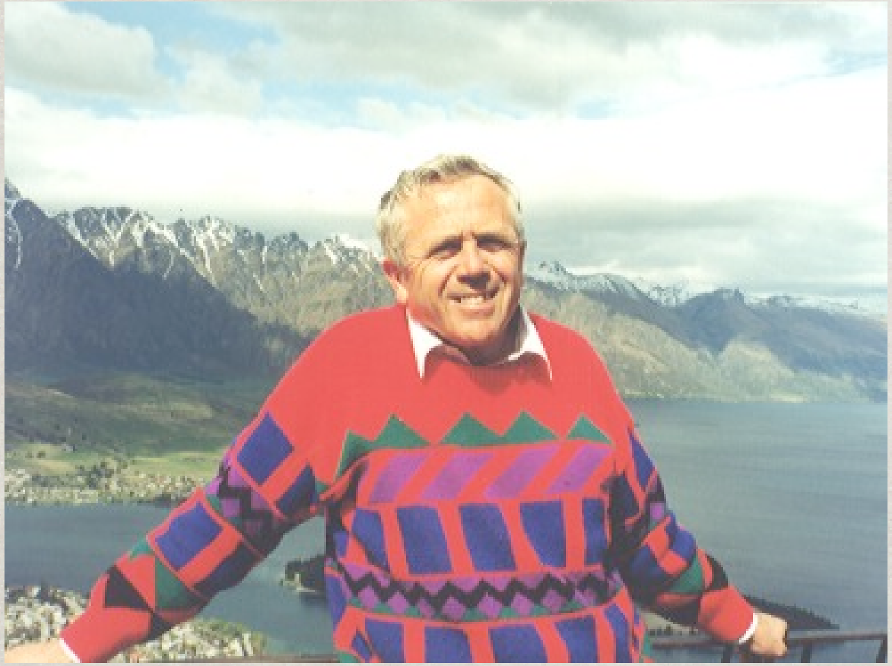

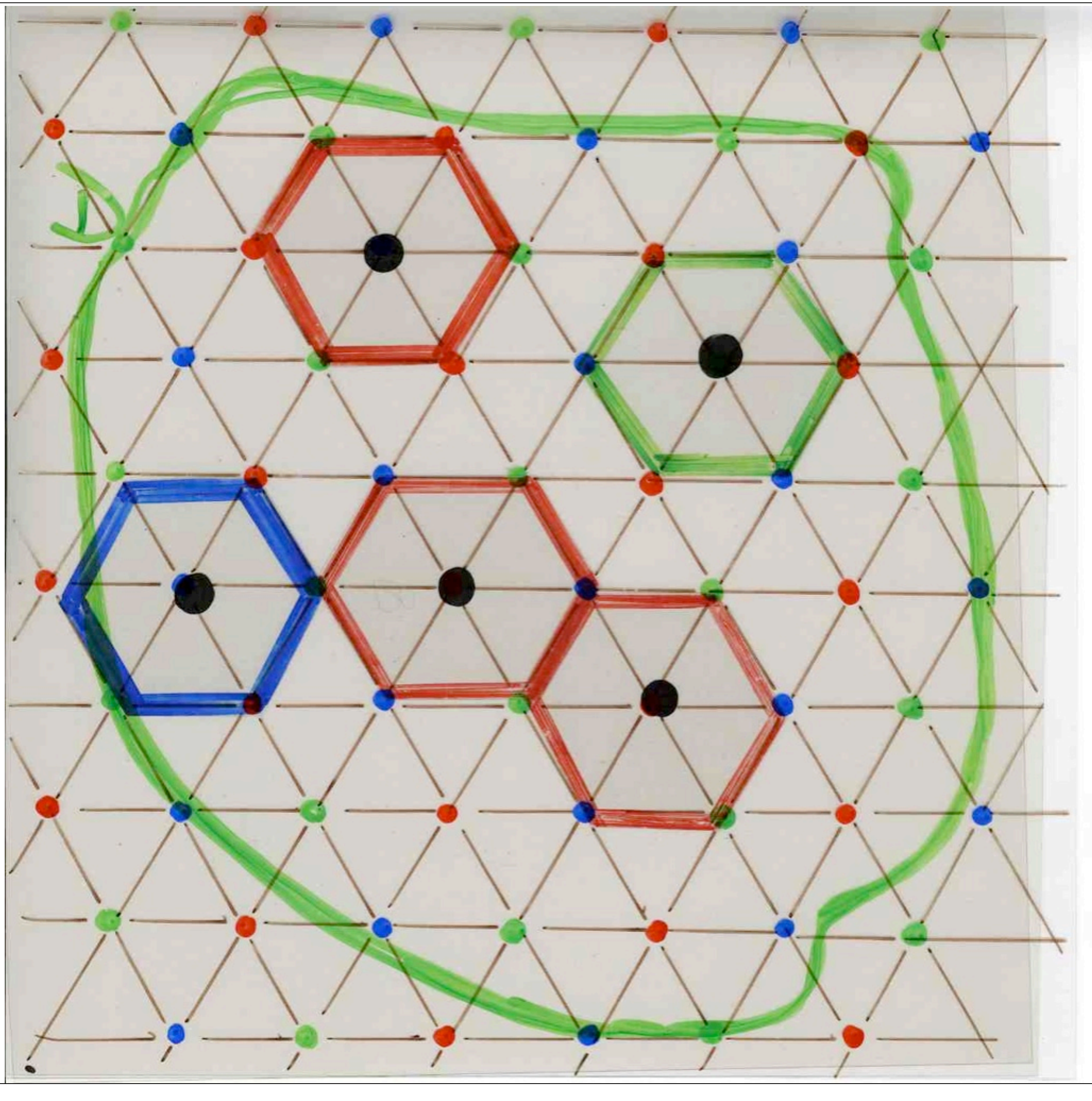

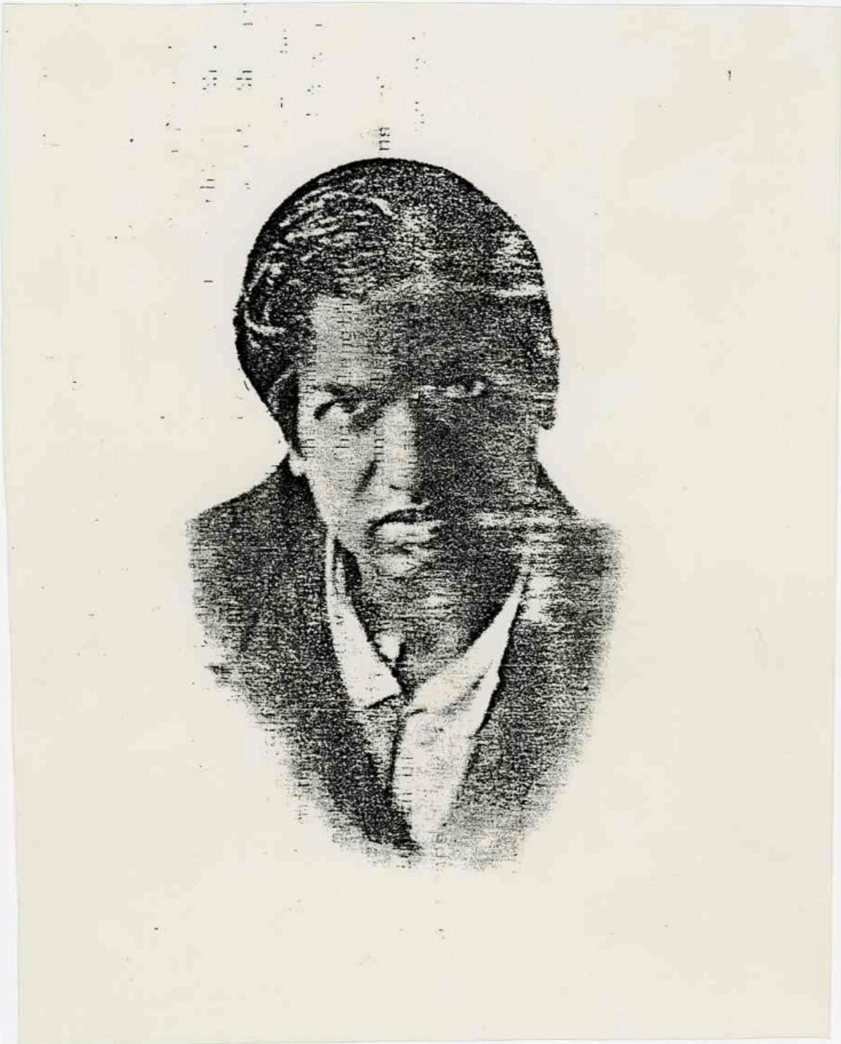

partition function

$$Z_D(t) = \sum_{n \geqslant 0} a_{n,D} t^n$$

$$Z(t) = \lim_{\text{"}D \to \infty\text{"}} \left(Z_D(t) \right)^{1/|D|}$$

thermodynamic limit

Rogers - Ramanujan identities

$$R_I \qquad \sum_{n\geq 0} \frac{q^{n^2}}{(1-q)(1-q^2)\cdots(1-q^n)} = \prod_{\substack{i\equiv 1,4 \\ \bmod 5}} \frac{1}{(1-q^i)}$$

$$R_{II} \qquad \sum_{n\geq 0} \frac{q^{n^2+n}}{(1-q)(1-q^2)\cdots(1-q^n)} = \prod_{\substack{i\equiv 2,3 \\ \bmod 5}} \frac{1}{(1-q^i)}$$

