Solving Strategic and Tactical Optimization Problems in City Logistics

Optimisation Stratégique et Tactique en Logistique Urbaine

2021

Plan

Multicommodity-Ring Location Routing Problem (MRLRP)

Branch & Cut for the VRP with Intermediate Replenishment Facilities (VRPIRF)

Branch & Price for the VRP with Intermediate Replenishment Facilities

Plan

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The Multicommodity-Ring Location Routing Problem

An instance: scenario and demands

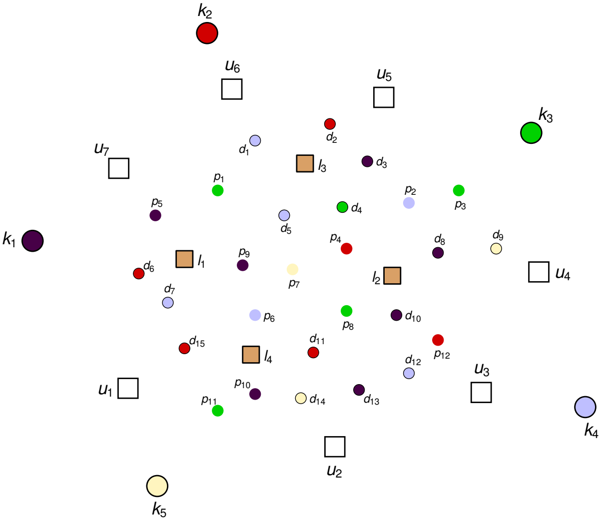

Figure 1: A MRLRP instance: gates, sites, delivery and pickup demands, SPLs

The Multicommodity-Ring Location Routing Problem

An instance: solution

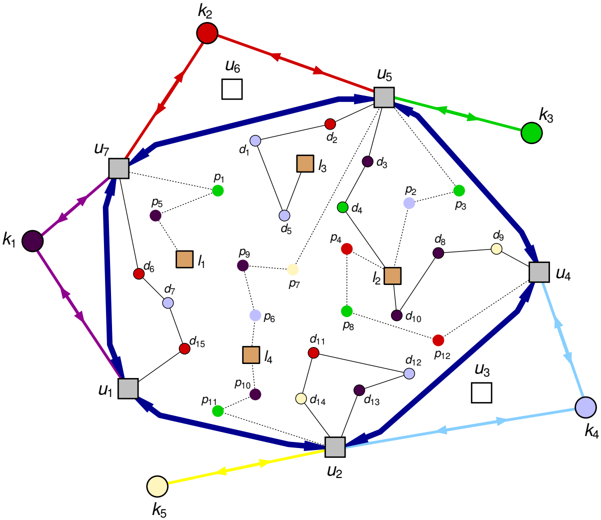

Figure 2: A solution to the previous instance

The Multicommodity-Ring Location Routing Problem

Problem definition

Gates

Sources/destinations of delivery/pick-up goods

The Multicommodity-Ring Location Routing Problem

Problem definition

Gates

Sources/destinations of delivery/pick-up goods

Ring

- UDCs with fixed installation cost and capacity
- ring arcs with
  - fixed installation cost and capacity
  - per-load-unit transportation cost

The Multicommodity-Ring Location Routing Problem

Problem definition

Gates

Sources/destinations of delivery/pick-up goods

Ring

- UDCs with fixed installation cost and capacity
- ring arcs with
  - fixed installation cost and capacity
  - per-load-unit transportation cost

Routing

- vehicles with maximum load and trip length
- separate service of pickup and delivery demands
- service routes may be open

  $\Rightarrow$ SPLs are additional ending points $\Rightarrow$ same fleet for UDCs and SPLs

  $\Rightarrow$ fleet rebalancing constraints on SPLs and UDCs

The Multicommodity-Ring Location Routing Problem

Problem definition

Strategic analysis

- Time-independent scenario
- Goods type is not considered

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Decisions

- install UDCs and ring
- gates-UDCs, ring flows
- demands assignment to UDCs

The Multicommodity-Ring Location Routing Problem

Problem definition

Strategic analysis

- Time-independent scenario
- Goods type is not considered

Decisions

- install UDCs and ring
- gates-UDCs, ring flows
- demands assignment to UDCs

Objective function

Minimize the sum of:

- installation costs
- flow transportation costs
- routing costs

MILP model for MRLRP

Decision variables

1. ring variables $y_u \in \{0, 1\}$, $z_{uv} \in \{0, 1\}$
2. service variables $\chi_{ku} \in \{0, 1\}$
3. second-level routing variables $x_r \in \{0, 1\}$
4. first-level flow variables
   - $\varphi_{ku} \geq 0$, $\varphi_{uk} \geq 0$ (gates-UDCs flows)
   - $\varphi_{uv}^{dk} \geq 0$, $\varphi_{uv}^{pk} \geq 0$ (ring flows)
   - $\phi_{ku} \geq 0$, $\phi_{uk} \geq 0$ (UDC capacity upper bound)

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4. first-level flow variables $\varphi_{ku} \geq 0$, $\varphi_{uk} \geq 0$, $\varphi_{uv}^{dk} \geq 0$, $\varphi_{uv}^{pk} \geq 0$, $\phi_{ku} \geq 0$, $\phi_{uk} \geq 0$

Objective function

$$\min \sum_{u \in U} F_u y_u + \sum_{\substack{u \in U \\ k \in K}} (c_{ku}\varphi_{ku} + c_{uk}\varphi_{uk}) + \sum_{uv \in A_U} g_{uv} z_{uv} + \sum_{\substack{uv \in A_U \\ k \in K}} c_{uv}(\varphi_{uv}^{pk} + \varphi_{uv}^{dk}) + \sum_{r \in \mathscr{R}} c(r) x_r$$

Cost terms:

- UDC installation
- UDC-gates flows
- ring installation
- ring flows
- Second level routing

MILP model for MRLRP

Objective function

$$\min \sum_{u \in U} F_u y_u + \sum_{\substack{u \in U\\ k \in K}} (c_{ku}\varphi_{ku} + c_{uk}\varphi_{uk}) + \sum_{uv \in A_U} g_{uv} z_{uv} + \sum_{\substack{uv \in A_U\\ k \in K}} c_{uv}(\varphi^{pk}_{uv} + \varphi^{dk}_{uv}) + \sum_{r \in \mathscr{R}} c(r) x_r$$

Constraints

s.t.

$$\sum_{u \in U} \varphi_{ku} = \sum_{i \in D_k} q_i \qquad \forall k \in K \qquad \text{gates-UDC flows}$$

$$\sum_{u \in U} \varphi_{uk} = \sum_{i \in P_k} q_i \qquad \forall k \in K$$

$$\varphi_{uk} + \varphi_{ku} \leq \chi_{ku} \sum_{i \in P_k \cup D_k} q_i \qquad \forall k \in K, u \in U$$

$$\chi_{ku} \leq y_u \qquad \forall k \in K, u \in U$$

$$\sum_{u \in U} \chi_{ku} \leq B \qquad \forall k \in K$$

$$\varphi_{ku} + \sum_{v \in U \setminus u} \varphi^{dk}_{vu} = \sum_{v \in U \setminus u} \varphi^{dk}_{uv} + \sum_{r \in \mathscr{R}^{+d}_u} q_k(r) x_r \qquad \forall k \in K, u \in U \qquad \text{UDC flow conservation}$$

$$\sum_{r \in \mathscr{R}^{-p}_u} q_k(r) x_r + \sum_{v \in U \setminus u} \varphi^{pk}_{vu} = \sum_{v \in U \setminus u} \varphi^{pk}_{uv} + \varphi_{uk} \qquad \forall k \in K, u \in U$$

