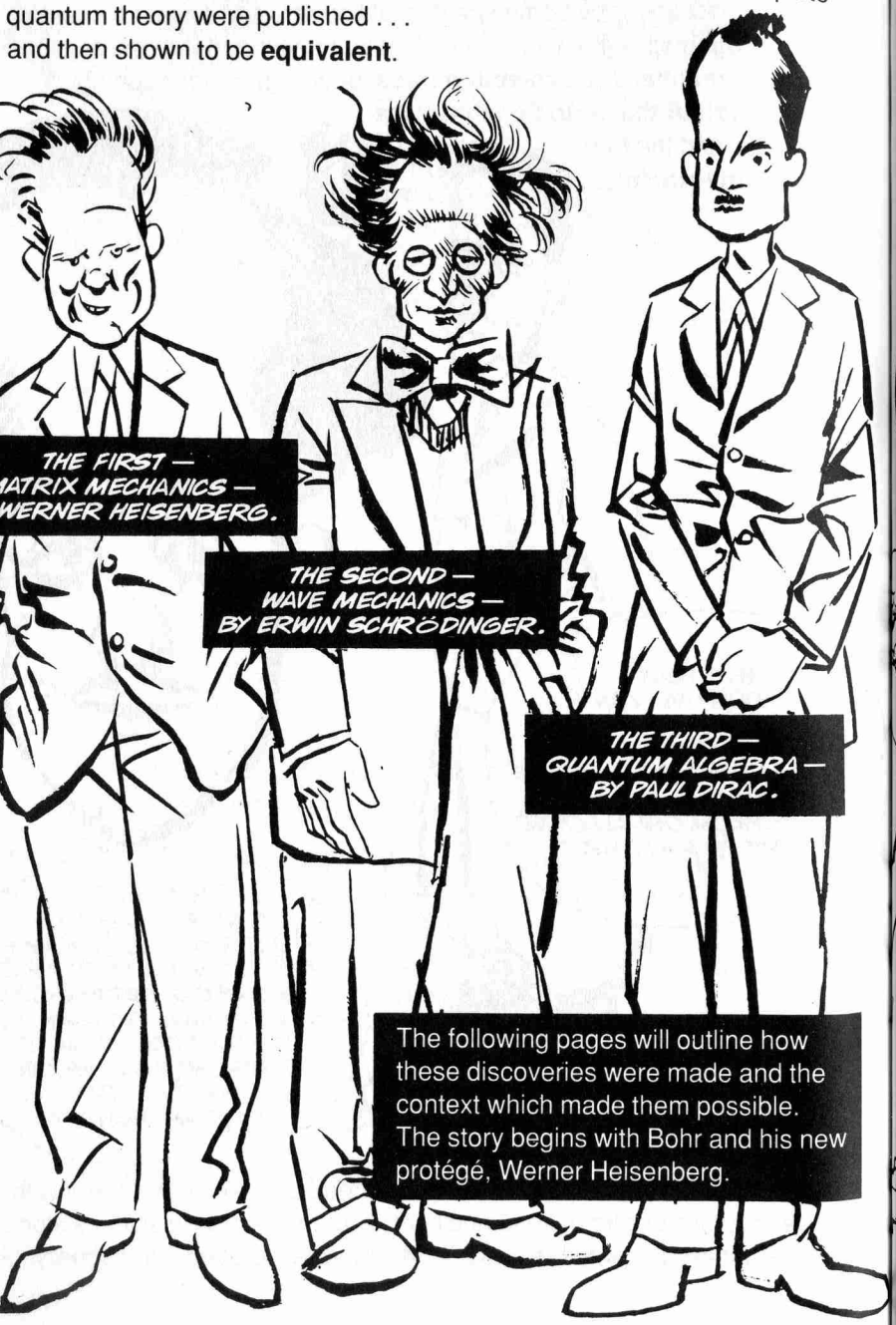
Triple Birth of the New Quantum Theory

Now a remarkable report on the end of 25 years of confusion.
During the twelve month period from June 1925 to June 1926, not one, not two, but **three** distinct and independent developments of a complete quantum theory were published . . .
and then shown to be **equivalent**.