"La fraction continue" de Ramanujan

$$\cfrac{1}{1+\cfrac{q}{1+\cfrac{q^2}{1+\cfrac{q^3}{\cfrac{\cdots\cdots}{1+\cfrac{q^k}{\cdots\cdots}}}}}} = \frac{\displaystyle\sum_{n\geq 0} \frac{q^{n^2+n}}{(1-q)(1-q^2)\cdots(1-q^n)}}{\displaystyle\sum_{n\geq 0} \frac{q^{n^2}}{(1-q)(1-q^2)\cdots(1-q^n)}}$$

$$R(q) = \prod_{n \geqslant 0} \frac{(1-q^{5n+1})(1-q^{5n+4})}{(1-q^{5n+3})(1-q^{5n+2})} = \frac{R_{II}}{R_{I}}$$

$$R(q) = \prod_{n\geq 0} \frac{(1-q^{5n+1})(1-q^{5n+4})}{(1-q^{5n+3})(1-q^{5n+2})} = \frac{R_{II}}{R_{I}}$$

$$t = -q\left[R(q)\right]^5$$

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$$t = -q\left[R(q)\right]^5$$

$$Y(q) = \prod_{n \geq 0} \frac{(1-q^{6n+2})(1-q^{6n+3})^2(1-q^{6n+4})(1-q^{5n+1})^2(1-q^{5n+4})^2(1-q^{5n})^2}{(1-q^{6n+1})(1-q^{6n+5})\,(1-q^{6n})^2\,(1-q^{5n+2})^3(1-q^{5n+3})^3}$$

$$R(q) = \prod_{n\geq 0} \frac{(1-q^{5n+1})(1-q^{5n+4})}{(1-q^{5n+3})(1-q^{5n+2})} = \frac{R_{II}}{R_{I}}$$

$$t = -q\left[R(q)\right]^5$$

$$Y(q) = \prod_{n\geq 0} \frac{(1-q^{6n+2})(1-q^{6n+3})^2(1-q^{6n+4})(1-q^{5n+1})^2(1-q^{5n+4})^2(1-q^{5n})^2}{(1-q^{6n+1})(1-q^{6n+5})(1-q^{6n})^2(1-q^{5n+2})^3(1-q^{5n+3})^3}$$

$$Z(t) = Y(q(t))$$

$$\rho(t) = t \frac{d}{dt} \log Z(t)$$

density of a "hard-core" lattice gas model

t is the "activity" of the gas

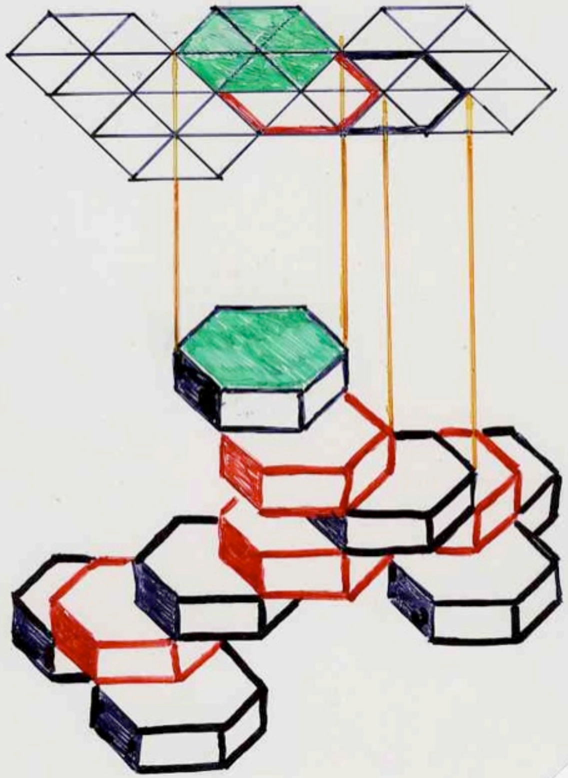
$$y\left(1+14t+97t^{2}+415t^{3}+1180t^{4}+2321t^{5}+3247t^{6}+3300t^{7}+2475t^{8}+1375t^{9}+550t^{10}+143t^{11}+18t^{12}\right)+$$

$$y^{3}\left(3+50t+381t^{2}+1715t^{3}+5040t^{4}+10130t^{5}+14062t^{6}+13002t^{7}+6930t^{8}+715t^{9}-1595t^{10}-988t^{11}-198t^{12}\right)+$$

$$y^{4}\left(1+17t+131t^{2}+595t^{3}+1765t^{4}+3574t^{5}+4939t^{6}+4356t^{7}+1815t^{8}-605t^{9}-1210t^{10}-616t^{11}-126t^{12}\right)$$

$$=\left(t+11t^{2}+55t^{33}+165t^{4}+330t^{5}+462t^{6}+462t^{7}+330t^{8}+165t^{9}+55t^{10}+11t^{11}+t^{12}\right)$$

$$-p(-t) = y$$



- critical temperature

$$T_c = \frac{11 + 5\sqrt{5}}{2}$$

- critical exponent $\frac{5}{6}$

$$= \left(\frac{1+\sqrt{5}}{2}\right)^5$$