COMPLEX SYSTEMS DYNAMICS IN AN ECONOMIC MODEL WITH MEAN FIELD INTERACTIONS

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ABSTRACT

In this paper the simulated dynamics of a simple agent based economic system are analyzed. These dynamics are of the complex systems type in the sense that the degree of self-organization changes with time. Indeed the attractor of the macroscopic dynamics changes with time from a cyclic situation to a stable situation and then back again to a cyclic one without changes in the parameters.

INTRODUCTION

One of the most important results of neoclassical Economics, the General Equilibrium Theory, relies on the existence of a coordination mechanism introduced using the elegant device of the walrasian auctioneer. This is probably a provoking sentence, but it opens an important debate in the economic profession: is the presence of coordination mechanisms a good approximation of the economic reality? A positive answer to this question would prevent studies in economics to enter roads already opened for other disciplines like those of the self-organization phenomena and complexity theory. Fortunately, in recent years, a small but growing number of economists became convinced that the economy is a complex system (see Anderson et al. (1988); Arthur et al. (1997); Blume and Durlauf (2005) for example) and therefore started to travel these roads.

Before preparing for the trip proposed in this paper (of course traveling the road we are talking about) some preliminary comments and definitions are useful. In our view, a complex system is composed of a high number of different elements that are interacting in some way, but not through a coordination device. Complex systems are interesting because under certain conditions they give rise to "unusual" dynamics. In particular it is possible that while one of the parameters changes smoothly, the behavior of some endogenous macroscopic variable changes in an unexpectedly organized way or, putting it another way, structures form in an unstructured environment. When such structures emerge without a coordination device researchers generally say that the system "self-organizes". Sometimes, the expression self-organization is used to denote the emergence of structures in the phase space of dynamical systems. Indeed systems composed of a low number of difference or differential equations can display more structured attractors on the phase space when a parameter is gradually moved. This is not the way the expression self-organization is used in this paper. Dynamical systems are intractable when their dimension increases and consequently they cannot be classified as complex systems (recall that according to our definition a complex systems has a very high dimension). Talking about dynamical systems a confusion may arise because a dynamical system (that, from our point of view, is not complex) can display chaotic dynamics that are usually referred to as "complex dynamics". Thus, despite the similarity of the expressions, in what follows, "complex systems dynamics" has a different meaning from "complex dynamics". In particular the latter are a subset of the former at least as long as one identifies chaotic dynamics with the complex dynamics. More interestingly complex systems may exhibit dynamics never detected in dynamical systems. In cellular automata systems, for instance, the existence of such a type of dynamics was found by Wolfram (1986), Langton (1986), and Packard (1988). The last one coined the expression "the edge of chaos" to identify them. We will refer to this type of dynamics as "complex systems dynamics".

The aim of this paper is to show how the economic system can give rise to "complex systems dynamics." The model presented below belongs to a set born out of a paper by Greenwald and Stiglitz (1993) (GS hereafter). The intent of these works is to show how the financial conditions of firms is a determinant of the aggregate production of countries, that is, of the Gross Domestic Product (GDP). It is well known that GDP has cyclical dynamics (indeed the explanation of this phenomenon is one of the main topics of macroeconomic theory) and, from a dynamical systems point of view, this calls for the presence of a limit cycle in some relevant variable. GS obtain a difference equation for the financial condition of firms (represented by the equity base) that, under certain parameterization, gives rise to limit cycles and to chaotic dynamics. From our point of view, GS's work has the serious inconveniece that the millions heterogeneous firms populating the economy are replaced by one of them that is supposed to be representative. This way to proceed is questionable because, among other drawbacks (see Kirman (1992) for example), it limits the analysis to the use of dynamical systems tools that, as maintained above, shuts out complex systems dynamics. The same criticism applies to a paper by Delli Gatti et al. (2000). Building on GS they go a step further recognizing the importance of heterogeneity, but they take it into account introducing a difference equation for the variance of the financial position ending up with the analysis of a two dimensional dynamical system.

