Random Tilings: Exercises

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Exercise 1: gauge transformation

Let $G = (V, E)$ be a finite, bipartite graph, with a weight function ν on edges.

1. How is modified the partition function if we multiply by a the weights of the edges incident to a given vertex v?
2. When G is planar, how is affected the Kasteleyn matrix by this operation?
3. How are affected probabilities?
4. If all edge weights are multiplied by a, how is modified the partition function?

Exercise 2: graph surgery

If G has a degree 2 vertex, define a new graph G' obtained from G by "fusing" this vertex with its two neighbors

- Find a bijection between dimer configurations on G and G'.
- Find weights on G' such that this bijection preserves the weights of dimer configurations.

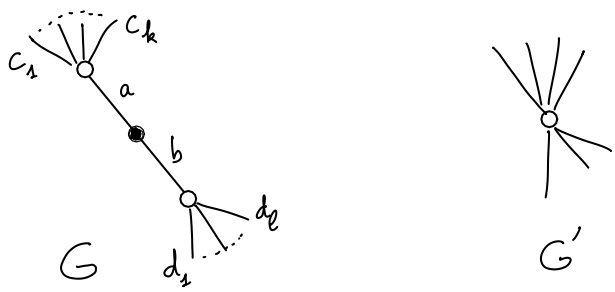

Figure 1: Fusion of a degree 2 vertex

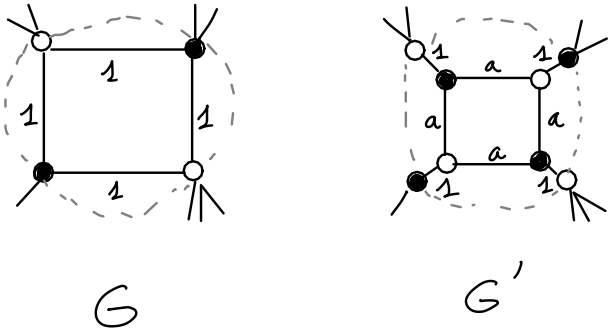

Figure 2: Urban renewal

Let G and G' be two graphs differing in a small neighborhood as in the second figure. This transformation is called *urban renewal*.

- Find a local correspondence between dimer configurations on G and G'.
- Find a and b such that $Z(G) = bZ(G')$.

Exercise 3: counting tilings of the Aztec diamond

Let Z_n be the number of dimer configurations of the Aztec diamond of size n. Using urban renewal and fusion, find a relation between Z_n and Z_{n-1}. Compute Z_n as a function of n.

Exercise 4

Computing using several techniques the number of tilings of $\mathbb{H}^*_{2,2,2}$ and $\mathbb{H}^*_{3,3,3}$.

Exercise 5

Compute the probability to have two rhombi in a corner of $\mathbb{H}^*_{n,n,n}$. Give a simple asymptotic equivalent for this probability as $n \to \infty$, using Stirling's formula.