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Smooth models of Linear Logic

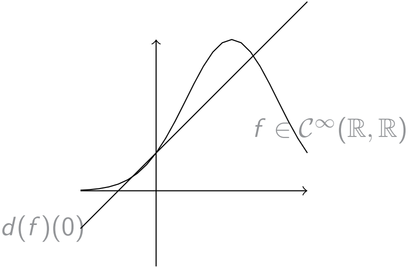
Marie Kerjean

IRIF, Université Paris Diderot
kerjean@irif.fr

Smoothness

Differentiation

Differentiating a function $f : \mathbb{R}^n \to \mathbb{R}$ at x is finding a linear approximation $d(f)(x) : v \mapsto d(f)(x)(v)$ of f near x.



Smooth functions are functions which can be differentiated everywhere in their domain and whose differentials are smooth.

Differentiating proofs

- Differentiation was in the air since the study of Analytic functors by Girard :

$$\bar{d}(x) : \sum f_n \mapsto f_1(x)$$

- DiLL was developed after a study of vectorial models of LL inspired by coherent spaces : Finiteness spaces (Ehrhard 2005), Köthe spaces (Ehrhard 2002).

Normal functors, power series and λ-calculus. Girard, APAL(1988)

Differential interaction nets, Ehrhard and Regnier, TCS (2006)

Differential Linear Logic

The rules of DiLL are those of MALL and :

co-dereliction

$$\bar{d} : x \mapsto f \mapsto df(0)(x)$$

Syntax

$$\frac{\vdash \Gamma}{\vdash \Gamma, ?A}\, w \qquad \frac{\vdash \Gamma, ?A, ?A}{\vdash \Gamma, ?A}\, c \qquad \frac{\vdash \Gamma, A}{\vdash \Gamma, ?A}\, d$$

$$\frac{\vdash}{\vdash !A}\, \bar{w} \qquad \frac{\vdash \Gamma, !A \quad \vdash \Delta, !A}{\vdash \Gamma, \Delta, !A}\, \bar{c} \qquad \frac{\vdash \Gamma, A}{\vdash \Gamma, !A}\, \bar{d}$$

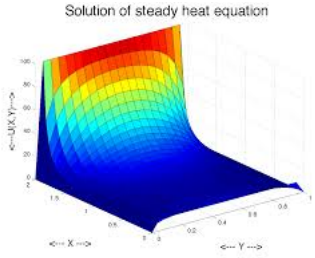
The computational content of differentiation

Historically, resource sensitive syntax and semantics

- Quantitative semantics : $f = \sum_n f_n$
- Resource λ-calculus and Taylor formulas : $M = \sum_n M_n$

Differentiation is inspired by the study of continuous systems :

- Differential Geometry and functional analysis
- Ordinary and Partial Differential Equations


Solution of steady heat equation

Smoothness of proofs

- Traditionally proofs are interpreted as graphs, relations between sets, power series on finite dimensional vector spaces, strategies between games.

- Differentiation appeals to differential geometry, manifolds, functional analysis : we want to find a denotational model of DiLL where proofs are smooth functions, and see what computational or categorical meaning it may have.

Smooth models of Linear Logic

$$A, B := A \otimes B | 1 | A \parr B | \bot | A \oplus B | 0 | A \times B | \top | !A | ?A$$

A decomposition of the implication

$$A \Rightarrow B \simeq !A \multimap B$$

A decomposition of function spaces

$$\mathcal{C}^\infty(E, F) \simeq \mathcal{L}(!E, F)$$

The dual of the exponential : smooth scalar functions

$$\mathcal{C}^\infty(E, \mathbb{R}) \simeq \mathcal{L}(!E, \mathbb{R}) \simeq !E'$$

A typical inhabitant of $!E$ is $ev_x : f \mapsto f(x)$.

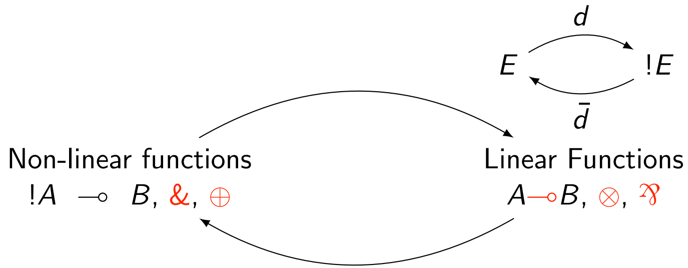
Interpreting DiLL in vector spaces

$$\begin{tikzcd}
 & & E \arrow[r, bend left, "d"] & !E \arrow[l, bend left, "\bar{d}"] \\
\begin{array}{c}\text{Non-linear functions}\\ !A \multimap B, \&, \oplus\end{array} \arrow[rr, bend left] & & \begin{array}{c}\text{Linear Functions}\\ A \multimap B, \otimes, \parr\end{array} \arrow[ll, bend left] &
\end{tikzcd}$$

