

ON THE USE OF GENERALIZED DERANGEMENTS FOR SCHELLING'S MODEL OF SEGREGATION

Rawan Ghnemat⁽¹⁾, Gérard H.E. Duchamp⁽²⁾, Cyrille Bertelle⁽¹⁾

⁽¹⁾LITIS - University of Le Havre, 25 rue Philippe Lebon, BP 540, 76058 Le Havre Cedex, France
email: rawan.ghnemat@litislab.eu, cyrille.bertelle@litislab.eu

⁽²⁾LIPN - University of Paris XIII, 99 avenue Jean-Baptiste Clément, 93430 Villetaneuse, France
email: ghed@lipn.univ-paris13.fr

KEYWORDS

self-organization, urban modelling, generalized derangement, Schelling

ABSTRACT

This paper proposes a definition of Schelling's model of segregation using generalized derangements. Many of urban or territorial modellings are based on decentralized approaches where rule-based systems have to be integrated inside a whole interaction system to describe complex phenomena. The goal of these decentralized modellings is to deal with emergent computing able to detect dynamically emergent organizations in an unsupervised way, thanks to complex systems theory. The convergence of these modern computings is generally hard to study because of the use of asynchronous processes dealing with a number of autonomous entities which are acting and interacting, in non linear way, during the whole simulation. Our approach is to define a non sequential-dependant algorithm, thanks to generalized derangements, and so to use this efficient tool to study some properties on the evolutive process.

INTRODUCTION

Urban or territorial systems are nowadays the topics of decentralized simulations which find roots on two majors advances in the development of scientific researches (Pumain 2006). The first one concerns the development of powerful distributed computer networks which allow to simulate in accurate way, detailed phenomena. The second one which finds major support from the first, is the development of decentralized approaches of modelling. This modern vision of modelling has been mainly enlightened by the complex systems theory (LeMoigne 1999) which allows to deal with the dynamic detection of emergent organizations using swarm optimization processes (Ghnemat et al. 2007), for example. The impressive current development of these simulations today starts with the development of Geographical Information Systems (GIS) which give accurate spatial data and

allow with specific platforms (Strout 2006) to link these spatial data with agent-based simulation and emergent computing.

We can observe that humans are more or less sorted into groups of similar people, according sometimes with racial and ethnic features but also in many others ways according to various characteristics. Thomas Schelling proposed in the 1970's a very simple rule-based model able to describe segregation phenomena (Schelling 1971). His description can easily be adapted to cellular automata models and it generates from random population, a spatial self-organization which is observed as patterns of clusters of similar persons. The cluster formation does not follow linear dependance from any parameter dealing with the evolution of the rule-based systems. More generally, Schelling model can be used as generic spatial self-organization concept for various kinds of systems: A. Singh and M. Haahr, for example, use a variant of this model to study the topology adaptation in P2P networks (Singh and Haahr 2006).

AN EXACT DEFINITION OF SCHELLING'S MODEL USING GENERALIZED DERANGEMENT

The original Schelling's model can be extended in concurrent version in order to obtain an exact definition of the implementation for each time step, not depending of way the grid is scanned.

In the following, we will make use of the (somewhat extended) notion of derangement. For the classical notion see (Stanley 1999):

<http://mathworld.wolfram.com/Derangement.html>
<http://en.wikipedia.org/wiki/Derangement>

Here it will be called *generalized derangement*.

Definition 0.1 *Let X, Y be two sets $X \subset Y$. A generalized derangement from X to Y is an into (*i. e.**

injective) mapping ¹ $\alpha : X \mapsto Y$ such that

$$(\forall x \in X)(\alpha(x) \neq x) \quad (1)$$

To describe Thomas Schelling’s concurrent model, we start with a cellular automata board which is a rectangle of $n \times m$ (n lines and m columns) points (each point will be located by its coordinates (x, y) with $1 \leq x \leq m$; $1 \leq y \leq n$). A *state* of the board will be simply a mapping $s : [1..m] \times [1..n] \mapsto \{0, A, B\}$ ² indicating whether a point at (x, y) has a value corresponding to

$$\left\{ \begin{array}{l} \bullet \text{ nothing } s(x, y) = 0 \\ \bullet \text{ an element of type } A, s(x, y) = A \\ \bullet \text{ an element of type } B, s(x, y) = B \end{array} \right. \quad (2)$$

the dynamics of the system will be described by a sequence of states $s_0, s_1, \dots, s_n, \dots$ generated by the following rules.

1. one fixes a threshold (in percent) $0 \leq t \leq 1$
2. the original state is s_0 (i.e. a distribution of A, B and empty cells along the board)
3. at each step, for each (filled) cell of type X at (x, y) , one counts the ratio $\rho(x, y)$ of neighbours of type X over the number of neighbours
4. if $\rho \geq t$ the cell is marked r (remain), if not it is marked m (move)
5. let M be the set of cells marked “move” and E the set of empty cells
6. choose randomly (uniform distribution) α among the generalized derangements $M \mapsto M \cup E$
7. then $s_{n+1}(x, y) = s_n(x, y)$ if the cell was marked r and $s_{n+1}(x, y) = \alpha(s_n(x, y))$ otherwise.