MILP model for MRLRP

Objective function

$$\min \sum_{u\in U} F_u y_u + \sum_{\substack{u\in U\\k\in K}} (c_{ku}\varphi_{ku} + c_{uk}\varphi_{uk}) + \sum_{uv\in A_U} g_{uv} z_{uv} + \sum_{\substack{uv\in A_U\\k\in K}} c_{uv}(\varphi^{pk}_{uv} + \varphi^{dk}_{uv}) + \sum_{r\in\mathscr{R}} c(r) x_r$$

Constraints

s.t. $\sum_{r\in\mathscr{R}_i} x_r = 1$	$\forall i \in P \cup D$	demands service
$x_r \le y_u \le \sum_{r\in\mathscr{R}^{+d}_u} x_r + \sum_{r\in\mathscr{R}^{-p}_u} x_r$	$\forall r \in \mathscr{R}^+_u \cup \mathscr{R}^-_u, u \in U$	logical constraints
$-\delta^-_h \le \sum_{r\in\mathscr{R}^-_h} x_r - \sum_{r\in\mathscr{R}^+_h} x_r \le \delta^+_h$	$\forall h \in U \cup L$	fleet rebalancing
$\sum_{\substack{v\in U\\u<v}} z_{uv} + \sum_{\substack{v\in U\\v<u}} z_{vu} = 2y_u$	$\forall u \in U$	SEC
$\sum_{\substack{u\in S\\v\notin S\\u<v}} z_{uv} + \sum_{\substack{u\notin S\\v\in S\\u<v}} z_{uv} \ge 2(y_w + y_{w'} - 1)$	$\forall S\in\mathcal{S}_U, w\in S, w'\in U\setminus S$	

MILP model for MRLRP

Objective function

$$\min \sum_{u \in U} F_u y_u + \sum_{\substack{u \in U \\ k \in K}} (c_{ku}\varphi_{ku} + c_{uk}\varphi_{uk}) + \sum_{uv \in A_U} g_{uv} z_{uv} + \sum_{\substack{uv \in A_U \\ k \in K}} c_{uv}(\varphi_{uv}^{pk} + \varphi_{uv}^{dk}) + \sum_{r \in \mathscr{R}} c(r) x_r$$

Constraints

s.t. $\sum_{k \in K} (\varphi_{uv}^{dk} + \varphi_{uv}^{pk}) \leq q_{uv}\, z_{uv}$	$\forall (u,v) \in A'_U$	ring arcs capacity
$\sum_{k \in K} (\varphi_{vu}^{dk} + \varphi_{vu}^{pk}) \leq q_{vu}\, z_{uv}$	$\forall (u,v) \in A'_U$	
$\sum_{k \in K} (\phi_{ku} + \varphi_{ku} + \phi_{uk} + \varphi_{uk}) \leq Q_u y_u$	$\forall u \in U$	UDC capacity
$\phi_{ku} \geq \sum_{r \in \mathscr{R}_u^{+d}} q_k(r) x_r - \varphi_{ku}$	$\forall k \in K, u \in U$	
$\phi_{uk} \geq \sum_{r \in \mathscr{R}_u^{-p}} q_k(r) x_r - \varphi_{uk}$	$\forall k \in K, u \in U$	
$\sum_{u \in U} y_u \leq N$		budget constraint

$$y_u, z_{uv}, \chi_{ku}, x_r \in \{0,1\},\ \varphi_{uv}^{pk}, \varphi_{uv}^{dk}, \varphi_{ku}, \varphi_{uk}, \phi_{uk}, \phi_{ku} \geq 0$$

The GALW Matheuristic

A Four-stage Decomposition Heuristic

1–Route Generator

Generation of a set of delivery/pick-up service routes

- construction + local search

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2–Assignment Subproblem

Solution of a MILP assignment subproblem to:

- choose a subset of UDC
- assign demands to UDCs at both first and second level
- constraints: fleet balance, UDCs capacity

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3–Ring Construction

The chosen UDCs are connected by solving a Symmetric TSP

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4–Ring MultiFlow Subproblem

Solution of a LP ring multiflow with capacities and demands problem

- routing of indirect shipments

GALW: a Matheuristic Approach

Stage 1: Route Generator

Generation of a route *m*

- first demand: random choice of one with a less-than-average # of visits
- nearest neighbor step considering both head and tail insertion:

$$d' = \arg\min_{d \in D \setminus m} p_Q \frac{q(m) + q(d)}{Q} + p_C \frac{c(m) + c(d)}{C} + p_\omega \frac{\omega(d)}{\gamma}$$

- 2-opt local search
- generation of *m* ends when no more demands can be added

GALW: a Matheuristic Approach

Stage 1: Route Generator

Generation of a route *m*

- first demand: random choice of one with a less-than-average # of visits
- nearest neighbor step considering both head and tail insertion:

$$d' = \arg\min_{d \in D \setminus m} p_Q \frac{q(m)+q(d)}{Q} + p_C \frac{c(m)+c(d)}{C} + p_\omega \frac{\omega(d)}{\gamma}$$

- 2-opt local search
- generation of *m* ends when no more demands can be added

How to obtain a *good* route set $\overline{\mathscr{R}}$

- many route subsets $\overline{\mathscr{R}}^t$ with different maximal length
- each demand is served by at least ω feasible sequences of demands per route subset
- endpoints combination

GALW: a Matheuristic Approach

Stage 2: Assignment Subproblem

Decision variables

1. UDC variables $y_u \in \{0,1\}$
2. service variables $\chi_{ku} \in \{0,1\}$
3. second-level routing variables $x_r \in \{0,1\}$
4. first-level flow variables
   - $\varphi_{ku} \geq 0$, $\varphi_{uk} \geq 0$ (gates-UDCs flows)
   - $\phi_{ku} \geq 0$, $\phi_{uk} \geq 0$ (UDC capacity upper bound)

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Objective function

$$\min \sum_{u \in U} F_u y_u + \sum_{\substack{u \in U \\ k \in K}} (c_{ku}\varphi_{ku} + c_{uk}\varphi_{uk}) + \sum_{r \in \mathscr{R}} c(r) x_r + \sum_{\substack{u \in U \\ o=3\ldots N}} F_{u,o}\, \xi_o^u + \sum_{\substack{u \in U \\ k \in K}} \overline{c_u}\, f_\phi(\phi_{ku}, \phi_{uk})$$

Cost terms:

- UDC installation
- UDC-gates flows
- Second level routing
- Ring construction costs estimate
- Ring flows costs estimate