$$!E \otimes !F \simeq !(E \times F)$$
$$d \circ \bar{d} = Id_E$$

$!E \otimes !F \simeq !(E \times F)$ allows to have a cartesian closed Co-Kleisli category

Interpreting DiLL in vector spaces



$$\begin{tikzcd} E \arrow[r, bend left, "d"] & !E \arrow[l, bend left, "\bar{d}"] \end{tikzcd}$$

Non-linear functions

$!A \multimap B$, $\&$, $\oplus$

Linear Functions

$A \multimap B$, $\otimes$, $\parr$

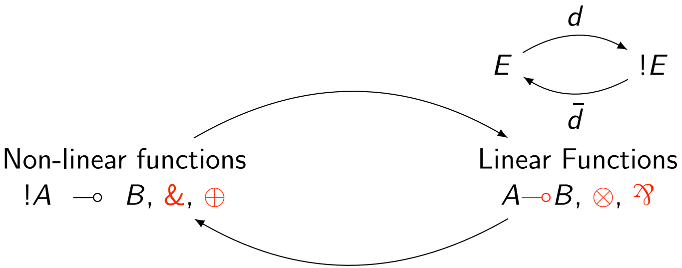
$$!E \otimes !F \simeq !(E \times F)$$

$$d \circ \bar{d} = Id_E$$

$d \circ \bar{d} = Id_E$: the differential at 0 of a linear function is the same linear function.

$\bar{c}$ and $\bar{w}$: an algebraic structure on $!A$ traditionally inherited from convolution.

Interpreting DiLL in vector spaces



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Linear Functions

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$$!E \otimes !F \simeq !(E \times F)$$

$$d \circ \bar{d} = Id_E$$

We want to find good spaces in which we can interpret all these constructions, and an appropriate notion of smooth functions.

Challenges

We encounter several difficulties in the context of topological vector spaces :

- Finding a category of general tvs and smooth functions which is Cartesian closed.
- Interpreting the involutive linear negation $(E^{\perp})^{\perp} \simeq E$

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📄 *Convenient differential category* Blute, Ehrhard Tasson Cah. Geom. Diff. (2010)

📄 *Mackey-complete spaces and Power series*, K. and Tasson, MSCS 2016.

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Weak topologies for Linear Logic, K. LMCS 2015.
Involves a topology which is an internal Chu construction.

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- *A model of LL with Schwartz' epsilon product*, K. and Dabrowski, *In preparation*.
- *Distributions and Smooth Differential Linear Logic*, K., *In preparation*

What's not working

A space of (non necessarily linear) functions between finite dimensional spaces is not finite dimensional.

$$\dim \mathcal{C}^0(\mathbb{R}^n, \mathbb{R}^m) = \infty.$$

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We can't restrict ourselves to finite dimensional spaces.

The tentative to have a normed space of analytic functions fails (Coherent Banach spaces).

- We want to use functions.
- For polarity reasons, we want the supremum norm on spaces of power series.
- But a power series can't be bounded on an unbounded space (Liouville's Theorem).
- Thus functions must depart from an open ball, but arrive in a closed ball. Thus they do not compose.
- This is why Coherent Banach spaces don't work.

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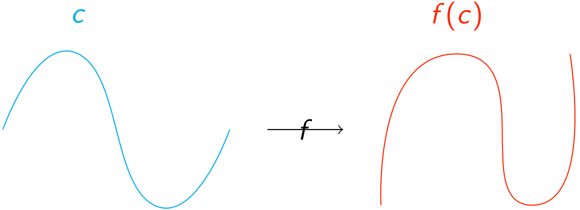
Smooth maps à la Frölicher,Kriegl and Michor

A smooth curve $c : \mathbb{R} \to E$ is a curve infinitely many times differentiable.



A smooth function $f : E \to F$ is a function sending a smooth curve on a smooth curve.

In Banach spaces, the definition coincides with the usual one (all iterated derivatives exists and are continuous).