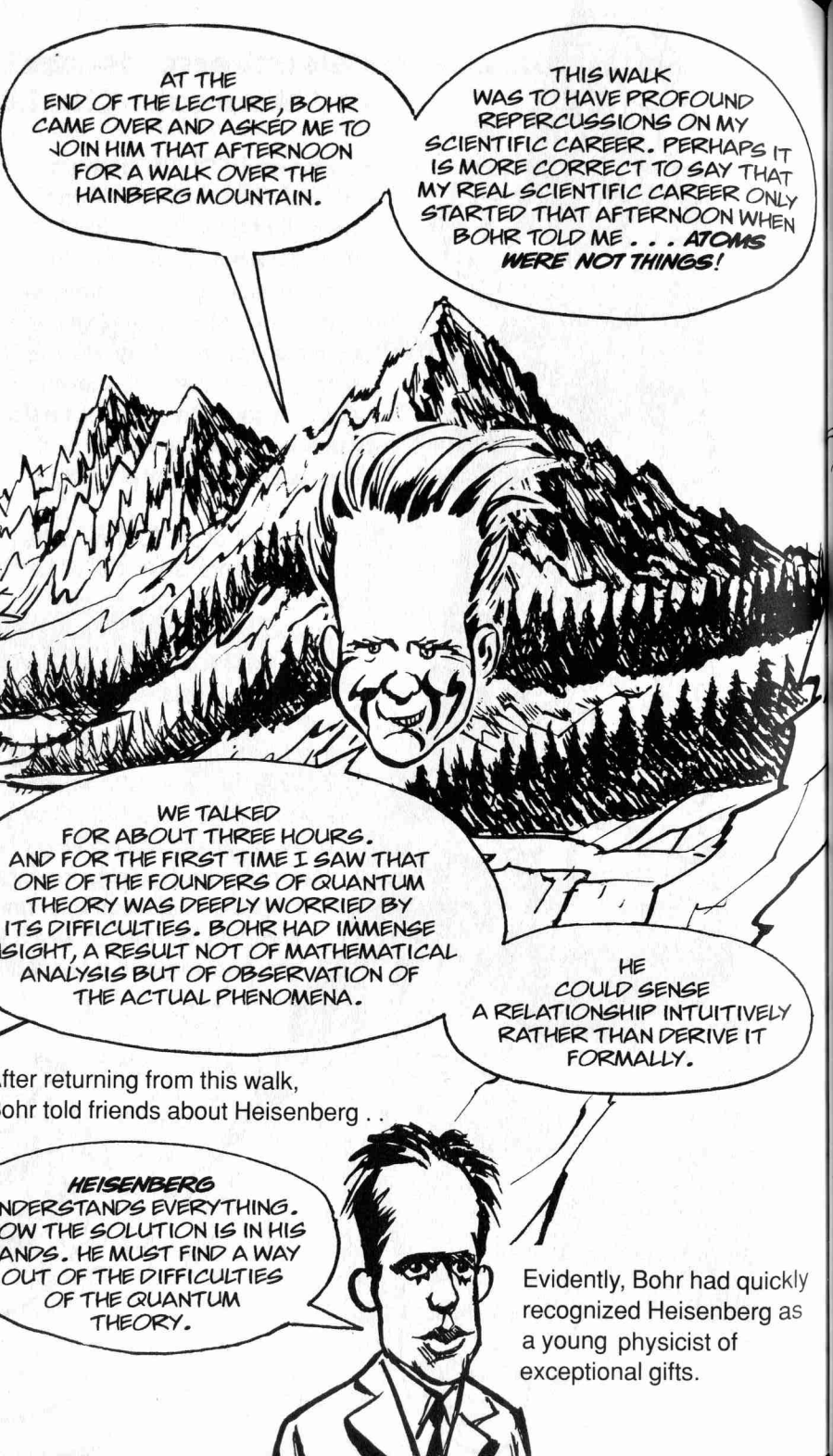


Heisenberg, Genius and Mountain-Climber

Heisenberg (1901–76) grew up in Munich, where his father was professor of Greek at the local university. Always interested in mountain walking, Heisenberg was fortunate that Munich is set at the foot of the Bavarian Alps. He was a brilliant student and an excellent pianist. At secondary school, he had already immersed himself in independent studies of physics.

In the autumn of 1920, immediately after he had enrolled in the University of Munich to study physics with Sommerfeld, he met Wolfgang Pauli. This was the beginning of a lifelong friendship.