GALW: a Matheuristic Approach

Stage 2: Assignment Subproblem

Objective function

$$\min \sum_{u \in U} F_u y_u + \sum_{\substack{u \in U \\ k \in K}} (c_{ku}\varphi_{ku} + c_{uk}\varphi_{uk}) + \sum_{r \in \mathscr{R}} c(r) x_r + \sum_{\substack{u \in U \\ o=3\dots N}} F_{u,o}\, \xi_o^u + \sum_{\substack{u \in U \\ k \in K}} \overline{c_u}\, f_\phi(\phi_{ku}, \phi_{uk})$$

Constraints

s.t. $\sum_{u \in U} \varphi_{ku} = \sum_{i \in D_k} q_i$	$\forall k \in K$	gates-UDC flows
$\sum_{u \in U} \varphi_{uk} = \sum_{i \in P_k} q_i$	$\forall k \in K$	
$\varphi_{uk} + \varphi_{ku} \leq \chi_{ku} \sum_{i \in P_k \cup D_k} q_i$	$\forall k \in K, u \in U$	
$\chi_{ku} \leq y_u$	$\forall k \in K, u \in U$	
$\sum_{u \in U} \chi_{ku} \leq B$	$\forall k \in K$	
$\sum_{r \in \mathscr{R}_i} x_r = 1$	$\forall i \in P \cup D$	demands service
$x_r \leq y_u \leq \sum_{r \in \mathscr{R}_u^{+d}} x_r + \sum_{r \in \mathscr{R}_u^{-p}} x_r$	$\forall r \in \mathscr{R}_u^+ \cup \mathscr{R}_u^-, u \in U$	logical constraints
$\sum_{u \in U} y_u \leq N$		budget constraint

GALW: a Matheuristic Approach

Stage 2: Assignment Subproblem

Objective function

$$\min \sum_{u\in U} F_u y_u + \sum_{\substack{u\in U\\k\in K}} (c_{ku}\varphi_{ku} + c_{uk}\varphi_{uk}) + \sum_{r\in\mathscr{R}} c(r)x_r + \sum_{\substack{u\in U\\o=3\dots N}} F_{u,o}\,\xi_o^u + \sum_{\substack{u\in U\\k\in K}} \overline{c_u}\, f_\phi(\phi_{ku}, \phi_{uk})$$

Constraints

s.t. $-\delta_h^- \leq \sum_{r\in\mathscr{R}_h^-} x_r - \sum_{r\in\mathscr{R}_h^+} x_r \leq \delta_h^+$	$\forall h \in U \cup L$	fleet rebalancing
$\sum_{k\in K} (\phi_{ku} + \varphi_{ku} + \phi_{uk} + \varphi_{uk}) \leq Q_u y_u$	$\forall u \in U$	UDC capacity
$\phi_{ku} \geq \sum_{r\in\mathscr{R}_u^{+d}} q_k(r)x_r - \varphi_{ku}$	$\forall k \in K, u \in U$	
$\phi_{uk} \geq \sum_{r\in\mathscr{R}_u^{-p}} q_k(r)x_r - \varphi_{uk}$	$\forall k \in K, u \in U$	
$\sum_{k\in K} f_\phi(\phi_{ku}, \phi_{uk}) \leq \overline{q_u} y_u$	$\forall u \in U$	outgoing flows bound
$\xi_o^u = f_\xi(y_u : u \in U)$	$\forall u \in U, o \in \{3,\dots,N\}$	ring estimate

$y_u, z_{uv}, \chi_{ku}, x_r \in \{0,1\}, \varphi_{ku}, \varphi_{uk}, \phi_{uk}, \phi_{ku} \geq 0$

GALW: a Matheuristic Approach

Stage 2: Assignment Subproblem

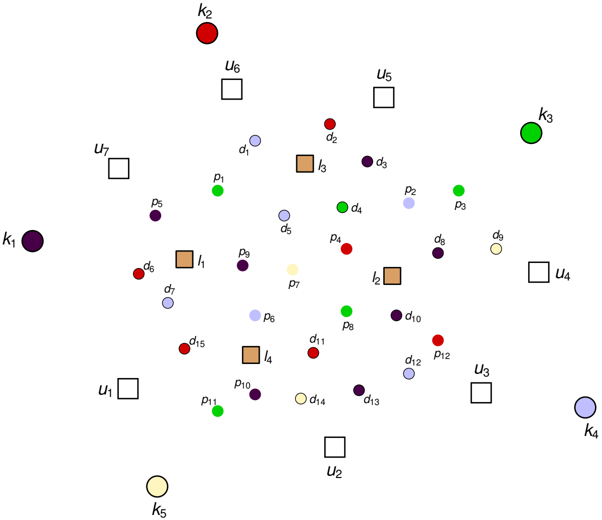

Figure 3: Solution of GALW stage2.

GALW: a Matheuristic Approach

Stage 2: Assignment Subproblem

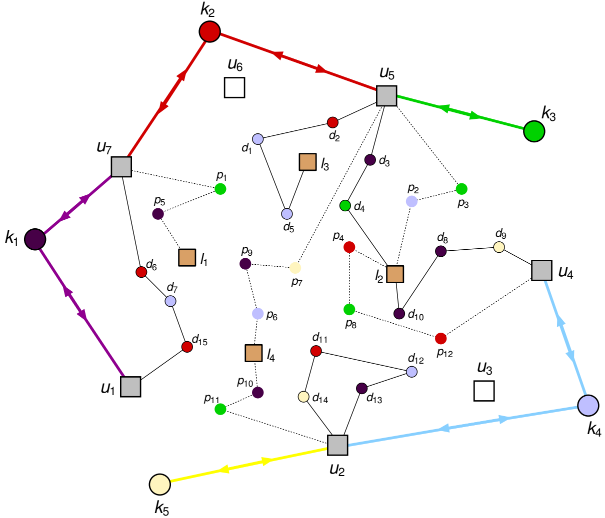

Figure 3: Solution of GALW stage2.

GALW: a Matheuristic Approach

Stage 3: Ring Construction

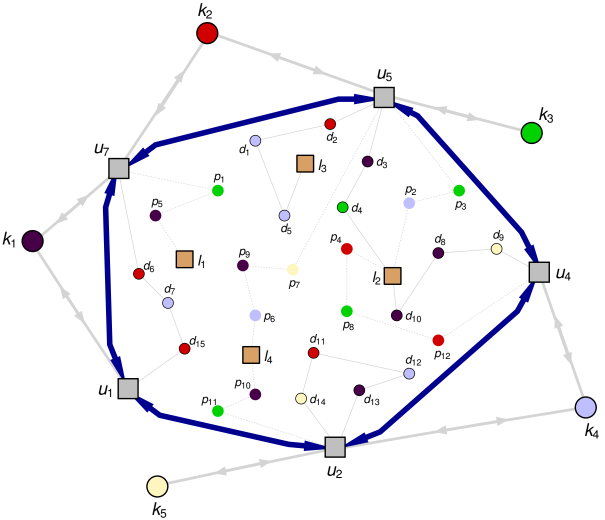

Figure 4: Solution of GALW stage3.