Smooth maps à la Frölicher,Kriegl and Michor

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A model of IDiLL

This definition leads to a cartesian closed category of Mackey-complete spaces and smooth functions, and to a first smooth model of Intuitionist DiLL [a].

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[a] A Convenient differential category, Blute, Ehrhard Tasson Cah. Geom. Diff. (2010)

A model with Distributions

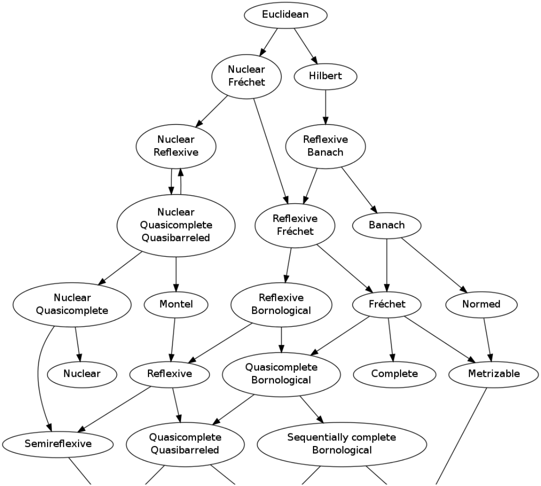
Topological vector spaces

We work with Hausdorff topological vector spaces : real or complex vector spaces endowed with a Hausdorff topology making addition and scalar multiplication continuous.

- The topology on E determines E'.
- The topology on E' determines whether $E \simeq E''$.

We work within the category $\textsc{TopVect}$ of topological vector spaces and continuous linear functions between them.

Topological models of DiLL



Euclidean

Nuclear Fréchet

Hilbert

Nuclear Reflexive

Reflexive Banach

Nuclear Quasicomplete Quasibarreled

Reflexive Fréchet

Banach

Nuclear Quasicomplete

Montel

Reflexive Bornological

Fréchet

Normed

Nuclear

Reflexive

Quasicomplete Bornological

Complete

Metrizable

Semireflexive

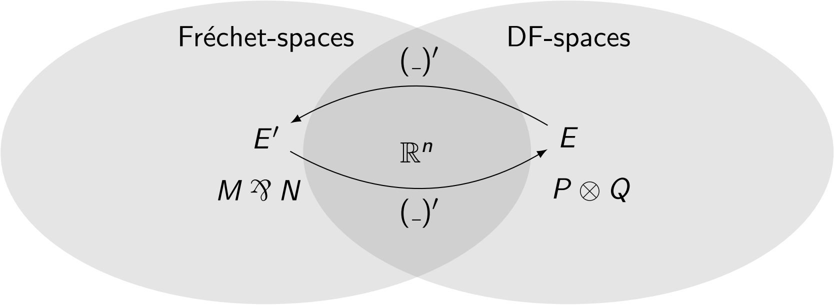
Quasicomplete Quasibarreled

Sequentially complete Bornological

Let us take the other way around, through Nuclear Fréchet spaces.

Fréchet and DF spaces

- Fréchet : metrizable complete spaces.
- (DF)-spaces : such that the dual of a Fréchet is (DF) and the dual of a (DF) is Fréchet.

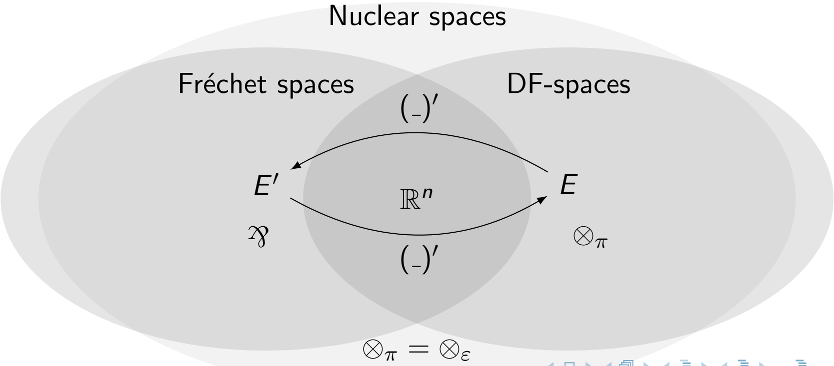


These spaces are in general not reflexive.

