Computing in Analysis Through Logic

by

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INTRODUCTION

Theoretical Computer Science consists in the study of computation, their design, and their models.

‣Results in **new computational concepts**.

‣Computer science is by tradition attached to **discrete mathematics** while used to model **continuous phenomena**.

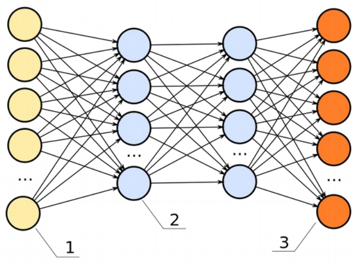

Figure : forward differentiation in a neural network

Differentiation is pervasive in computer science, through Numerical Analysis, Computer Algebra, or through Deep Learning.

Through Type Theory, **Logic verifies** that **programs** are by construction **correct** : they can be executed safely.


Figure : TT by D. Hirschkoff

SEMANTICS AND TYPE THEORY

The Curry-Howard Correspondence (**proofs-as-programs**): *from logic to programming.*

Categorical Semantics (**proofs-as-functions**): *from mathematics to logic.*

Differential Linear Logic [1]: gives logical rules for differentiation, and has a a semantics in Distribution Theory **[3]**.


Picture by Hannah

Through this correspondence, Type Theory is also a **language for writing verified mathematics.**

‣Proof assistant as **Coq** led to highly successful algebra formalizations in group theory **[2]**.

‣These methods may now be extended to analysis, through the Mathematical Components Analysis project.

LINES OF RESEARCH

Logic acts as a bridge between Computer Science and Mathematics : it allows to extract computational concepts from preexisting mathematical theories and to encode mathematics in a programming language. My work focuses on Analysis in Dependant Type Theory and Linear Logic.

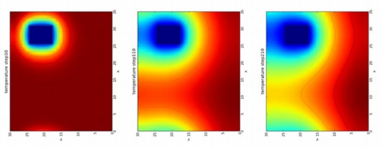
Figure : The heat equation

Methods in **numerical analysis as computational paradigms** .

A library of **formalized mathematics**, useful for verification (e.g for robotics).

```
Lemma continuousR_atP x (f : V → R) :
  (continuousR_at x f) ↔
  ∀ eps : posreal, \∀ y \near f @ x, ball (f x) eps%:num y.
Proof.
  rewrite /continuousR_at. split ; by move ⇒ /flim_ballPpos.
Qed.

Lemma continuousR_bounded0 (f : {scalar V}) :
  (continuousR_at 0 f) →
  ( ∃ r , (r > 0 ) ∧ (∀ x : V, (`|f x| ) ≤ (`|[x]| ) * r)) .
```

Figure : A sample of code in ssreflect

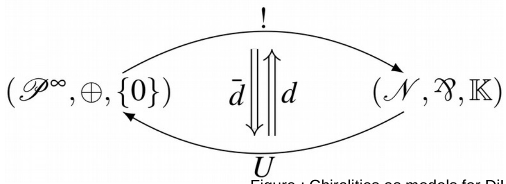

Figure : Chiralities as models for DiLL

Towards a typed differential programming language.

CONCLUSION

The search for adequate models of computations has an influence over the way we design our computation.

Mathematical structures are an inspiration for computer science, and **computer science in turn reveals new worthy mathematical structures.**

The theoretical study of computations should use the mathematical objects which are being modeled.

REFERENCES

[1] Thomas Ehrhard & Laurent Regnier, Differential Interaction Nets, Theoretical Computer Science, 2006.
[2] A Machine-Checked Proof of the Odd Order Theorem, Gonthier & al., Interactive Theorem Proving, 2013.
[3] Marie Kerjean, A Logical Account for Linear Partial Differential Equations, Logic in Computer Science, 2018.