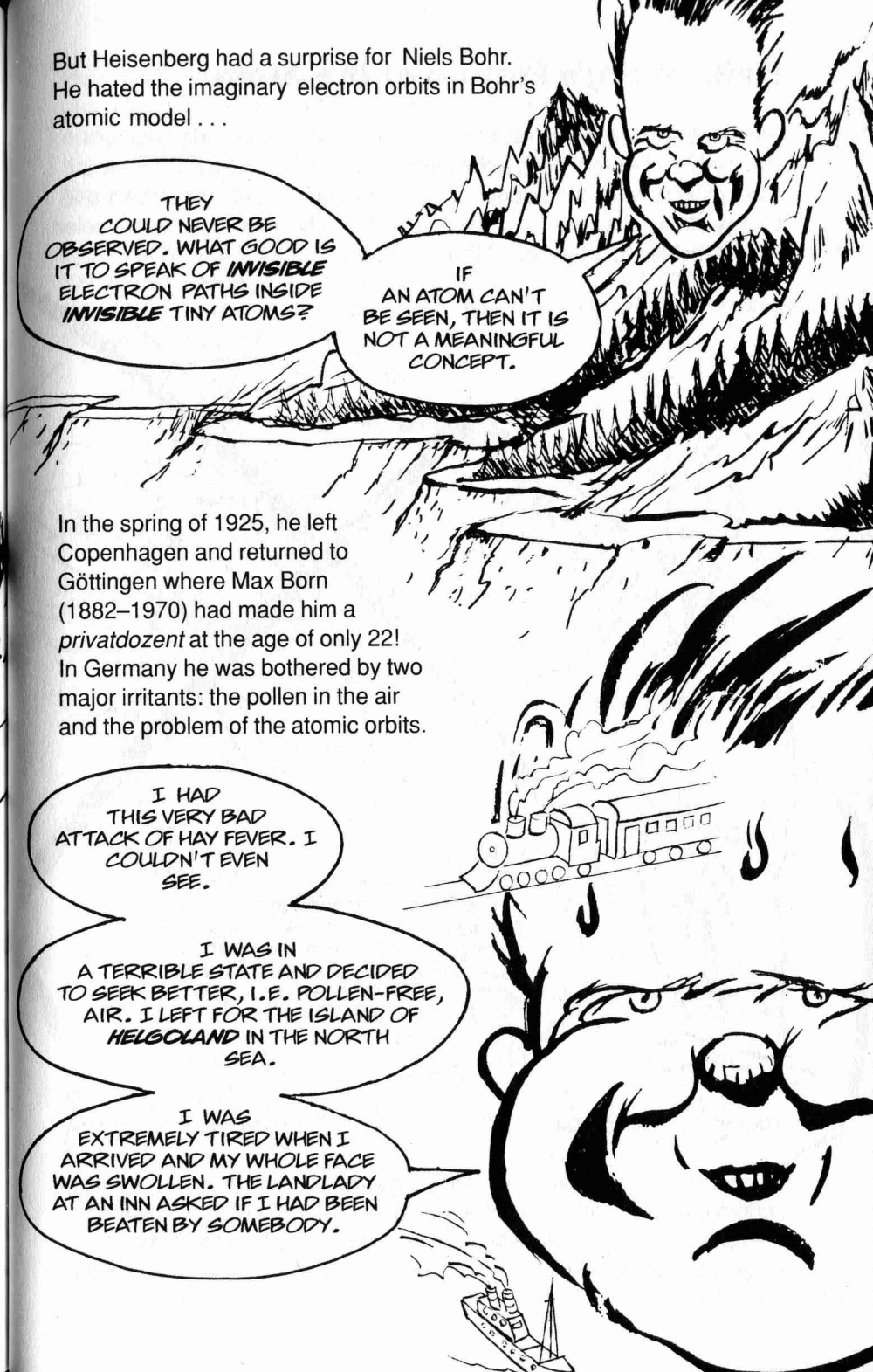
Pauli and Heisenberg were both at Göttingen in June 1922 when Heisenberg first met Bohr. Only 20 years old and still working toward his Ph.D, Heisenberg rose to make an objection after one of Bohr's lectures, to which Bohr replied somewhat hesitantly . . .



fter returning from this walk, ohr told friends about Heisenberg . .

Evidently, Bohr had quickly recognized Heisenberg as a young physicist of exceptional gifts.