Nuclear spaces

Nuclear spaces are spaces in which one can identify the two canonical topologies on tensor products :

$$\forall F, E \otimes_\pi F = E \otimes_\varepsilon F$$



Nuclear spaces

Fréchet spaces

DF-spaces

$(\_)'$

E'

$\mathbb{R}^n$

E

$(\_)'$

$⅋$

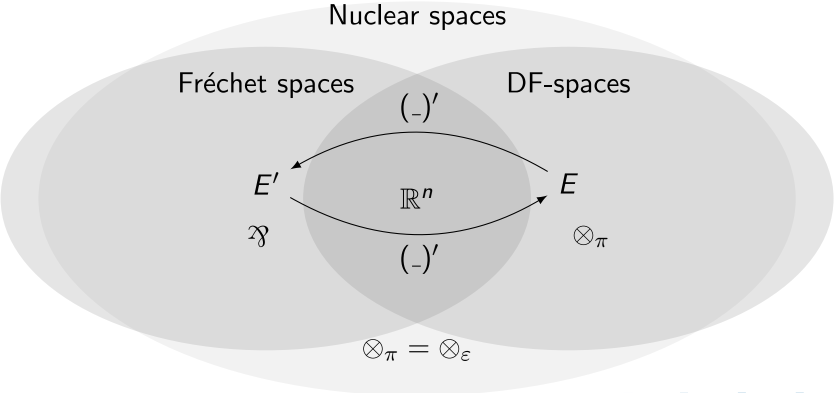
$\otimes_\pi$

$\otimes_\pi = \otimes_\varepsilon$

Nuclear spaces

A polarized $\star$-autonomous category

A Nuclear space which is also Fréchet or dual of a Fréchet is reflexive.



Nuclear spaces

Fréchet spaces

DF-spaces

$$\begin{tikzcd} E' \arrow[r, bend right, "(\_)'"'] & E \arrow[l, bend right, "(\_)'"'] \end{tikzcd}$$

$\mathbb{R}^n$

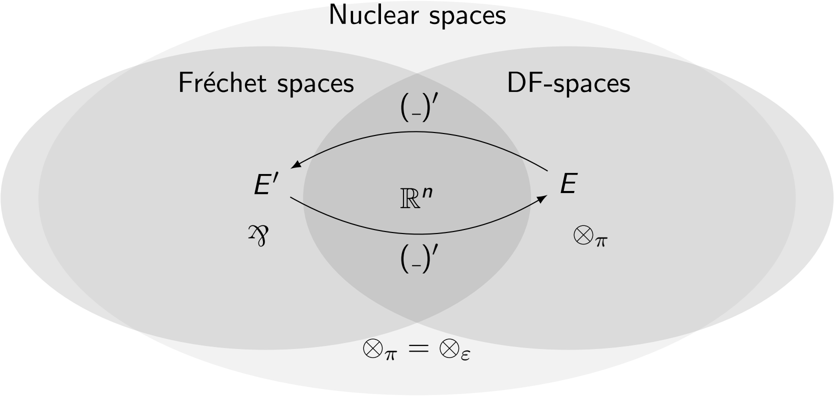
$⅋$

$\otimes_\pi$

$\otimes_\pi = \otimes_\varepsilon$

Nuclear spaces

We get a polarized model of MALL : involutive negation $(\_)^{\perp}$, $\otimes$, $\parr$, $\oplus$, $\times$.



Nuclear spaces

Fréchet spaces

DF-spaces

$(\_)'$

E'

$\mathbb{R}^n$

E

$\parr$

$\otimes_\pi$

$(\_)'$

$\otimes_\pi = \otimes_\varepsilon$

Distributions and the Kernel theorems

A typical Nuclear Fréchet space is the space of smooth functions on $\mathbb{R}^n$: $\mathcal{C}^\infty(\mathbb{R}^n, \mathbb{R})$.

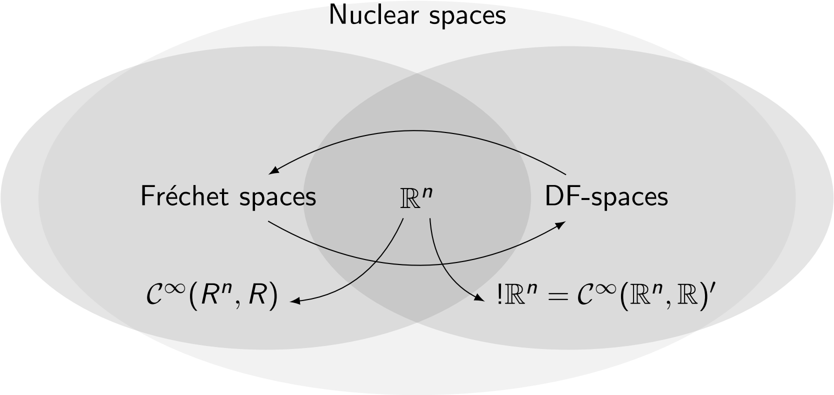
A typical Nuclear DF spaces is Schwartz' space of distributions with compact support : $\mathcal{C}^\infty(\mathbb{R}^n, \mathbb{R})'$.

