Exercises : Semi-Definite Programming

MOPA

Exercice 1 — *Semi-Definite Programming duality*

For each program, find its dual :

1.

$$(SDP_1)\begin{cases} \min 2x + y \\ \text{s.t} \\ \quad \begin{pmatrix} 1-x & y \\ y & 1+x \end{pmatrix} \succeq 0 \end{cases}$$

2.

$$(SDP_2)\begin{cases} \min \langle Q, X \rangle \\ \text{s.t} \\ \quad X_{ii} = 1 \quad \forall i = 1, \dots, n \\ \quad X \in \mathcal{S}_+^n \end{cases}$$

Exercice 2 — *Duality gap* Consider the following SDP problems [1] :

1.

$$(SDP_1)\begin{cases} \max -X_{33} \\ \text{s.t} \\ \quad X_{12} + X_{21} + X_{33} = 1 \\ \quad X_{22} = 0 \\ \quad X \succeq 0 \end{cases}$$

2.

$$(SDP_2)\begin{cases} \max -\varepsilon X_{11} - X_{33} \\ \text{s.t} \\ \quad X_{12} + X_{21} + X_{33} = 1 \\ \quad X_{22} = 0 \\ \quad X \succeq 0 \end{cases}$$

For each of them, calculate the dual and indicate (and justify), whether these problems have a nonzero duality gap and, if so, characterize it.

1. These examples come from : Anupam Gupta and Ryan O'Donnell. Linear and semidefinite programming (advanced algorithms) fall 2011. Lecture 12