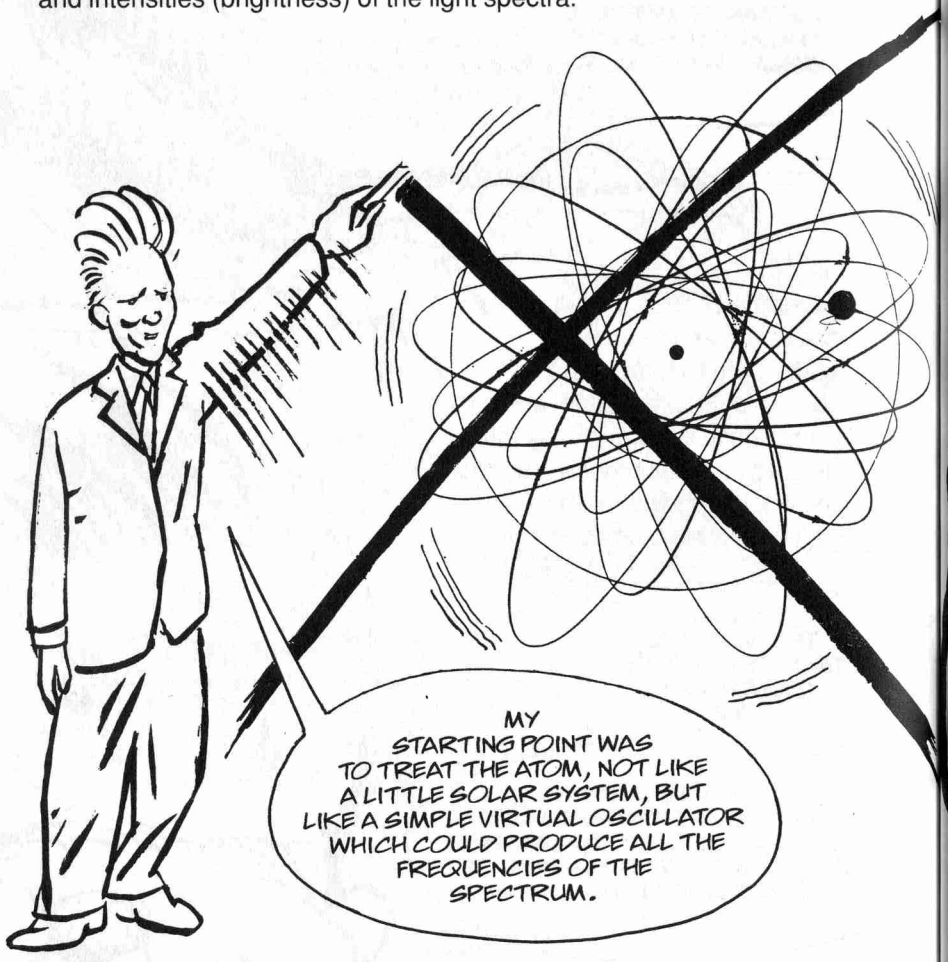
But Heisenberg had a surprise for Niels Bohr. He hated the imaginary electron orbits in Bohr's atomic model . . .



In the spring of 1925, he left Copenhagen and returned to Göttingen where Max Born (1882–1970) had made him a *privatdozent* at the age of only 22! In Germany he was bothered by two major irritants: the pollen in the air and the problem of the atomic orbits.

Heisenberg's Picture of the Atom

Heisenberg hardly slept, dividing his time between inventing quantum mechanics, climbing rocks and memorizing poems by Goethe. He was attempting to work out a **code** for connecting the quantum numbers and energy states in an atom with the experimentally determined frequencies and intensities (brightness) of the light spectra.