The Kernel Theorems

$$\mathcal{C}^\infty(E, \mathbb{R})'\hat{\otimes}\mathcal{C}^\infty(F, \mathbb{R})' \simeq \mathcal{C}^\infty(E \times F, \mathbb{R})'$$

$$!\mathbb{R}^n = \mathcal{C}^\infty(\mathbb{R}^n, \mathbb{R})'.$$

A model of Smooth differential Linear Logic



Nuclear spaces

Fréchet spaces

$\mathbb{R}^n$

DF-spaces

$\mathcal{C}^\infty(R^n, R)$

$!\mathbb{R}^n = \mathcal{C}^\infty(\mathbb{R}^n, \mathbb{R})'$

A Smooth differential Linear Logic

A graded semantic

Finite dimensional vector spaces:

$$R^n, R^m := \mathbb{R} | R^n \otimes R^m | R^n \parr R^m | R^n \oplus R^m | R^n \times R^m.$$

Nuclear spaces :

$$U, V := R^n | !R^n | ?R^n | U \otimes V | U \parr V | U \oplus V | U \times V.$$

$$!\mathbb{R}^n = \mathcal{C}^\infty(\mathbb{R}^n, \mathbb{R})' \in \textsc{Nucl}$$

$$!\mathbb{R}^n \otimes !\mathbb{R}^m \simeq !(\mathbb{R}^{n+m})$$

We have obtained a smooth classical model of DiLL, to the price of Digging $!A \multimap !!A$.

Smooth DiLL, a failed exponential

A new graded syntax

Finitary formulas : $A, B := X | A \otimes B | A \parr B | A \oplus B | A \times B.$
General formulas : $U, V := A | !A | ?A | U \otimes V | U \parr V | U \oplus V | U \times V$

For the old rules

$$\frac{\vdash \Gamma}{\vdash \Gamma, ?A}\, w \qquad \frac{\vdash \Gamma, ?A, ?A}{\vdash \Gamma, ?A}\, c \qquad \frac{\vdash \Gamma, A}{\vdash \Gamma, ?A}\, d$$

$$\frac{\vdash}{\vdash !A}\, \bar{w} \qquad \frac{\vdash \Gamma, !A \qquad \vdash \Delta, !A}{\vdash \Gamma, \Delta, !A}\, \bar{c} \qquad \frac{\vdash \Gamma, A}{\vdash \Gamma, !A}\, \bar{d}$$

The categorical semantic of smooth DiLL is the one of LL, but where ! is a monoidal functor and d and $\bar{d}$ are to be defined independently.

Linear Partial Differential Equations as Exponentials

Linear functions as solutions to an equation

$$
\begin{aligned}
f \in \mathcal{C}^\infty(\mathbb{R}^n,\mathbb{R}) \text{ is linear} \quad & \text{iff } \forall x, f(x) = D(f)(0)(x) \\
& \text{iff } f = \bar{d}(f) \\
& \text{iff } \exists g \in \mathcal{C}^\infty(\mathbb{R}^n,\mathbb{R}), f = \bar{d}g
\end{aligned}
$$

Another definition for $\bar{d}$

A linear partial differential operator D acts on $\mathcal{C}^\infty(\mathbb{R}^n, R)$:

$$D(f)(x) = \sum_{|\alpha| \leq n} a_\alpha(x) \frac{\partial^\alpha f}{\partial x^\alpha}.$$

Another exponential is possible

$$!_D A = (D(\mathcal{C}^\infty(A,\mathbb{R})))'$$

that is the space of linear functions acting on functions $f = Dg$, for $g \in \mathcal{C}^\infty(A,\mathbb{R})$, when $A \subset \mathbb{R}^n$ for some n.