GALW: a Matheuristic Approach

Stage 4: Ring Multiflow Subproblem

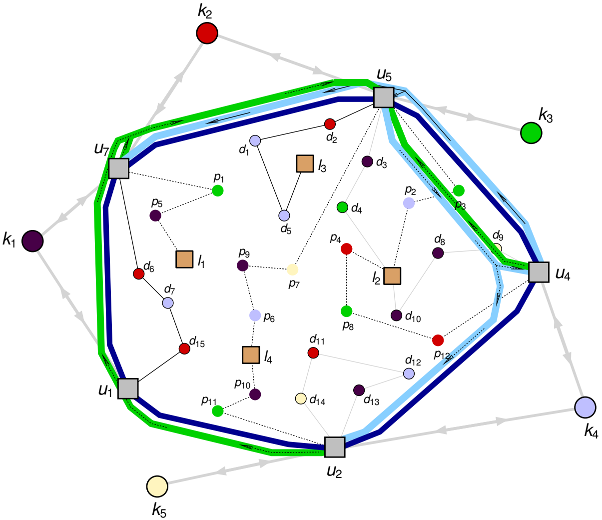

Figure 5: Solution of GALW stage4.

Computational Results

MRLRP instances

Derived from benchmark CLRP instances taken from [5] with MRLRP additional features

[5] C.Prins, C.Prodhon, R.Wolfler Calvo, *Solving the capacitated location-routing problem by a GRASP complemented by a learning process and a path relinking.* 4OR, 4(3):221–238, 2006.

Computational Results

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Computational evaluation

- aim: to establish how:
  - the hardness of an instance is affected its dimensional features
  - the decomposition process affects GALW performance
- collections of MRLRP instances, subdivided in scenarios:
  - different scenarios $\Rightarrow$ different $|U|$ or $|K|$ or $|L|$ or $|D| = |P|$
  - in a scenario: different ring costs

Computational Results

MRLRP instances

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  - in a scenario: different ring costs

Methods

Three methods are evaluated and compared:

- Exact method (X): Branch & Bound on MILP model with complete route set
- GALW (H)
- Hybrid method (Y): Branch & Bound on MILP model with GALW's route set

Computational Results

Small-sized instances

Instance	Features							Exact(X)			GALW				Hybrid(Y)	
	$\lvert K\rvert$	$\lvert U\rvert$	$\lvert L\rvert$	$\lvert D\rvert$	N	q	M	$\%^r$	$T/\%$	#ps	$\%^X$	t	#ps	$\%^Y$	$\%^X$	$T/\%$
galwc01-0-L|L	5	5	5	15	5	70	50	3.5	24.8	21503	2.1	3.6	3122	0.0	2.1	3.6
galwc01-1-L|L	5	5	5	20	5	70	50	3.3	48.7	37839	3.3	5.2	4033	0.1	3.2	4.8
galwc01-2-L|L	5	10	5	15	10	70	50	4.5	642.5	64737	6.2	29.2	8388	2.4	3.9	37.3
galwc01-3-L|L	10	5	5	15	5	70	50	4.7	16.8	21503	3.8	4.7	3068	2.5	1.4	2.3
galwc01-4-L|L	5	5	10	15	5	70	50	4.4	31.0	31503	2.5	5.0	4348	0.0	2.5	2.7
average								4.1	152.8	35417	3.6	9.5	4592	1.0	2.6	10.1
galwc02-0-L|L	5	5	5	15	5	70	50	5.7	8.0	14066	1.4	2.2	2319	0.0	1.4	1.6
galwc02-1-L|L	5	5	5	20	5	70	50	6.0	13.3	33616	0.9	3.4	3253	0.1	0.8	1.9
galwc02-2-L|L	5	10	5	15	10	70	50	5.0	59.7	39438	3.6	5.3	4707	0.1	3.5	7.5
galwc02-3-L|L	10	5	5	15	5	70	50	3.2	2.8	14066	0.3	2.7	2324	0.1	0.2	1.4
galwc02-4-L|L	5	5	10	15	5	70	50	6.3	8.0	23190	0.2	3.1	3478	0.0	0.2	2.0
average								5.2	18.4	24875	1.3	3.3	3216	0.1	1.2	2.9
galwc03-0-L|L	5	5	5	15	5	150	65	18.6	10.8	18761	2.1	2.7	2082	0.0	2.1	2.2
galwc03-1-L|L	5	5	5	20	5	150	65	16.0	113.8	68987	3.7	4.0	3513	1.5	2.2	3.8
galwc03-2-L|L	5	10	5	15	10	150	65	19.2	181.9	79140	0.6	27.1	8449	0.0	0.6	13.7
galwc03-3-L|L	10	5	5	15	5	150	65	18.6	11.3	18761	2.7	2.5	2085	0.0	2.7	1.8
galwc03-4-L|L	5	5	10	15	5	150	65	18.1	22.7	31590	1.9	3.2	3319	0.0	1.9	2.7
average								18.1	68.1	43448	2.2	7.9	3890	0.3	1.9	4.8

Table 1: Numerical results of the three methods.

Computational Results

Medium-sized instances

Instance	Features							GALW			Hybrid(Y)
	$\|K\|$	$\|U\|$	$\|L\|$	$\|D\|$	N	q	M	t	#ps	$\%^Y$	$T/\%$
galwc04-0-L|L	5	5	5	25	5	70	60	4.0	3180	3.0	3.9
galwc04-1-L|L	5	5	5	40	5	70	60	10.0	7376	2.0	8.3
galwc04-2-L|L	5	10	5	25	10	70	60	12.7	9428	1.4	33.2
galwc04-3-L|L	10	5	5	25	5	70	60	4.4	3155	1.1	4.1
galwc04-4-L|L	5	5	10	25	5	70	60	4.7	4448	0.6	4.2
average								7.2	5517	1.6	10.7
galwc05-0-L|L	5	5	5	25	5	150	65	8.9	4791	0.7	7.4
galwc05-1-L|L	5	5	5	40	5	150	65	26.2	10902	0.3	23.1
galwc05-2-L|L	5	10	5	25	10	150	65	25.7	13286	3.0	31.1
galwc05-3-L|L	10	5	5	25	5	150	65	8.7	4752	0.0	8.9
galwc05-4-L|L	5	5	10	25	5	150	65	10.6	6936	0.0	9.0
average								16.0	8133	0.8	15.9

Table 2: Numerical results of GALW and the hybrid method.