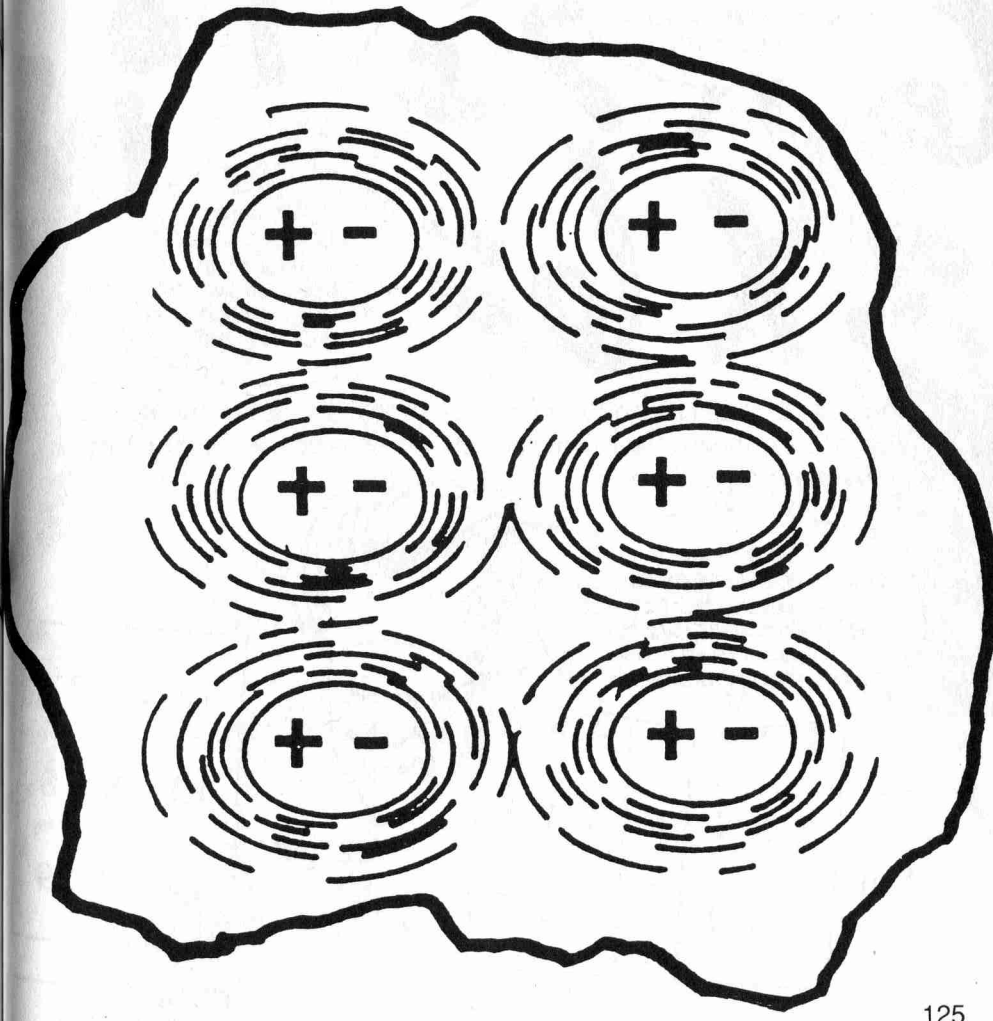


This was similar to what Planck had done on black-body radiation in 1900.

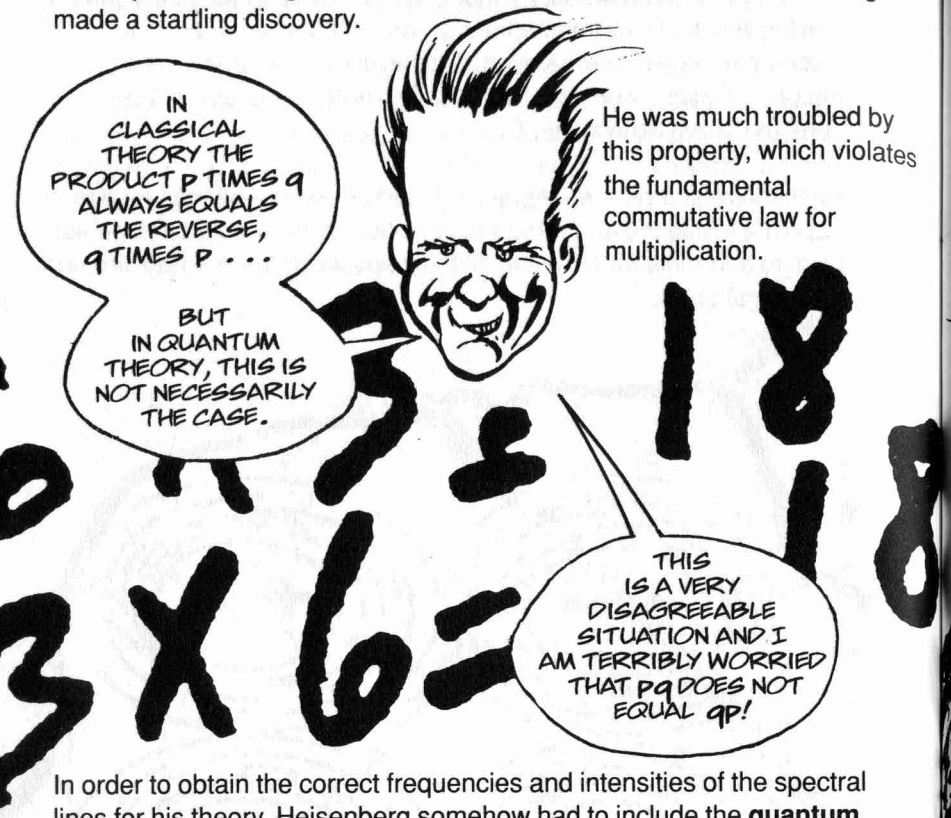
Using the concept which Bohr had called the *correspondence principle*, (where quantum and classical regions overlap), Heisenberg imagined the Bohr atom at very large orbits. There the orbital frequency would equal the radiation frequency and the atom would be like a simple linear oscillator.