$$\bar{d}_D :!_D A \rightarrow !A; \phi \mapsto (f \mapsto \phi(D(f)))$$

$$d_D :!A \rightarrow !_D A; \phi \mapsto \phi_{|D(\mathcal{C}^\infty(A)}$$

Functions	E'	$D(\mathcal{C}^\infty(A))$	$\mathcal{C}^\infty(A)$
$!$	$E'' \simeq E$	$!_D A = D(\mathcal{C}^\infty(A))'$	$!A = \mathcal{C}^\infty(A)'$
d	$\phi \mapsto \phi_{\vert(A)'}$	$\phi \mapsto \phi_{\vert D(\mathcal{C}^\infty(A))}$	
$\bar{d}$	$x \mapsto (f \mapsto d(f)(0)(x))$	$\phi \mapsto (f \mapsto \phi(D(f)))$	

Recall : The structural morphisms on $!E$

- The codereliction $\bar{d}_E : E \rightarrow !E = \mathcal{C}^\infty(E, \mathbb{R})'$ encodes the differential operator.
- In a $\star$-autonomous category $d_E : E \rightarrow ?E$ encode the fact that linear functions are smooth.

- $c : !E \rightarrow !E \otimes !E \rightarrow$ is deduced from the Seely isomorphism and maps $ev_x \otimes ev_x$ to ev_x.
- $\bar{c} !E \otimes !E \rightarrow !E$ is the convolution $\star$ between two distributions
- $w : !E \rightarrow \mathbb{R}$ maps ev_x to 1.
- $\bar{w} : \mathbb{R} \rightarrow !E$ maps 1 to $ev_0 : f \mapsto f(0)$, the neutral for $\star$.

Another exponential $!_D$

Consider D a LPDO with constant coefficients :

$$D = \sum_{\alpha, |\alpha| \leq n} a_\alpha \frac{\partial^\alpha}{\partial x^\alpha}.$$

Existence of a fundamental solution

For such D there is $E_0 \in \mathcal{C}^\infty(A)'$ such that $DE_0 = ev_0$.

D commutes with convolution

If $f \in D(\mathcal{C}^\infty(A))$ and $g \in \mathcal{C}^\infty(A)$, then $f * g \in D(\mathcal{C}^\infty(A))$.

The coalgebra structure

$D(E_0) * f = f$

Functions	E'	$D(\mathcal{C}^\infty(A))$	$\mathcal{C}^\infty(A)$
$!$	$E'' \simeq E$	$!_D A = D(\mathcal{C}^\infty(A))'$	$!A = \mathcal{C}^\infty(A)'$
d	$\phi \mapsto \phi_{\|(A)'}$	$\phi \mapsto \phi_{\|D(\mathcal{C}^\infty(A))}$	
$\bar{d}$	$x \mapsto$ $(f \mapsto d(f)(0)(x))$	$\phi \mapsto$ $(f \mapsto \phi(D(f)))$	
$?A^\perp$	E'	$D(\mathcal{C}^\infty(A, \mathbb{R}))$	$\mathcal{C}^\infty(A, \mathbb{R})$
$!A$	$E'' \simeq E$	$D(\mathcal{C}^\infty(A, \mathbb{R}))'$	$\mathcal{C}^\infty(A, \mathbb{R})'$
$\bar{c}$		$* :!A \otimes !_D A \to !_D A$	$* :!A \otimes !A \to !A$
$\bar{w}$		$1 \mapsto E_0$	$1 \mapsto ev_0$

and a co-algebra structure : $c :!_D A \to !A \otimes !_D A$ and $w :!A \to \mathbb{R}$

Solving Linear PDE's with constant coefficient

$\bar{w}$ is the fundamental solution

E_0 is the fundamental solution, such that $DE_0 = ev_0$. Its existence is guaranteed when D has constant coefficients.

Solving Linear PDE through $\bar{w}$ and $\bar{c}$

If $f \in \mathcal{C}^\infty(A)$, then $D(E_0 * f) = f$.

Solving Linear PDE's with constant coefficient

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E_0 is the fundamental solution, such that $DE_0 = ev_0$. Its existence is guaranteed when D has constant coefficients.

Solving Linear PDE through $\bar{w}$ and $\bar{c}$

If $f \in \mathcal{C}^\infty(A)$, then $D(E_0 * f) = f$.
If $f \in E'$, then $d(ev_0 * f) = f$.

