mid u_n{=}v_n \mid s_1{=}t_1 \mid \ldots \mid s_m{=}t_m \mid x{=}y$$

... or Laneve and Victor's ***solos calculus***? Well... up-to barbed congruence...

Substitution lemma

For any process P and any two names a, b,

$$P \mid W_{a,b} \simeq^c P\{a/b\} \mid W_{a,b} \simeq^c P\{b/a\} \mid W_{a,b}$$

So one can get rid of many useless bounded names, or...

Corollary and definition

For any process P, there exists an *expansion* Q s.t. $P \simeq^c Q$ and

- every bounded name appears exactly twice in Q,
- every free name appears exactly once in Q.

... or Laneve and Victor's ***solos calculus***? Well... up-to barbed congruence...

Substitution lemma

For any process P and any two names a, b,

$$P \mid W_{a,b} \simeq^c P\{a/b\} \mid W_{a,b} \simeq^c P\{b/a\} \mid W_{a,b}$$

So one can get rid of many useless bounded names, or...

Corollary and definition

For any process P, there exists an *expansion* Q s.t. $P \simeq^c Q$ and

- every bounded name appears exactly twice in Q,
- every free name appears exactly once in Q.

... or Laneve and Victor's ***solos calculus***? Well... up-to barbed congruence...

Substitution lemma

For any process P and any two names a, b,

$$P \mid W_{a,b} \ \simeq^c \ P\{a/b\} \mid W_{a,b} \ \simeq^c \ P\{b/a\} \mid W_{a,b}$$

So one can get rid of many useless bounded names, or...

Corollary and definition

For any process P, there exists an *expansion* Q s.t. $P \simeq^c Q$ and

- every bounded name appears exactly twice in Q,
- every free name appears exactly once in Q.

Famous subsystems

Uniport (multiwire) solos

- for any $\alpha \in \Lambda$, $\mathtt{coar}(\alpha) = 1$.

The real new idea is:

Simply wired (multiport) solos

- wires are of size at most 2.

Lafont solos

- both uniport and simply wired interaction solos.

Additional property:

Multirule solos

- a rule can have several reduction rules.

Simplifying the wiring

$\alpha(\ldots, a, \ldots; \ldots)$

$\beta(\ldots, b, \ldots; \ldots)$

$\{a, b, c\}$

$\gamma(\ldots, c, \ldots; \ldots)$

Simplifying the wiring

$A_{\bar{1}}(\ldots, a, \ldots; \ldots)$

$B_{\bar{1}}(\ldots, b, \ldots; \ldots)$

$\{a, b, c\}$

$C_{\bar{1}}(\ldots, c, \ldots; \ldots)$

Simplifying the wiring

$A_{\bar{1}}(\dots, a, \dots; \dots)$ $B_{\bar{1}}(\dots, b, \dots; \dots)$

$$(\nu a_1, a_2, b_1, b_2, c_1, c_2)$$
$$\delta(a; a_1, a_2) \mid \delta(b; b_1, b_2) \mid \delta(c; c_1, c_2)$$
$$a_1{=}b_2 \mid b_1{=}c_2 \mid c_1{=}a_2$$

$C_{\bar{1}}(\dots, c, \dots; \dots)$

Simplifying the wiring

$(\nu a_1, a_2, b_1, b_2, c_1, c_2)$

$A_{\bar{1}}(\ldots, a, \ldots; \ldots)$

$B_{\bar{1}}(\ldots, b, \ldots; \ldots)$

$$\delta(a; a_1, a_2) \mid \delta(b; b_1, b_2) \mid \delta(c; c_1, c_2)$$
$$a_1 = b_2 \mid b_1 = c_2 \mid c_1 = a_2$$

$C_{\bar{1}}(\ldots, c, \ldots; \ldots)$

Simplifying the wiring

$(\nu a_1, a_2, b_1, b_2, c_1, c_2)$

$A_{\bar{1}2\bar{1}}(\ldots, a_1, a_2, \ldots; \ldots)$

$B_{\bar{1}}(\ldots, b, \ldots; \ldots)$

$\delta(b; b_1, b_2) \mid \delta(c; c_1, c_2)$

$a_1 = b_2 \mid b_1 = c_2 \mid c_1 = a_2$

$C_{\bar{1}}(\ldots, c, \ldots; \ldots)$

Simplifying the wiring

$(\nu a_1, a_2, b_1, b_2, c_1, c_2)$

$A_{\bar{1}2\bar{1}}(\dots, a_1, a_2, \dots; \dots)$ $B_{\bar{1}2\bar{1}}(\dots, b_1, b_2, \dots; \dots)$

$\delta(c; c_1, c_2)$

$a_1 = b_2 \mid b_1 = c_2 \mid c_1 = a_2$

$C_{\bar{1}}(\dots, c, \dots; \dots)$

Simplifying the wiring

$(\nu a_1, a_2, b_1, b_2, c_1, c_2)$

$A_{\bar{1}2\bar{1}}(\dots, a_1, a_2, \dots; \dots)$

$B_{\bar{1}2\bar{1}}(\dots, b_1, b_2, \dots; \dots)$

$a_1 = b_2 \mid b_1 = c_2 \mid c_1 = a_2$

$C_{\bar{1}2\bar{1}}(\dots, c_1, c_2, \dots; \dots)$

Lemma

*If P is an expanded process, than $[\![P]\!]$ is **simply wired**.*

Since every process has an extended congruent version,

Theorem (informal)

***Simply wired** interaction solos are operationnaly as expressive as general ones.*

Translations

Gave us some properties on translations, that one can wish to have.
A *translation* is