He knew how to analyse this problem from classical physics. Familiar quantities like the linear momentum (**p**) and the displacement from equilibrium (**q**) could now be used. Classically, he could solve the equation of motion, then calculate the energy of the particle in the state **n**, the quantized values, $\mathbf{E_n}$.

From the largest orbit – where he could get answers – he then tried to extrapolate *inside* the atom. Here his intuition, some would call it genius, led him to a formula for including all the possible states. **He had broken the spectral code.**

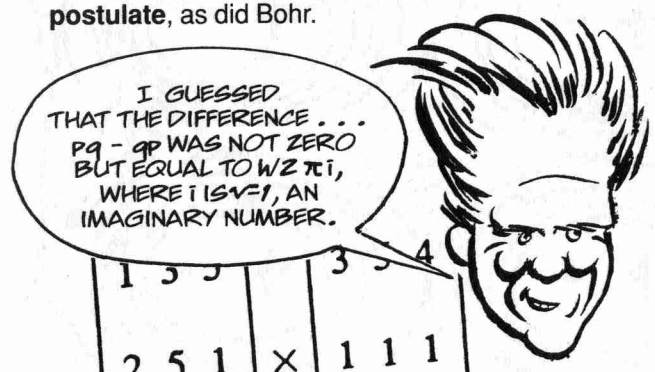


At this point, knowing he was close to something quite new, Heisenberg made a startling discovery.



He was much troubled by this property, which violates the fundamental commutative law for multiplication.

In order to obtain the correct frequencies and intensities of the spectral lines for his theory, Heisenberg somehow had to include the **quantum postulate**, as did Bohr.



For

2 5 1	×	1 1 1
4 3 2		2 3 5

we get:

16	23	32
13	18	18
19	29	29

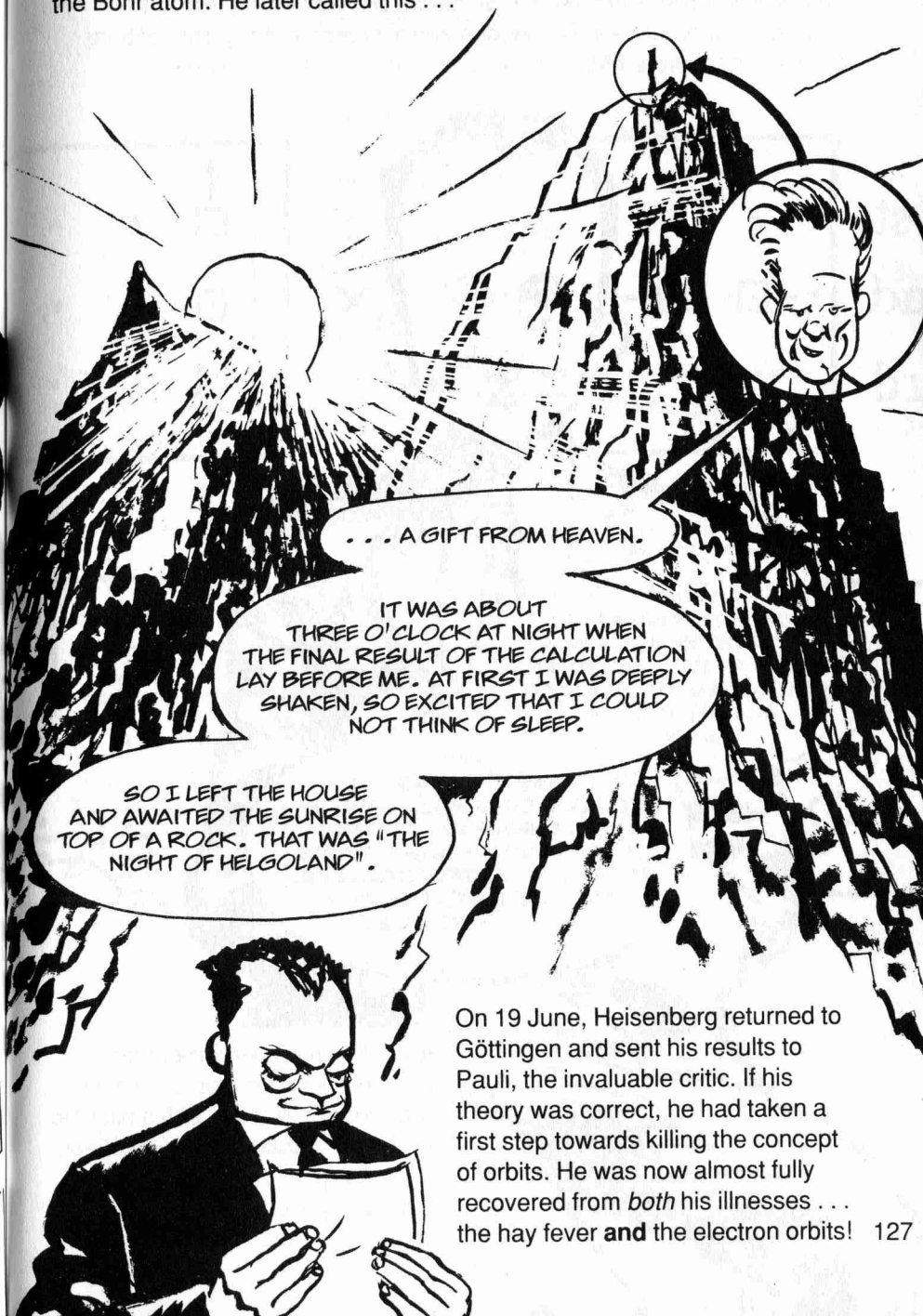
For

3 5 4		1 3 5

we get:

29	46	28

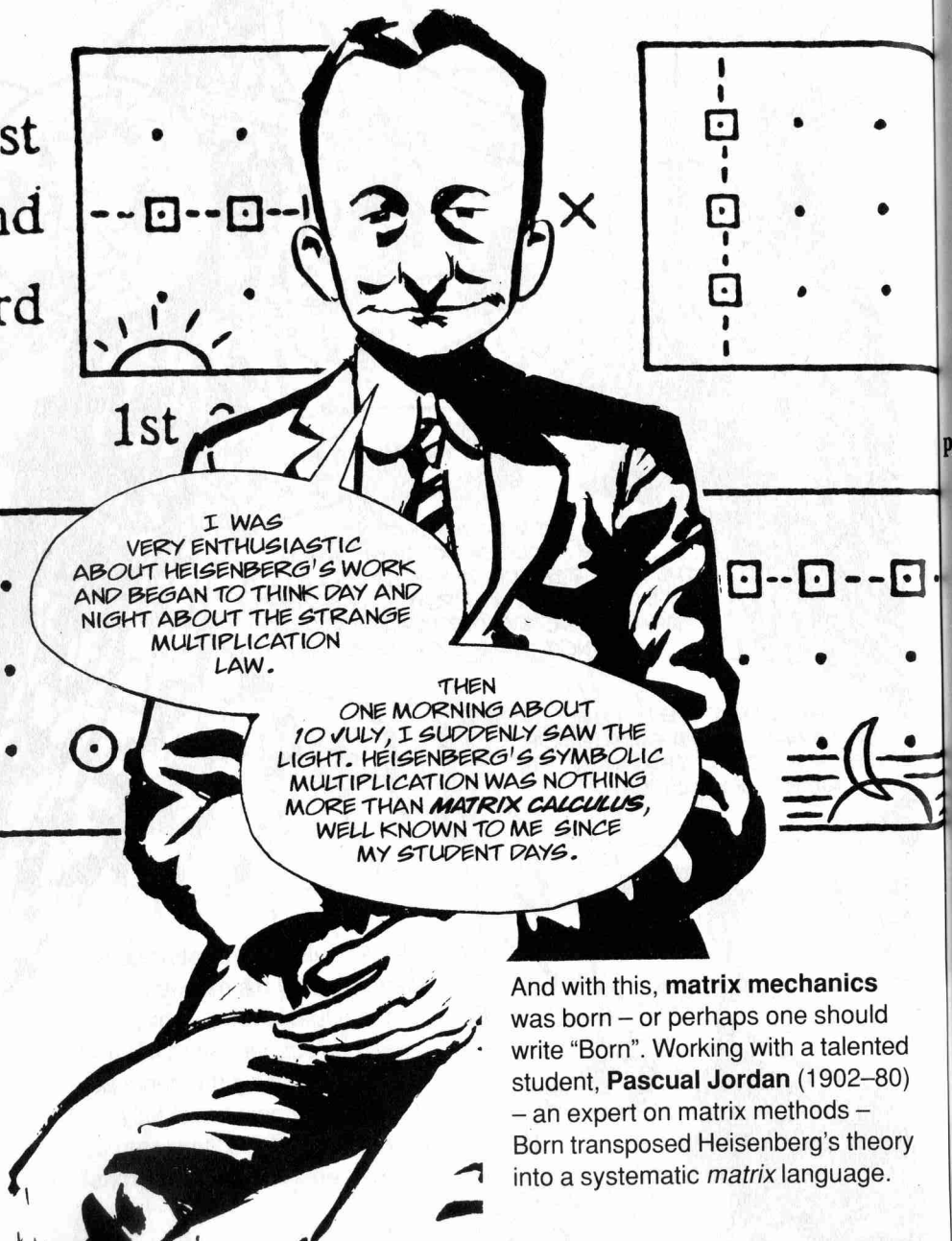
That very night on Helgoland, he was able to show that the energy states were **quantized** and **time independent**, i.e. they were *stationary* as in the Bohr atom. He later called this . . .



