Exercise 1: gauge transformation

Let $G = (V,E)$ be a finite, bipartite graph, with a weight function $\nu$ on edges.

1. How is modified the partition function if we multiply by $a$ the weights of the edges incident to a given vertex $v$?

2. When $G$ is planar, how is affected the Kasteleyn matrix by this operation?

3. How are affected probabilities?

4. If all edge weights are multiplied by $a$, how is modified the partition function?

Exercise 2: graph surgery

If $G$ has a degree 2 vertex, define a new graph $G'$ obtained from $G$ by “fusing” this vertex with its two neighbors

- Find a bijection between dimer configurations on $G$ and $G'$.
- Find weights on $G'$ such that this bijection preserves the weights of dimer configurations.

Figure 1: Fusion of a degree 2 vertex
Let $G$ and $G'$ be two graphs differing in a small neighborhood as in the second figure. This transformation is called urban renewal.

- Find a local correspondence between dimer configurations on $G$ and $G'$.
- Find $a$ and $b$ such that $Z(G) = bZ(G')$.

**Exercise 3: counting tilings of the Aztec diamond**

Let $Z_n$ be the number of dimer configurations of the Aztec diamond of size $n$. Using urban renewal and fusion, find a relation between $Z_n$ and $Z_{n-1}$. Compute $Z_n$ as a function of $n$.

**Exercise 4**

Computing using several techniques the number of tilings of $\mathbb{H}^{2,2}$ and $\mathbb{H}^{3,3}$.

**Exercise 5**

Compute the probability to have two rhombi in a corner of $\mathbb{H}^{n,n,n}$. Give a simple asymptotic equivalent for this probability as $n \to \infty$, using Stirling’s formula.