

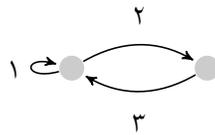
Exercises

Symbolic dynamics

Isfahan, Aug. 28 and 31, 2015

Exercise 1

Let \mathbf{X} be the set of biinfinite paths over the following graph.



1. Give a sentence describing the configurations of \mathbf{X} .
2. Is there a periodic configuration in \mathbf{X} ?
3. Is \mathbf{X} a subshift? SFT?
4. Describe a family of forbidden patterns.
5. Write the matrix corresponding to the graph (entry M_{ij} is the number of edges from vertex i to vertex j).
6. In general for the matrix of the graph, what is represented by M_{ij}^2 ? by M_{ij}^n ? by $\sum_{i,j} M_{i,j}^n$? by M_{ii}^n ? by the trace of M^n ?
7. Find a recurrence relation for the number of paths of length n in the graph of \mathbf{X} .
8. Deduce the entropy.
9. What are the eigenvalues of the matrix?
10. † Is there a transitive configuration?

Exercise 2

Let \mathbf{Y} be the set of configurations involving infinitely many φ s on the left and on the right, and such that two φ s are always separated by between 1 and 3 λ s (this corresponds to a common constraint in data storage on hard drives).

1. Is \mathbf{Y} a subshift? a SFT?
2. Describe a family of forbidden patterns.
3. Consider the graph whose vertices are allowed patterns of length 3, and whose edges are allowed patterns $u = u_0u_1u_2u_3$ of length 4 that link vertices $u_0u_1u_2$ to $u_1u_2u_3$.

4. What is the set of biinfinite paths over that graph, compared to \mathbf{Y} ? is it a factor? the contrary?
5. Give the matrix N representing the graph.
6. Does \mathbf{Y} admit a configuration? a periodic one? † a transitive one?
7. † What is the entropy of \mathbf{Y} ?

Exercise 3

Let \mathbf{X}' be the set of configurations such that there is an even number of γ s between two γ s.

1. Is \mathbf{X}' a subshift? a SFT?
2. Can \mathbf{X}' be read somehow on some graph?
3. Is there a SFT that factors onto \mathbf{X}' ?
4. What is the link between the previous two questions?
5. Is there a periodic configuration? † a transitive one?
6. What is the number of patterns of length n ?
7. What is the entropy?

Exercise 4

Take the set $\mathbf{Y}_{\mathbb{N}_+}$ of configurations such that two γ s never follow each other.

1. Same questions.
2. What is the link with \mathbf{X} ?
3. Let $S \subset \mathbb{N} \setminus \{0\}$ and \mathbf{Y}_S the set of configurations such that the distances between two consecutive γ s are always in S , and additionally, if S is finite, there are infinitely many γ s on the left and on the right.
4. What conditions on S are necessary or sufficient for \mathbf{Y}_S to be a SFT?