Computational Results

Big-sized instances

Instance	$\|K\|$	$\|U\|$	$\|L\|$	$\|D\|$	N	q	M	GALW t	GALW #ps	GALW $\%^Y$	Hybrid(Y) $T/\%$
galwc06-0-L|L	5	5	10	50	5	70	50	20.7	6350	0.5	20.1
galwc06-1-L|L	5	5	10	80	5	70	50	59.7	16593	0.7	271.0
galwc06-2-L|L	5	10	10	50	10	70	50	64.2	15019	0.0	69.8
galwc06-3-L|L	10	5	10	50	5	70	50	21.6	6412	0.1	16.9
galwc06-4-L|L	5	5	15	50	5	70	50	22.7	9334	0.5	21.0
average								37.8	10742	0.4	79.8
galwc07-0-L|L	5	5	10	50	5	70	50	25.2	10814	0.0	18.0
galwc07-1-L|L	5	5	10	80	5	70	50	131.2	28020	0.6	768.3
galwc07-2-L|L	5	10	10	50	10	70	50	250.1	26212	0.2	281.1
galwc07-3-L|L	10	5	10	50	5	70	50	39.7	10912	0.2	55.7
galwc07-4-L|L	5	5	15	50	5	70	50	27.7	13963	0.1	23.8
average								94.8	17984	0.2	229.4
galwc08-0-L|L	5	10	10	50	10	70	50	474.8	14835	2.0	2681.4
galwc08-1-L|L	5	10	10	80	10	70	50	499.0	36996	$-\infty$	$(+\infty)$
galwc08-2-L|L	5	15	10	50	15	70	50	621.3	27451	-18.6	(28.9%)
galwc08-3-L|L	10	10	10	50	10	70	50	842.8	14691	-0.1	(2.0%)
galwc08-4-L|L	5	10	15	50	10	70	50	399.7	19571	5.8	879.2
average								567.5	22709	-2.7	2690.2

Table 3: Numerical results of GALW and the hybrid method

Plan

Multicommodity-Ring Location Routing Problem (MRLRP)

Branch & Cut for the VRP with Intermediate Replenishment Facilities (VRPIRF)

Branch & Price for the VRP with Intermediate Replenishment Facilities

VRP with Intermediate Replenishment Facilities (VRPIRF)

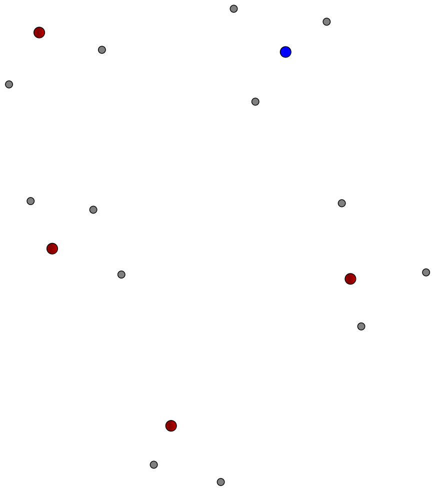

Figure 6: A VRPIRF instance: the depot (blue), the facilities (red), and the customers

VRP with Intermediate Replenishment Facilities (VRPIRF)

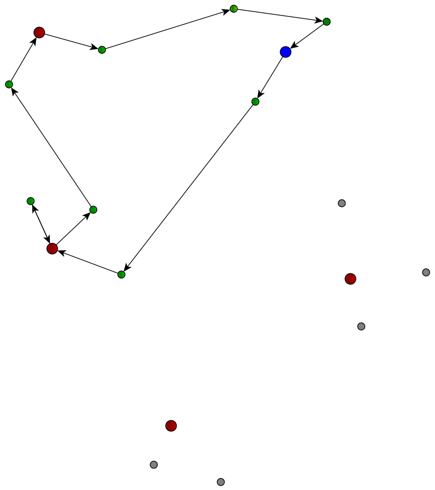

Figure 6: A solution to the previous instance: rotation of first vehicle

VRP with Intermediate Replenishment Facilities (VRPIRF)

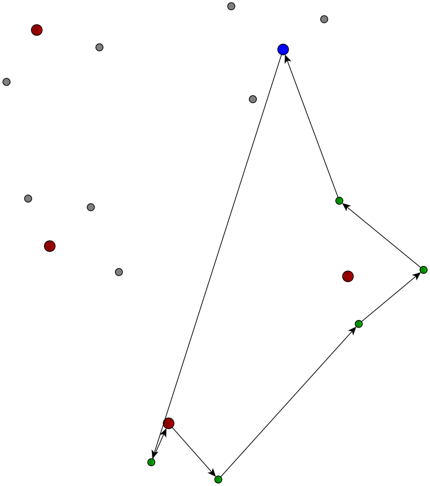

Figure 6: A solution to the previous instance: rotation of second vehicle

VRP with Intermediate Replenishment Facilities (VRPIRF)

Problem Definition

Actors

- set of customers with a demand and a service time
- a depot, the base of a fleet of homogeneous vehicles with fixed capacity
- a set of replenishment facilities, with a recharge time each

VRP with Intermediate Replenishment Facilities (VRPIRF)

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Distinctive Features

- multi-trip: vehicles can replenish at facilities when empty $\Rightarrow$ rotation
- the depot has no replenishment purposes
- a vehicle's rotation must:
  - start and end at the depot
  - not exceed a given maximum shift length

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- multi-trip: vehicles can replenish at facilities when empty ⇒ rotation
- the depot has no replenishment purposes
- a vehicle's rotation must:
  - start and end at the depot
  - not exceed a given maximum shift length

Objective

Find a minimum-cost set of rotations that visit each client exactly once

Main features of the MILP model for VRPIRF

Replenishment arcs

Replenishment arcs

They represent stops at facilities to recharge in between two customers
$\Rightarrow$ no facility nodes
$\Rightarrow$ the depot is the only node with in/outdegree greater than 1
$\Rightarrow$ a rotation becomes very similar to a classical CVRP route
$\Rightarrow$ connectivity of rotations can be assured in a much stronger way

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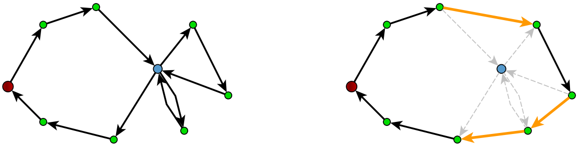

Figure 7: The same rotation with replenishment arcs and without.

Main features of the MILP model for VRPIRF

Arrival times

Arrival times

They allow to keep track of time on a partial path from the depot:

$$(\forall i \in C) \sum_{j \in V \setminus \{i\}} z_{ij} = \sum_{j \in V \setminus \{i\}} z_{ji} + \sum_{j \in V \setminus \{i\}} t_{ij} x_{ij} + \sum_{j \in C \setminus \{i\}} u_{ij} w_{ij} \; ; \quad z_{i\Delta} \leq T x_{i\Delta}$$

$\Rightarrow$ maximal shift length is enforced without vehicle index
$\Rightarrow$ dramatic reduction of symmetry issues
$\Rightarrow$ connectivity of integer solutions as a side effect

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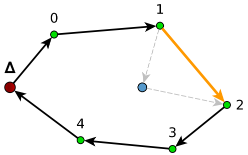

Figure 8: In this rotation, $z_{4\Delta} \leq T$ is sufficient to enforce the maximum shift length.

