Substitutions: Exercises I
CIMPA School, Isfahan

N. Bédaride *

The questions with a ✠ are much more difficult. Do not start with them.

Exercice 1. *Consider a factorial language. A Rauzy graph G_n is a graph where the vertices are the words of length n of the language and there is an edge from u to v if there exists two letters a, b, such that $ua = bv$ and ua is a word of the language.*

For the Fibonacci substitution draw the Rauzy graph of the subshift if $n = 2, 3, 4$. Do the same for the Thue Morse substitution. What are the differences ?

Exercice 2. *Fix an orthonormal basis (e_0, e_1) of $\mathbb{R}^2$.*

- *Consider the Fibonacci substitution :Consider the fixed point u. Draw the sequence of points $\sum_{k=0}^{n} e_{u_k}$. What is the asymptotic direction v of this set ?*
- *Show that this set is included in $[0,1]^2 + \mathbb{R}v$*
- *Do the same exercice for the Thue Morse fixed point. What do you remark ?*

Exercice 3. *Compute the complexity of the Fibonacci subshift.*

Exercice 4. *Consider the Fibonacci substitution.*

- *Compute the measure of the cylinders $[0], [1]$.*
- *Find some equalities between the measures of the cylinders of sizes two and one.*
- *Deduce the measures of the cylinders of size two.*

For the Thue Morse fixed point, find the frequency of the letters $0, 1$.

Exercice 5. ✠ *Compute the complexity of the Tribonacci subshift. Use the following questions :*

- *How can be extended each letter ?*
- *Is it possible to have three consecutives 0 ?*
- *How can you prolongate 00 ?*

Exercice 6. ✠ *Consider the map*

$$T(x) = \begin{cases} x + \varphi - 1 & [0, 2 - \varphi) \\ x + \varphi - 2 & [2 - \varphi, 1) \end{cases}$$

- *Show that there is no periodic point.*

*Aix Marseille Université, CNRS, Centrale Marseille, I2M, UMR 7373, 13453 Marseille, France. Email: nicolas.bedaride@univ-amu.fr

— *What is the link with a translation on the torus* $\mathbb{T}^1$ *?*
— *We define the induction of* T *on* $[2-\varphi, 1)$ *as the map* S *defined by*

$$S(x) = T^k x, k = \inf\{n, T^n x \in [2-\varphi)\}$$

Compute S*. What is the link between* S *and* T *?*
— *Consider a coding of* T, S *on a two letters alphabet (we associate one letter to each interval, the orbit of a point becomes an infinite word on this alphabet.) Show that the codings of* $x \in [2-\varphi, 1)$ *for* T *and* S *are related by a substitution.*