CIMPA School: Tilings and Tessellations

1 Markov chain for random tilings

Exo 1

1. Consider a 4×2 rectangle R with its lower left corner in position $(0, 0)$. How many domino tilings has it?
2. Now, consider the Markov chain over this tilings that begin with the tiling with only vertical dominoes and the following evolution: selection uniformly at random a point x in $\{(1, 1), (1, 2), (1, 3)\}$, if it possible to make a flip centered in x then do it, otherwise let the tiling unchanged. Describe the automaton associated to this Markov chain.
3. Give the transposition matrix associated to this Markov chain.
4. Is this Markov chain is aperiodic and irreducible?
5. Give the stationary distribution.

Exo 2

1. Consider a regular hexagon H with sides of length 2. What is the number of rhombus tilings of H?
2. Show that we have a bijection between tilings and 2×2 matrices such that the rows and the columns are decreasing and the values are in $\{0, 1, 2\}$.
3. Describe a Markov chain on these 2×2 matrices, that admits a uniform stationary distribution.

Exo 3 [Hard-core model Let C be a chessboard, a configuration on C is an assignment of 0's and 1's on the cells in such a way that two adjacent cells cannot have both 1's.
1) Find a set of wang tiles which simulate exactly the constraints above in the sense that we can partition the set of tiles, the back ones and the white ones, and the resulting colorations of the chessboard are exactly the possible configurations in the hard-core model.
2) Find a certification chain that allow to draw uniformly at random a configuration on the hard-core model.

Exo 4 † Consider a regular hexagon H with sides of length n. Describe a CFTP that admits a uniform stationary distribution.
Assume that the number of tilings is equivalent to $\alpha e^{\beta n^2} n^{-1/12}$ for some constant α and β. Prove that the expected time of the coalescence is greater than $\gamma n^2 \ln(n)$ for some constant γ.