A MILP 2-Index Formulation for VRPIRF

Decision variables

1. base arc variables $x_{ij} \in \{0, 1\}$
2. replenishment arc variables $w_{ij} \in \{0, 1\}$
3. arrival time variables $z_{ij} \geq 0$

A MILP 2-Index Formulation for VRPIRF

Decision variables

1. base arc variables $x_{ij} \in \{0, 1\}$
2. replenishment arc variables $w_{ij} \in \{0, 1\}$
3. arrival time variables $z_{ij} \geq 0$

Objective function

$$\min \sum_{ij \in A_0} d_{ij} x_{ij} + \sum_{ij \in A_P} f_{ij} w_{ij}$$

A MILP 2-Index Formulation for VRPIRF

Objective function

$$\min \sum_{ij \in A_0} d_{ij} x_{ij} + \sum_{ij \in A_P} f_{ij} w_{ij}$$

Constraints

s.t. $x(\delta_0^-(\Delta)) + w(A_P) \geq \kappa(C)$		min nr of routes
$x(\delta_0^+(i)) + w(\delta_P^+(i)) = x(\delta_0^-(i)) + w(\delta_P^-(i))$	$\forall i \in C$	clients service
$x(\delta_0^+(i)) + w(\delta_P^+(i)) = 1$	$\forall i \in C$	
$x(\delta_0^-(\Delta)) \leq n_K$		depot degree
$x(\delta_0^+(\Delta)) = x(\delta_0^-(\Delta))$		
$x(A_0(S)) \leq \lvert S \rvert - \kappa(S)$	$\forall S \in \mathcal{S}(C)$	capacity
$z_{\Delta i} = t_{\Delta i} x_{\Delta i}$	$\forall i \in C$	arrival times
$(t_{\Delta i} + t_{ij}) x_{ij} + (t_{\Delta i} + u_{ij}) w_{ij} \leq z_{ij} \leq (T - t_{j\Delta})(x_{ij} + w_{ij})$	$\forall i \in C, j \in C \setminus \{i\}$	
$\sum_{j \in V \setminus \{i\}} z_{ij} = \sum_{j \in V \setminus \{i\}} z_{ji} + \sum_{j \in V \setminus \{i\}} t_{ij} x_{ij} + \sum_{j \in C \setminus \{i\}} u_{ij} w_{ij}$	$\forall i \in C$	
$(t_{\Delta i} + t_{i\Delta}) x_{i\Delta} \leq z_{i\Delta} \leq T x_{i\Delta}$	$\forall i \in C$	max shift length

$x_{ij}, w_{ij} \in \{0, 1\}, z_{ij} \geq 0$

Separation

Capacity constraints

Separated as classical capacity constraints on base arcs only

- solution transformation to symmetric, one-depot support graph is required
- separated using CVRPSEP (see [18]) routines and imposed as rounded capacity inequalities (RCI)

$$x(A_0(S)) \leq |S| - r(S) = |S| - \lceil \frac{1}{Q} \sum_{i \in S} q_i \rceil$$

[18] J.Lysgaard, A.N.Letchford, R.W.Eglese, *A new branch-and-cut algorithm for the capacitated vehicle routing problem.* Mathematical Programming, 100(2):423–445, 2004.

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Valid inequalities

- Connectivity valid inequalities exploit the rotation structure, imposed as SECS on both base and replenishment arcs:

$$(\forall S \in \mathcal{S}(C)) \quad x(A_0(S)) + w(A_P(S)) \leq |S| - 1$$

- Multistar inequalities: given nucleus $N \subset C$ and satellites $S \subseteq C \setminus N$ (see [19])

$$\alpha(N,S) \sum_{ij \in A(N)} x_{ij} + \beta(N,S) \sum_{ij \in A(C:S)} x_{ij} \leq \gamma(N,S)$$

[19] J.Lysgaard, A.N.Letchford, R.W.Eglese, *Multistars, partial multistars and the capacitated vehicle routing problem.* Mathematical Programming, 94(1):21–40, 2002.

Computational Results

Benchmark Instances

- instances taken from Crevier et al. (see [**?**]) and Tarantilis et al. (see [**?**])
- features: from 48 to 288 customers, 2 to 7 facilities, 2 to 8 vehicles

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- features: from 48 to 288 customers, 2 to 7 facilities, 2 to 8 vehicles

Computational Strategy

- smaller instances (48 to 75 customers):
  - complete computation
  - time limit: 3600–5400s on both root node and Branch & Bound search
  - Branch & Bound search fed with the best known solution (see [?]) as initial UB
- bigger instances (96+ customers):
  - cutting plane algorithm at root (time limit: 3600–7200s)
  - the gap with the best known solution is reported

Computational Results

instance	n	f	n_K	$T_1 = T_{23}$	t_1	t_{23}	BKS	$\%^1$	$\%^{23}$
50c3d2v	50	2	2	3600	4.51	42.44	<u>2209.83</u>	1.89	0.00
50c3d4v	50	2	4	3600	1.37	3600.12	2368.33	11.07	9.61
50c3d6v	50	2	6	3600	2.07	3600.01	2999.29	11.57	11.11
50c5d2v	50	4	2	3600	3.75	386.21	<u>2608.25</u>	2.18	0.00
50c5d4v	50	4	4	3600	2.01	3600.01	3086.58	8.27	6.98
50c5d6v	50	4	6	3600	1.97	3600.01	3548.88	12.44	9.90
50c7d2v	50	6	2	3600	5.78	437.26	<u>3353.08</u>	2.11	0.00
50c7d4v	50	6	4	3600	3.92	3600.01	3380.27	2.88	0.25
50c7d6v	50	6	6	3600	2.33	3600.01	4074.43	11.94	10.08
75c3d2v	75	2	2	5400	36.19	4101.01	<u>2678.79</u>	1.62	0.00
75c3d4v	75	2	4	5400	26.44	5400.31	2746.73	2.94	1.44
75c3d6v	75	2	6	5400	59.33	5400.04	3393.88	7.85	7.65
75c5d2v	75	4	2	5400	40.71	5400.28	3373.68	3.48	2.43
75c5d4v	75	4	4	5400	16.78	5400.25	3553.46	6.07	5.54
75c5d6v	75	4	6	5400	20.24	5400.06	4184.65	8.13	7.97
75c7d2v	75	6	2	5400	42.71	5400.01	3569.01	1.90	0.63
75c7d4v	75	6	4	5400	13.64	5400.22	3822.09	4.99	4.22
75c7d6v	75	6	6	5400	12.49	5400.10	4239.76	7.62	6.76

Table 4: Results on Tarantilis et al. instances with 50 to 75 customers.

Computational Results

instance	n	f	n_K	T_{12}	t_1	t_2	BKS	$\%^1$	$\%^2$
100c3d3v	100	2	3	3600	170.76	34.84	3123.51	2.47	2.47
100c3d5v	100	2	5	3600	62.75	76.81	3548.44	13.58	13.58
100c3d7v	100	2	7	3600	372.76	2692.72	4235.30	9.40	9.32
100c5d3v	100	4	3	3600	244.81	84.93	4053.95	2.60	2.47
100c5d5v	100	4	5	3600	17.12	25.49	4413.16	9.36	9.23
100c5d7v	100	4	7	3600	84.26	49.56	5142.52	13.71	13.62
100c7d3v	100	6	3	3600	158.96	62.10	4207.79	5.41	5.41
100c7d5v	100	6	5	3600	54.55	54.67	4412.85	10.25	10.25
100c7d7v	100	6	7	3600	94.58	61.94	4869.65	10.46	10.46
125c4d3v	125	3	3	3600	465.86	158.00	3916.01	2.47	2.36
125c4d5v	125	3	5	3600	117.57	156.57	4308.44	9.36	9.36
125c4d7v	125	3	7	3600	289.77	162.71	4664.38	10.87	10.87
125c6d3v	125	5	3	3600	329.12	103.28	4063.25	2.55	2.55
125c6d5v	125	5	5	3600	791.67	1041.44	4760.46	6.25	6.15
125c6d7v	125	5	7	3600	265.55	148.37	5164.02	7.83	7.81
125c8d3v	125	7	3	3600	992.42	135.43	4534.14	3.97	3.97
125c8d5v	125	7	5	3600	1839.76	1765.75	4947.00	5.08	5.08
125c8d7v	125	7	7	3600	1184.34	2489.37	5334.91	6.94	6.94

Table 5: Results on Tarantilis et al. instances with 100 and 125 customers.

