A monadic approach to ressource calculus

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Location LIPN, Université Sorbonne Paris Nord, Villetaneuse

Advisors Marie Kerjean, with the possibility of remote discussions with Jean-Simon Lemay (McQuarie University, Australia). Jean-Simon will visit in May for two weeks.

Funding Possibilities for funding both the intership and the PHD

Language The internship can be done in French or in English

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General context and objectives Monads are a common object in the semantics of programming language. They are used to express a variety of effects and usage on typed functional programming languages. On the other hand, the quantitative point of view for programming languages consists of measuring their usage in time, space, or resources through syntax, types, or models. This has in particular led to refined results in λ-calculus [2][1][7] and innovations in probabilistic programming [4][3].

In denotational semantics, this consists of interpreting programs by power series, whose coefficients represent the quantitative information one would like to retrieve. In an analytic context, power series are, in particular, those functions that equal their Taylor series at 0:

$$\forall x, f(x) = \sum_{n\in\mathbb{N}} \frac{1}{n!} D_0^{(n)}(f)(x).$$

The introduction of differentiation as a core primitive of the λ-calculus was made possible by Linear Logic. Together with Jean-Simon [6], we recently showed that Taylor expansion can be expressed in terms of a *monad* structure on the main connective ! in Linear Logic [5].

The monad unit represents *differentiation* at 0, while the monad multiplication will correspond to the *convolutional exponential*, a preexisting object in functional analysis. This work took place in Differential Linear Logic, a sequent calculus extending Linear

Logic with a differentiation rule. The goal of the internship and the potential Ph.D. thesis would be to make the monadic point of view work in λ-calculus and to express the syntax and theorems of quantitative λ-calculus thanks to the new monadic point of view. This internship will be the occasion to learn much about lambda-calculus, linear logic, and their semantics, and what you like before starting a PhD thesis. The subject can of course according to your liking be adapted before or during the internship.

Prerequisite No formal prerequisite is necessary, although having a come across categories and λ-calculus would be easier. It would also help to have a good taste in analysis or good memories of what you did in math before specializing to Computer Science. However, everything can be picked up when you arrive. This subject would be a good fit for someone with a good taste for logic and subjects at the frontier of Computer Science and Maths. Nevertheless, do not hesitate to contact me; we can chat when in doubt.

Scientific Environment The LIPN is a young and dynamic laboratory. It is located in Villetaneuse, approximately a 30mn commute from Gare du Nord by train and tram. Our team is centered around Categories, Linear Logic, Type theory, and lambda-calculi. Other teams work on verification, combinatorics, algorithmic, machine learning, natural language processing. We have weekly team seminars, and you will have the occasion to return to Lyon for the monthly Chocola Seminar. Our team has around ten permanent members and 10 PhD or postdocs.

References

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