This algorithm is typically controlled by grid diffusion processus using elementary rule-based systems like most of the cellular automata models do. The rules leading to each move is based on local computation of neighbourhood, however the moves are global, holistic self-organizational. The following section proposes more sophisticated behavior modelled by agent systems.

RANDOM GENERATION FOR THE STUDY OF THE PROPERTIES OF THE DYNAMIC PROCESS

In order to make experiences for the description of emerging phenomena varying the threshold. We have to be able to draw “at random” a generalized derangement $M \mapsto M \cup E$ (step 6). This can be done very efficiently using a step by step random generation of the

images. But one must be aware that, given a fixed element of M , say m_0 the set of generalized derangements α such that $\alpha(m_0) \in E$ has not the same cardinality that the set of generalized derangements α such that $\alpha(m_0) \in M$. More precisely, if one denotes $gd(M, E)$ the set of generalized derangements $M \mapsto M \cup E$ and $gd(m, e)$ its cardinality ($m = |M|$ and $e = |E|$), one has

- $gd(0, e) = 1$
- $gd(m, 0) = d_m$, the number of classical derangements, given by the formula

$$\sum_{n=0}^{\infty} d_n \frac{z^n}{n!} = \exp\left(\sum_{n=2}^{\infty} \frac{z^n}{n!}\right) \quad (3)$$

- $gd(m, e + 1) = gd(m, e) + m gd(m - 1, e + 1)$

Moreover, the graphs of generalized derangements are of “exponential class” (Duchamp et al. 2003, Stanley 1999, Wilf 1994) and therefore, the multivariate statistics of them can be treated by the exponential formula (a formula invented by the Physicist G. E. Uhlenbeck, see (Goulden and Jackson 1983)). One can then harness elegantly the random generation by computing explicitly the branching quantities. The combinatorial study of these probabilities does not meet the purpose of this contribution and will be the subject of a forthcoming paper.

CONCLUSION AND PERSPECTIVES

This paper presents a definition of Schelling’s model using generalized derangements. This approach gives an efficient tool to study the properties of the system dynamic. Combinatorial studies and implementation of this segregation model description are in progress.

REFERENCES

- Benenson I. and Torrens P.M., 2004. *Geosimulation*. Wiley.
- Daudé E. and Langlois P., 2006. *Modélisation et simulation multi-agents*, Hermès Sciences, Lavoisier, chap. Comparaison de trois implémentations du modèle de Schelling. 411–442.
- Duchamp G.; Solomon A.; Penson K.; Horzela A.; and lasiak P.B., 2003. *One-parameter groups and combinatorial physics*. In M.N.H. J. Govaerts and A.Z.M. (eds.) (Eds.), *Proceedings of the Symposium Third International Workshop on Contemporary Problems in Mathematical Physics (COPROMAPH3)*. World Scientific Publishing 2004, Port-Nove, Benin, 436. ArXiv: quant-ph/04011262.
- Ghnemat R.; Bertelle C.; and Duchamp G.H., 2007. *Community Swarm Optimization*. In *workshop on*

¹see http://en.wikipedia.org/wiki/Injective_function

²We can extend the model using greater alphabet cardinal

Emergent Properties for Natural and Artificial Complex Systems, ECCS'07. Dresden, Germany.

Goulden I.P. and Jackson D.M., 1983. *Combinatorial Enumeration.* John Wiley & Sons Inc.

Laurie A.J. and Jaggi N.K., 2003. *Role of 'Vision' in neighborhood racial segregation: a variant of the Schelling segregation model.* *Urban Studies*, 40(14), 2687–2704.

LeMoigne J.L., 1999. *La modélisation des systèmes complexes.* Dunod.

Omer I., 2005. *How ethnicity influences residential distributions: an agent-based simulation.* *Environment and Planning B: Planning and Design*, 32, 657–672.

Pumain D. (Ed.), 2006. *Hierarchy in Natural and Social Sciences.* Springer.

Schelling T.C., 1971. *Dynamic Models of Segregation.* *Journal of Mathematical Sociology*, 1, 143–186.

Singh A. and Haahr M., 2006. *Creating an adaptive network of hubs using Schelling's model.* *Communication of the ACM*, 49(3), 69–73.

Stanley R., 1999. *Enumerative combinatorics.* Cambridge University Press.

Strout N., 2006. *What is Agent Analyst.* In *The 2006 ESRI International User Conference.* San Diego.

Weiss G. (Ed.), 1999. *Multiagent Systems.* MIT Press.

Wilf H., 1994. *Generating Functionology.* Academic Press, New York.