On 19 June, Heisenberg returned to Göttingen and sent his results to Pauli, the invaluable critic. If his theory was correct, he had taken a first step towards killing the concept of orbits. He was now almost fully recovered from *both* his illnesses . . . the hay fever **and** the electron orbits!

Max Born and Matrix Mechanics

Pauli's reaction was favourable. So before setting off for a visit to the Cavendish Laboratory in Cambridge and a walking holiday, Heisenberg set the paper before Max Born.



And with this, **matrix mechanics** was born – or perhaps one should write "Born". Working with a talented student, **Pascual Jordan** (1902–80) – an expert on matrix methods – Born transposed Heisenberg's theory into a systematic *matrix* language.

Now the frequencies of the optical spectrum could be represented by an infinite matrix which looks like this . . .

$$f_{m,n} = \begin{vmatrix} f_{11} & f_{12} & f_{13} & f_{14} & f_{15} & f_{16} & \text{etc.} \\ f_{21} & f_{22} & f_{23} & f_{24} & f_{25} & f_{26} & \text{etc.} \\ f_{31} & f_{32} & f_{33} & f_{34} & f_{35} & f_{36} & \text{etc.} \\ f_{41} & f_{42} & f_{43} & f_{44} & f_{45} & f_{46} & \text{etc.} \\ \text{etc.} & \text{etc.} & \text{etc.} & \text{etc.} & \text{etc.} & \text{etc.} & \text{etc.} \end{vmatrix}$$

Since Heisenberg's idea was that the individual oscillators with momentum **p(t)** and displacement **q(t)** vibrate with these frequencies, they will also be infinite matrices.

$$\mathbf{p} = \begin{vmatrix} p_{11} & p_{12} & p_{13} & p_{14} & \text{etc.} \\ p_{21} & p_{22} & p_{23} & p_{24} & \text{etc.} \\ p_{31} & p_{32} & p_{33} & p_{34} & \text{etc.} \\ \text{etc.} & \text{etc.} & \text{etc.} & \text{etc.} & \text{etc.} \end{vmatrix} \quad \text{and} \quad \mathbf{q} = \begin{vmatrix} q_{11} & q_{12} & q_{13} & q_{14} & \text{etc.} \\ q_{21} & q_{22} & q_{23} & q_{24} & \text{etc.} \\ q_{31} & q_{32} & q_{33} & q_{34} & \text{etc.} \\ \text{etc.} & \text{etc.} & \text{etc.} & \text{etc.} & \text{etc.} \end{vmatrix}$$

Heisenberg's quantum postulate was introduced to obtain the correct frequencies and intensities, each represented by a set of two numbers in *matrix* form.

pq – qp = (h/2πi) I (quantum condition)

I is the unit matrix which looks like this . . .

$$\mathbf{1} = \begin{vmatrix} 1 & 0 & 0 & \text{etc.} \\ 0 & 1 & 0 & \text{etc.} \\ 0 & 0 & 1 & \text{etc.} \\ \text{etc.} & \text{etc.} & \text{etc.} & \text{etc.} \end{vmatrix}$$