Computational Results

instance	n	f	n_K	T_{12}	t_1	t_2	BKS	%[1]	%[2]
150c4d3v	150	3	3	3600	2060.68	464.98	4049.47	2.09	2.09
150c4d5v	150	3	5	3600	1566.35	2933.70	4618.71	7.28	7.28
150c4d7v	150	3	7	3600	1311.58	3827.35	5118.40	9.82	9.82
150c6d3v	150	5	3	3600	940.12	138.51	4057.08	4.12	4.09
150c6d5v	150	5	5	3600	3620.32		4855.28	5.96	
150c6d7v	150	5	7	3600	3798.86		5695.25	7.95	
150c8d3v	150	7	3	3600	1749.28	247.99	4641.29	3.15	3.08
150c8d5v	150	7	5	3600	424.25	251.98	5065.10	6.41	6.41
150c8d7v	150	7	7	3600	444.10	327.18	5605.82	9.08	9.08
175c4d4v	175	3	4	3600	3652.25		4692.53	3.40	
175c4d6v	175	3	6	3600	3458.49	142.12	4816.54	4.13	4.13
175c4d8v	175	3	8	3600	3664.08		5830.62	9.87	
175c6d4v	175	5	4	3600	3813.19		5000.89	4.53	
175c6d6v	175	5	6	3600	2685.38	1304.21	5291.62	5.38	5.38
175c6d8v	175	5	8	3600	2708.57	1089.71	6034.21	9.94	9.94
175c8d4v	175	7	4	3600	3731.42		5747.72	5.29	
175c8d6v	175	7	6	3600	2836.71	961.12	5914.00	5.00	5.00
175c8d8v	175	7	8	3600	3764.96		6766.54	8.39	

Table 6: Results on Tarantilis et al. instances with 150 and 175 customers.

Computational Results

instance	n	f	n_K	$T_1 = T_{23}$	t_1	t_{23}	BKS	$\%^1$	$\%^{23}$
a1	48	2	6	3600	5.77	3600.22	1179.79	7.70	6.91
d1	48	3	5	3600	7.93	3600.02	1059.42	7.67	6.44
a2	48	4	4	3600	0.76	3600.04	997.94	6.96	5.71
g1	72	4	5	5400	60.28	5400.02	1181.13	4.77	4.59
j1	72	5	4	5400	33.40	5400.08	1115.77	5.46	4.46
g2	72	6	4	5400	7.85	5400.06	1152.92	5.76	4.65

Table 7: Results on Crevier et al. instances with 48 to 72 customers.

Computational Results

instance	n	f	n_K	T_{12}	t_1	t_2	BKS	$\%^1$	$\%^2$
b1	96	2	4	3600	115.95	54.56	1217.07	3.38	3.37
e1	96	3	5	3600	59.40	25.51	1309.12	2.48	2.05
b2	96	4	4	3600	56.38	28.79	1291.18	3.88	3.87
h1	144	4	4	3600	1487.26	588.42	1545.50	4.26	4.25
k1	144	5	4	3600	622.59	187.87	1573.20	3.89	3.89
c2	144	4	4	3600	1016.70	195.45	1715.59	3.86	3.86
h2	144	6	4	3600	1160.07	133.82	1575.27	4.34	4.34
c1	192	2	5	3600	2755.35	871.72	1866.75	3.61	3.55
f1	192	3	4	3600	3543.95	61.81	1570.40	2.36	2.36
d2	192	4	3	3600	3808.18		1854.03	3.92	
i1	216	4	4	7200	6090.99	1823.72	1922.17	2.85	2.85
l1	216	5	4	7200	7751.10		1863.27	3.23	
i2	216	6	3	7200	7341.02		1919.73	3.89	
e2	240	4	3	7200	7394.10		1916.67	4.43	
f2	288	4	3	7200	7201.17		2230.30	7.28	
j2	288	6	3	7200	8472.73		2247.68	3.13	

Table 8: Results on Crevier et al. instances with 96 to 288 customers.

Plan

Multicommodity-Ring Location Routing Problem (MRLRP)

Branch & Cut for the VRP with Intermediate Replenishment Facilities (VRPIRF)

Branch & Price for the VRP with Intermediate Replenishment Facilities

A MILP Extended Formulation

Decision variables

1. route variables $x_r^k \in \{0, 1\}$
2. usage variables $\widetilde{x}^k \in \{0, 1\}$
3. activity variables $y_p^k \in \{0, 1\}$

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Objective function

$$\min \sum_{k \in K} \sum_{r \in \mathscr{R}} c_r x_r^k$$

A MILP Extended Formulation

Objective function

$$\min \sum_{k \in K} \sum_{r \in \mathscr{R}} c_r x_r^k$$

Constraints

s.t. $\sum_{r \in \mathscr{R}} \sum_{k \in K} a_r^i \, x_r^k \geq 1$	$\forall i \in C$	clients service
$\sum_{r \in \mathscr{R}} a_r^i \, e_r'^p \, x_r^k \leq y_p^k \leq \widetilde{x}^k$	$\forall k \in K, p \in F, i \in C$	vehicle activity at facilities
$\sum_{r \in \mathscr{R}} (e_r'^p - e_r''^p) \, x_r^k = 0$	$\forall k \in K, p \in F$	depot degree
$\sum_{r \in \mathscr{R}} t_r \, x_r^k \leq T \, \widetilde{x}^k$	$\forall k \in K$	max shift length
$\sum_{r \in \mathscr{R}} e_r'^\Delta \, x_r^k = \sum_{r \in \mathscr{R}} e_r''^\Delta \, x_r^k = \widetilde{x}^k$	$\forall k \in K$	vehicle activity and usage
$\sum_{r \in \mathscr{R}} b_r'^s \, x_r^k \geq y_p^k$	$\forall k \in K, p \in F, s \in \mathcal{S}_p$	foreconnectivity
$\sum_{r \in \mathscr{R}} b_r''^s \, x_r^k \geq y_p^k$	$\forall k \in K, p \in F, s \in \mathcal{S}_p$	backconnectivity

$\widetilde{x}^k, x_r^k, y_p^k \in \{0, 1\}$

A MILP Extended Formulation

Main features

Route variables

- embedded structural information
  - $\Rightarrow$ better representation of connectivity constraints w.r.t. a 3-index compact model
- on the other hand: they are in exponential number
  - $\Rightarrow$ the solving calls for a Column Generation approach

