Cut and Projection Tilings

Send your solutions to `fernique@lipn.fr` *before December 10 at 23:59 and then look here:*

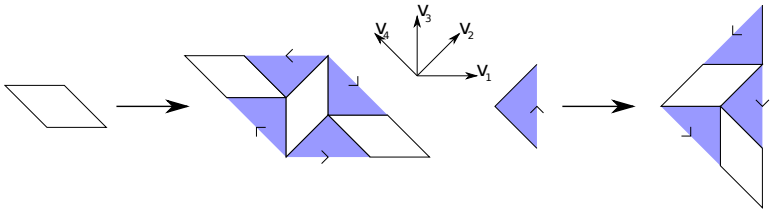
`http://www.lipn.fr/~fernique/info/exos_M2_sol.pdf`

All the pictures can be found in scalable vector graphic format on the following link, feel free to use them with your favorite graphic editor (e.g., *inkscape*) *to investigate and solve the problems:*

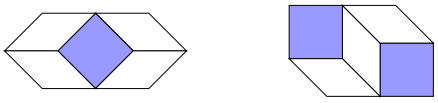
`http://www.lipn.fr/~fernique/info/exos_M2_fig.tgz`

Exercise 1 *Find a planar* $4 \to 2$ *tiling which admits local rules.*

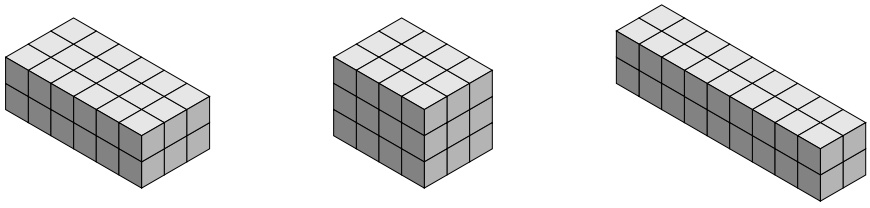
Exercise 2 *Consider the transformation on tiles depicted below.*



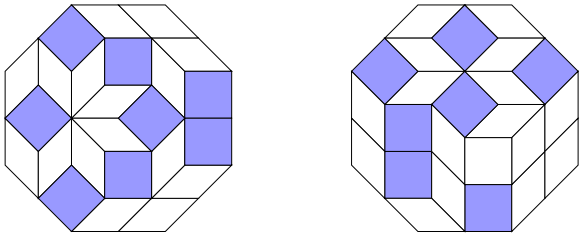
1. *Draw the next iteration of this transformation.*
2. *Show that it generates a* $4 \to 2$ *tiling.*
3. *Assuming that this tiling is planar, determine its slope.*
4. *Show that this tiling is indeed planar and has thickness* 1.
5. *Does it admit local rules?*
6. *Which of these patterns does it contain?*



Exercise 3 *Which of these tilings has the largest entropy?*



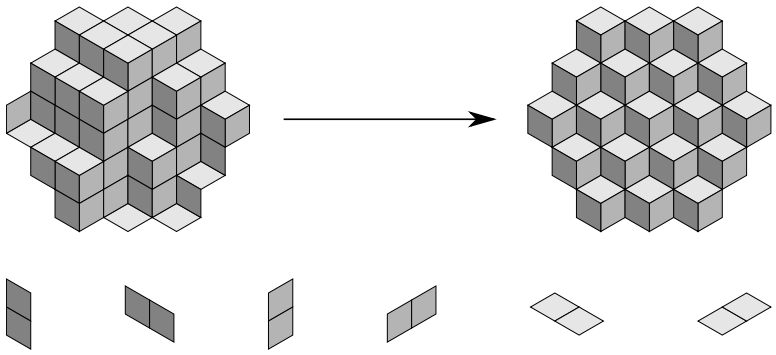
Exercise 4 *Find a shortest path of flips between these two tilings.*[1]



Exercise 5 *Order the following n-vertices graphs by increasing mixing time of a random walk on them. No proof is required, but you can explain your intuition in some words (you can also make simulations).*

1. *a clique*
2. *a path graph*
3. *a cycle*
4. *two cycles connected by an edge*
5. *two cliques connected by an edge*
6. *a complete binary tree*

Exercise 6 *Show that any $3 \to 2$ tiling with the same boundary as the left tiling can be transformed by flips into the right tiling so that the number of identical adjacent tiles (the configurations below tilings) never increases*[2].



[1] Hint: number the tiles and look how they travel through the tilings...

[2] increasing function: fonction *strictement* croissante.