The rules of Differential Linear Logic encode the resolution of a Linear Partial Differential Equation

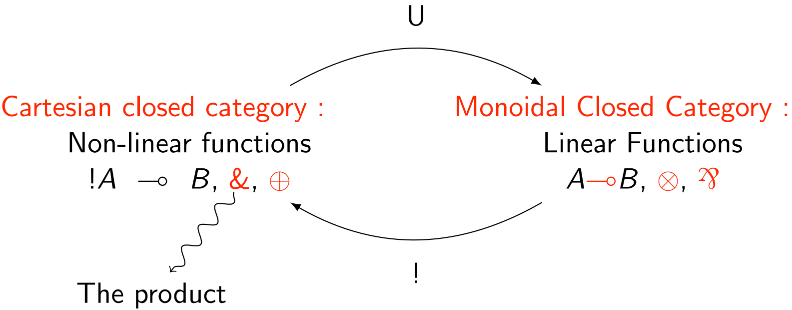
Models based on ε

Joint work with Y. Dabrowski

A typical lesson on the semantic of LL

$$\begin{tikzcd}
\begin{array}{c}\text{Cartesian closed category :}\\ \text{Non-linear functions}\\ !A \multimap B, \&, \oplus\end{array} \arrow[r, bend left, "U"] & \begin{array}{c}\text{Monoidal Closed Category :}\\ \text{Linear Functions}\\ A \multimap B, \otimes, \parr\end{array} \arrow[l, bend left, "!"]
\end{tikzcd}$$

A typical lesson on the semantic of LL



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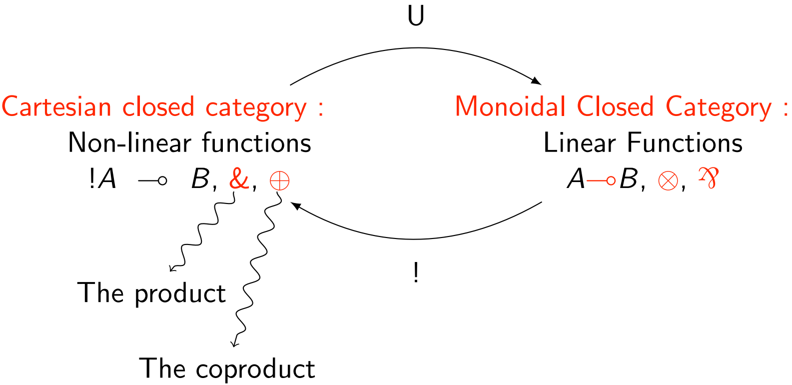
!

The product

A typical lesson on the semantic of LL



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The coproduct

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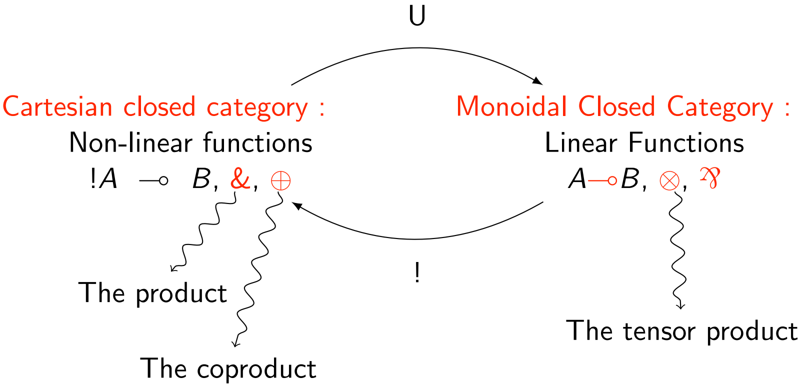

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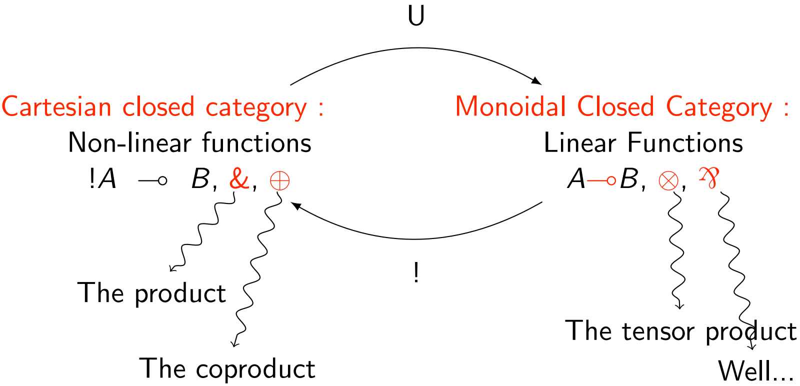
The product

The coproduct

The tensor product

A typical lesson on the semantic of LL



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Cartesian closed category :

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Monoidal Closed Category :

Linear Functions

$A \multimap B$, $\otimes$, ⅋

!