1. "functorial"

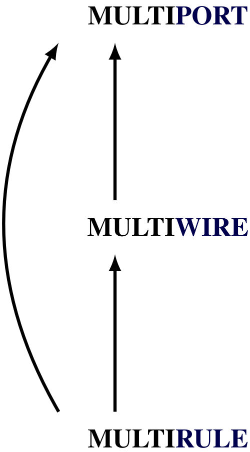
$$[\![\alpha(\tilde{x};\tilde{y})]\!] = R^{\alpha}, \quad [\![W]\!] = R^{w}$$

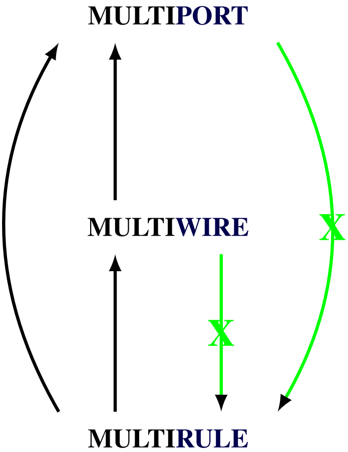
2. compositional

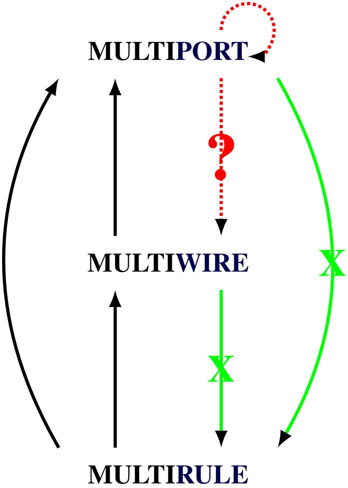
$$[\![0]\!] = 0, \quad [\![P \mid Q]\!] = [\![P]\!] \mid [\![Q]\!], \quad [\![\nu z\, P]\!] = \nu z\, [\![P]\!]$$

3. operationnaly cool

$$P \stackrel{\bullet}{\approx} [\![P]\!]$$


MULTIPORT
MULTIWIRE
MULTIRULE


MULTIPORT
MULTIWIRE
X
X
MULTIRULE


MULTIPORT
?
MULTIWIRE
X
X
MULTIRULE

Non-determinisms

In a rewriting system, any concept of **residue** yields the following definitions.

For any coexisting interactions r and s in a process P;

- r and s are independant if it's still possible to execute one after executing the other.
- r and s are separated if any coexisting interaction t is independant with r or s.
- r and s are contemporary if... they "appeared" at the same time (for any reduction path such that r already existed, so did s).
- r and s are in simple conflict if contemporary and not independant.

Confusion-free rewriting system

... if any coexisting interactions are either separated **or** in simple conflict.

Non-determinisms

In a rewriting system, any concept of **residue** yields the following definitions.

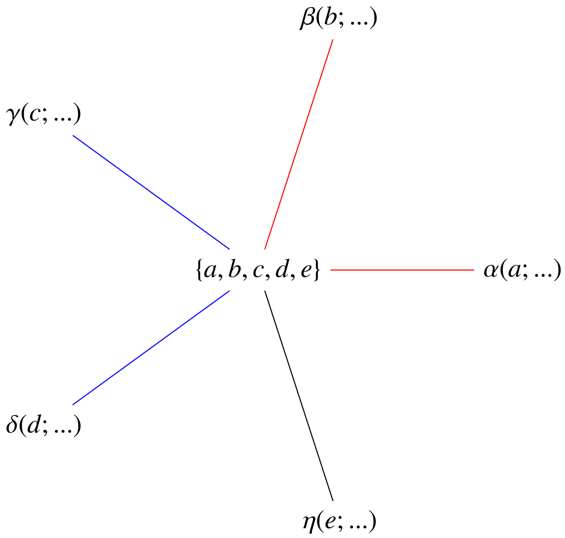
For any coexisting interactions r and s in a process P;

- r and s are independant if it's still possible to execute one after executing the other.
- r and s are separated if any coexisting interaction t is independant with r or s.
- r and s are contemporary if... they "appeared" at the same time (for any reduction path such that r already existed, so did s).
- r and s are in simple conflict if contemporary and not independant.

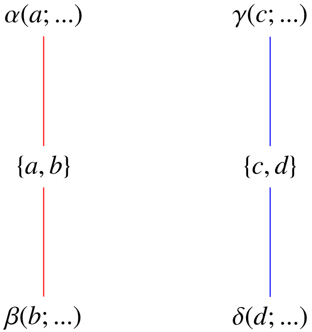
Confusion-free rewriting system

... if any coexisting interactions are either separated **or** in simple conflict.

Conflict in multiwires

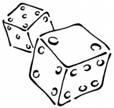


No conflict in multirule



Divergence

Dice and chips.



vs.

Best of both worlds

Process calculus

- Bisimilarities and congruences.
- Any "translation" from multiport to multiwire seems to introduce divergence. Actually, reducing the maximal number of principal ports does so already.
- Replication is just a particular reduction rule.

Graph rewriting

- Nice intuitions.
- Natural label transition system.
- Work out some semantic?
  (denotational, path theory, GoI...)

Best of both worlds

Process calculus

- Bisimilarities and congruences.
- Any "translation" from multiport to multiwire seems to introduce divergence. Actually, reducing the maximal number of principal ports does so already.
- Replication is just a particular reduction rule.

Graph rewriting

- Nice intuitions.
- Natural label transition system.
- Work out some semantic?
  (denotational, path theory, GoI...)

Thank you.