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Facility Graph

- connectivity constraints defined on subset of facilities (rather than nodes)
- connectivity constraints can be statically generated

A Branch & Price algorithm for the VRPIRF

Pricing Problem

Pricing Problem

- the Pricing Problem delivers routes (and not rotations)
- it can be seen as an Elementary Shortest Path Problem with Resource Constraint (ESPPRC)
- solved with a Dynamic Programming (DP) algorithm inspired by that of [20]
- the expression of the reduced costs

$$
\begin{aligned}
\overline{c}_r^k = & \sum_{ij \in A} b_r^{ij} \cdot (d_{ij} - \tau_{ij} \cdot \beta_k^\star) - \sum_{i \in C} a_r^i \cdot (\alpha_i^\star + \tau_i \cdot \beta_k^\star + \sum_{p \in F} e_r'^p \cdot \varphi_{kpi}^\star) \\
& - e_r'^\Delta \cdot \mu_k'^\star - e_r''^\Delta \cdot \mu_k''^\star - \sum_{p \in F} \tfrac{1}{2}(e_r'^p + e_r''^p) \cdot \tau_p \cdot \beta_k^\star - \sum_{p \in F} (e_r'^p - e_r''^p) \cdot \theta_{kp}^\star \\
& - \sum_{s \subseteq F} \sum_{p \in s} \Big((e_r'^\Delta + \sum_{q \notin s} e_r'^q) \big(\sum_{q \in s} e_r''^q \big) \cdot \delta_{kps}'^\star + \big(\sum_{q \in s} e_r'^q \big) (e_r''^\Delta + \sum_{q \notin s} e_r''^q) \cdot \delta_{kps}''^\star \Big)
\end{aligned}
$$

calls for the solving of a pricing problem per vehicle and per starting point

[20] D.Feillet, P.Dejax, M.Gendreau, C.Gueguen, *An exact algorithm for the elementary shortest path problem with resource constraints: Application to some vehicle routing problems*. Networks, 44(3):216–229, 2004.

A Branch & Price algorithm for the VRPIRF

Pricing Problem

Pricing Problem

- the Pricing Problem delivers routes (and not rotations)
- it can be seen as an Elementary Shortest Path Problem with Resource Constraint (ESPPRC)
- solved with a Dynamic Programming (DP) algorithm inspired by that of [20]
- the expression of the reduced costs calls for the solving of a pricing problem per vehicle and per starting point

[20] D.Feillet, P.Dejax, M.Gendreau, C.Gueguen, *An exact algorithm for the elementary shortest path problem with resource constraints: Application to some vehicle routing problems*. Networks, 44(3):216–229, 2004.

Dominance Level

The dominance rule of [20] compares cost, resources, unreachable nodes. Different dominance levels are used to accelerate ESPPRC convergence

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q-paths

A completion bound based on q-paths and through-q-routes is used (see [21]). A new label is discarded if the sum of its cost and such bound is nonnegative.

[21] N.Christofides, A.Mingozzi, P.Toth, *Exact algorithms for the vehicle routing problem, based on spanning tree and shortest path relaxations*. Mathematical Programming, 20(1):255–282, 1981.

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ng-paths

ng-paths allow to forbid n-loops, $n \geq 2$ (see [22]).
They are used to further restrain combinatorial explosion:

- an initial neighborhood for each customer
- we replace the unreachable nodes with the forbidden nodes of an ng-path $\Rightarrow$ the number of potential label on a node decreases dramatically
- if a nonelementary path occurs $\Rightarrow$ enlarge the neighborhood and repeat. Experimentally this rarely happens $\Rightarrow$ the use of ng-paths pays off

[22] R.Baldacci, A.Mingozzi, R.Roberti, *New Route Relaxation and Pricing Strategies for the Vehicle Routing Problem*. Operations Research, 59(5):1269–1283, 2011.

A Branch & Price algorithm for the VRPIRF

Branching

Branching rules

Three Problem-tailored Branching rules are defined and applied in this order:

1. branching on the number of vehicles: either we use no more than $\lfloor \sum_{k \in K} \widetilde{x}^k \rfloor$ vehicles, or at least $\lceil \sum_{k \in K} \widetilde{x}^k \rceil$
2. branching on the activity variables $y_p^k \Rightarrow$ the Pricing Problems related to vehicle k and facility p are affected
3. branching on arc variables:

$$x_{ij}^{\star} = \sum_{r \in \mathscr{R}} b_r^{ij} \cdot x_r^{\star}$$

We seek for the node with the highest number of fractional outgoing arc variables and impose one half of them to be 0 in each child node

A Branch & Price algorithm for the VRPIRF

Preliminary computational results

instance	B&P t_r	B&P $\%_r$	B&C t_r	B&C $\%_r$
50c3d2v	182	4.87	9	1.65
50c3d4v	655	17.83	9	10.72
50c3d6v	386	39.40	14	11.32
50c5d2v	138	5.10	10	1.81
50c5d4v	146	16.39	12	7.61
50c5d6v	216	31.86	15	10.41
50c7d2v	36	3.48	15	1.92
50c7d4v	66	4.31	11	2.38
50c7d6v	111	24.25	15	11.08
a1	55	7.07	41	7.59
d1	89	6.12	39	7.62
a2	1144	11.33	6	6.68

instance	B&P t_r	B&P $\%_r$	B&C t_r	B&C $\%_r$
75c3d2v		$+\infty$	54	1.62
75c3d4v		$+\infty$	50	2.84
75c3d6v		$+\infty$	178	7.78
75c5d2v	866	1.45	64	3.39
75c5d4v		$+\infty$	40	6.01
75c5d6v	708	13.10	58	8.13
75c7d2v		$+\infty$	57	1.73
75c7d4v		$+\infty$	36	4.88
75c7d6v		$+\infty$	34	7.14
g1	916	4.83	136	4.76
j1		$+\infty$	57	5.40
g2		$+\infty$	30	5.26

Figure 9: Comparison of the Branch & Cut and the Branch & Price algorithms. Both are asked to perform a cutting plane at the root node on a sample of small instances, under a 1200s time limit.

A Branch & Price algorithm for the VRPIRF

A weakness of the model

The connectivity constraints have a weaker impact than the connectivity valid inequalities of the Branch & Cut algorithm

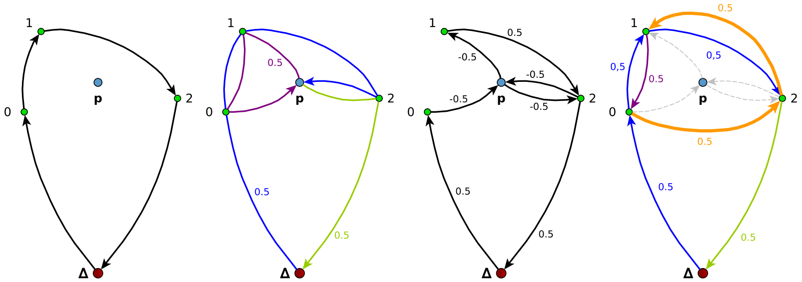

Figure 10: Detection of a fractional solution of a small instance that would be cut in the Branch & Cut algorithm but not in the Branch & Price algorithm.