The product

The coproduct

The tensor product

Well...

Have you heard about the ⅋?

Many topological tensor product

$\otimes_\pi$, $\otimes_i$, $\otimes_\varepsilon$, $\otimes_\gamma \dots$

Grothendieck problème des topologies

Some tensor products may form a monoidal closed category on some specific spaces.

Only one good ⅋

$E \varepsilon F := \mathcal{L}_\varepsilon(E'_c, F)$, where E'_c is E' with the topology compact-open, and the whole space is endowed with the topology of uniform convergence on equicontinuous sets of E'_c.

The ε product and tensor

Only one good $\parr$

$E\varepsilon F := \mathcal{L}_{\varepsilon}(E'_c, F)$, where E'_c is E' with the topology compact-open, and the whole space is endowed with the topology of uniform convergence on equicontinuous sets of E'_c.

$\mathcal{C}^{\infty}(E, F) \simeq \mathcal{C}^{\infty}(E, \mathbb{R})\varepsilon F$ when E and F are complete.

A monoidal category by Schwartz

ε is associative and commutative on quasi-complete spaces.

Duality as an orthogonality

The topology on E' determines whether $E \simeq E''$

The topological linear duality is in general not an orthogonality :

$$E'_\beta \neq ((E''_\beta)'_\beta)_\beta$$

However, when choosing on E' the topology compact open, one always has :

$$E'_c \simeq ((E'_c)'_c)'_c$$

This allows for the construction of a $\star$-autonomous category.

A *-autonomous category with ε

Completing E'_c does not lead to an orthogonality : one need to find a completion condition strong enough for ε to be associative but weak enough to have a good linear duality.

k-refl

We have a smooth model of MALL where spaces E are k-complete and $E^\perp = \widehat{E'_c}^k$.

A smooth model of LL with ϵ

We adapt the notion of smooth function to $\mathcal{C}^\infty_{co}$ in order to have an exponential and a model of LL.

Towards a general construction for smooth models of LL

Consider $\mathcal{C}$ a small cartesian category contained in k-ref.

Smooth functions with parameters

$\mathcal{C}^\infty_{\mathcal{C}}(E, F) := \{f : E \to F, \forall X \in \mathcal{C} \forall c \in \mathcal{C}^\infty_{co}(X, E), f \circ c \in \mathcal{C}^\infty_{co}(X, F)\}$

A new induced topology

For any tvs E there is an injection $E \hookrightarrow \mathcal{C}^\infty_{\mathcal{C}}(E'_\mu, \mathbb{R})$ which induces a new topology $\mathscr{S}_{\mathcal{C}}(E)$ on E.

Then when E is Mackey-complete :

$\mathcal{C}$	$\mathscr{S}_{\mathcal{C}}(E)$
Fin	The Schwartzification of E
Ban	The Nuclearification of E
$\{0\}$	The weak topology on E

Towards a general construction for smooth models of LL

A new induced topology

For any tvs E there is an injection $E \hookrightarrow \mathcal{C}^{\infty}_{\mathcal{C}}(E'_{\mu}, \mathbb{R})$ which induces a new topology $\mathscr{S}_{\mathcal{C}}(E)$ on E.

Then when E is Mackey-complete :

$\mathcal{C}$	$\mathscr{S}_{\mathcal{C}}(E)$
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Then when E is Mackey-complete :

$\mathcal{C}$	$\mathscr{S}_{\mathcal{C}}(E)$	
Fin	The Schwartzification of E	$+$ Mco $\Rightarrow$ LL $+ \mathcal{C}^\infty + \varepsilon$
Ban	The Nuclearification of E	SDiLL ($+$ LL ?)
$\{0\}$	The weak topology on E	LL ($+ \mathcal{C}^\infty$?)

Conjecture

Any $\mathscr{S}_{\mathcal{C}}$ gives us a model of LL.

Conclusion

What we have :

- Several smooth models of Classical Linear Logic
- An interpretation of the exponential in terms of distributions.
- The first steps towards for a generalization of DiLL to linear *PDE*'s.
- The first steps for a general understanding of smooth models of linear logic.

What we could get :

- A constructive Type Theory for differential equations.
- Logical interpretations of fundamental solutions, specific spaces of distributions, Fourier transformations or operation on distributions.
- A categorical framework for understanding smooth models of linear logic.

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Thank you .

I welcome questions, comments, or remarks later or at `kerjean@irif.fr`.