Linear Logic and Optimal Reductions
MPRI. Linear logic and logical paradigms of computation

— Sample exercices for the exam —

Stefano Guerrini

06/02/2010

1 Lévy's otpimal reductions

Given the λ-term

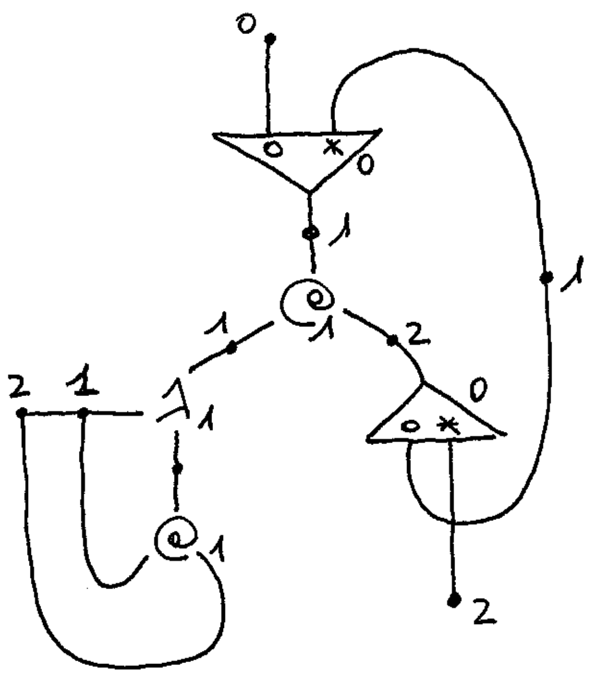
$$T = (\lambda x.\Delta(xIy))I$$

where $\Delta = \lambda x.xx$ and $I = \lambda x.x$.

1. Draw the reduction graph of the term T.
2. Given the redexes Ix in the reducts $M_1 = Ix(IIx)$ and $M_2 = IIx(Ix)$ of T, prove by applying the zig-zag relation that they are in the same family.
3. Find all the Lévy's families of the term T.
4. Give the canonical representative of each family.
5. Give the histories of the redexes Ix in M_1 and M_2 and, by applying the extraction reduction, find their canonical representative.

2 Sharing graphs

1. Using the optimal rules for sharing graphs reduce the following graph G to its normal form.



2. The following graph is the λ-tree T corresponding to the unfolding U of G, that is

   (a) all the muxes in the unshared graph U are lifts;

   (b) T is obtained from U by removing its lifts;

   (c) there is a sharing morphism (a graph morphism) that maps each link/node of U to a link/node of G and such that

      i. every link is mapped to a link of the same type;

      ii. the map preserves the levels of nodes and links;

      iii. each port of an @ or of λ-link is mapped to a port of the same type;

      iv. each lift with a port of name a is mapped to a port of a mux with the same name;

      v. every node u connected to a port of a link l is mapped to a port of the same type of the image of l.

   Reconstruct the unshared graph U.