Cut and Projection: Exercises

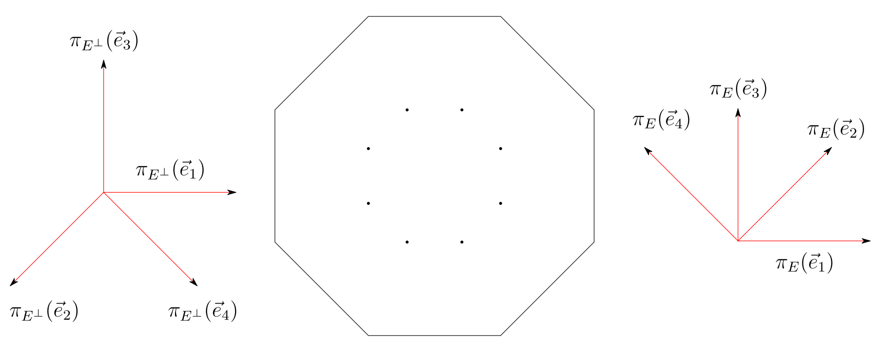
Exercise 1 *Draw the* $2 \to 1$ *planar tiling whose slope is generated by* $(\varphi, 1)$.

Exercise 2 *Consider the multigrid depicted on the seperate sheet. Draw its dualization on the grid, using sides and diagonals of squares as tile edges.*

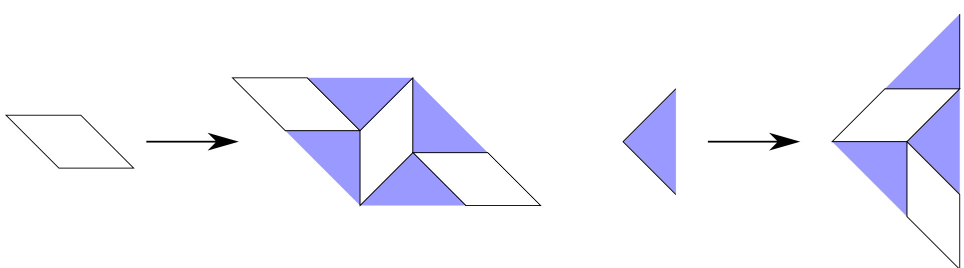
Exercise 3 *Compute the Grassmann coordinates of the plane generated by* $(1, 0, -1, -\sqrt{2})$ *and* $(0, 1, \sqrt{2}, 1)$. *Check the Plücker relations.*

Exercise 4 *Use the window to compute the exact complexity of the word encoding the tiling of the first exercise.*

Exercise 5 *Find the* 0*-atlas of a planar tiling with the following window:*



Exercise 6 *Check that the substitution depicted below generates a tiling by squares and rhombi. Assuming that this tiling is planar, determine its slope.*



Hint*: look how are modified either the tile frequencies or the tile edges.*

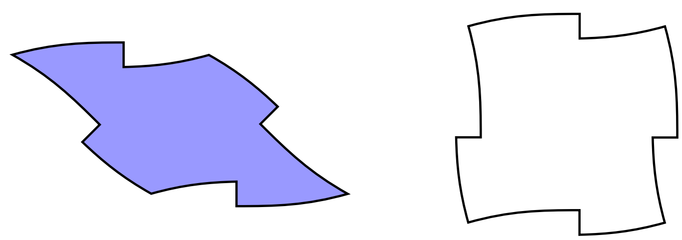
Exercise 7 *Do you think that the previous tiling is planar? Why?*

Exercise 8 *Show that rational planar tilings have local rules.*

Exercise 9 *Show that irrational codim.* 1 *planar tilings have no local rules.*

Exercise 10 *Find an irrational* $4 \to 2$ *planar tiling which has local rules.*
Hint*: use Theorem 6.5.*

Exercise 11 *Consider the tiles below: the left one is free*[1] *but the right one costs* 4030 *tomans per square meter. How cheap can you tile your bathroom?*



Hint*: try planar tilings and find the subperiods enforced by the tile shapes.*

Exercise 12 *We want to prove that the subperiods of a* n*-fold tiling characterize its slope iff* n *is not a multiple of* 4*.*

1. *Use suitable Plücker relations to show that the Grassmann coordinates* (G_{ij}) *of a* 2*-dim. plane are determined by the* G_{ij}*'s such that* $j-i \leq 2$*.*
2. *Write the* n*-fold tiling subperiods involving the* G_{ij}*'s such that* $j-i \leq 2$*.*
3. *Use suitable Plücker relations together with these subperiods to obtain an order two recurrence on these Grassmann coordinates.*
4. *Use Chebyshev polynomials of the second kind to conclude.*

[1] as a free beer...