In more recent times, GS's type of model have been analyzed using agent based simulation techniques that is, according to us, a more convenient way to deal with complex systems. There are no equations for the macroscopic variables, but only equations governing the individuals' behavior. The values of the macroscopic variables are recovered by simply summing or averaging the individual' ones. Consequently it may happen that the dynamics at the macroscopic level are completely different from those at the individual level identifying genuine emergent phenomena. Delli Gatti et al. (2005) for example show how one can recover particular statistical distributions (basically they are fat tailed distributions like power laws or Weibull) out of the individual data or from the aggregate time series obtained from simulations, and that the same distributions characterize real data. The important observation is that according to a number of scientists the presence of these distributions is common in complex systems dynamics (see Bak (1997) for example).

In what follows we build a model using some "ingredients" from the above cited papers. We then report some simulation results showing how the model produces peculiar dynamics that could be defined as "complex systems dynamics".

THE MODEL

The economy is populated by a large number of firms. As in the GS's type of model we concentrate our attention on the firm. Consumers and others economic agents are supposed to passively accommodate firms' decisions. In these supply side models the production function has a very important role (a large part of the macroeconomic theory is of the supply side type, think for example of the exogenous and endogenous growth and of the real business cycle theories). In the present model, the production function is linear:

$$Y_{it} = \nu_{it} K_{it}$$

where Y_{it} is the production, K_{it} the capital and ν_{it} its productivity.

We dedicate the remainder of this section to the two determinants of the production: ν and K. As mentioned before, we are interested in the emergent properties of the aggregate production dynamics $Y_t = \sum_i Y_{it}$ that we'll recover using a bottom-up approach, that is, through agent based simulations.

Some preliminary notions on the firms variables will be useful and are given here. The balance sheet of a firm is $K_{it} = D_{it} + A_{it}$. where D_{it} is debt and A_{it} the equity base. The fraction $\frac{A_{it}}{K_{it}} = a_{it}$ is the equity ratio that is a signal of the financial soundness of the firm. The dynamics of the balance sheet variables are strictly relative to the economic result of the firm (π_{it}). In these preliminary notions, we restrict ourselves to note that this variable directly affects the dynamics of the equity base in the following way: $A_{it} = A_{it-1} + (1 - \eta_{it})\pi_{it}$ where η_{it} is the fraction of the economic result that does not affect the equity base (more detailed explanation below). The important aspect is that the economic result can be negative and this decreases the equity base. As a consequence, the equity base of a firm could become negative and, if this happens, the firm must leave the market. Another exit mechanism will be considered and we'll come back to this issue below, but what is important to note here is that the presence of exit mechanisms calls for the existence of an entry process. These considerations serve to highlight that an important aspect of this kind of model is the firms turnover (Delli Gatti et al., 2003). However, in this paper we avoid such complication adopting the one-to-one replacement assumption (each exiting firm is replaced by a new one).

Now we can look at the model description starting from the economic result of the firm. In the following steps we use the economic result to determine the dynamics of the two variables we are interested in: the capital (K_{it}) and the productivity (ν_{it}).

Economic result

The economic result (π_{it}) is given by revenues (R_{it}) minus costs (C_{it}):

$$\pi_{it} = R_{it} - C_{it} \tag{1}$$

All the variables are in real terms so that prices will never appear in our equations.

Revenues. Firms sell all their product, but their real revenue from sales may be different from the production due to unforeseen external events (in GS for example this is due to an unknown selling price). We formulate this as follows

$$R_{it} = \nu_{it} K_{it} + u_{it} K_{it} \tag{2}$$

where u is a random variable with mean equal to 0 and finite variance.

Costs. Costs are of two types: production costs (C^L) and adjustment costs (C^K)

$$C_{it} = C^L_{it} + C^K_{it} \tag{3}$$

Production costs. Production costs are due to labor. We use the simplifying assumption that firms need one worker for each unit of capital (Leontief type production function) so that $L_{it} = K_{it}$. Labor costs are

$$C^L_{it} = w_{it} L_{it} = w_{it} K_{it} \tag{4}$$

where w_{it} is the wage and L_{it} the number of employed workers.

Adjustment costs. The adjustment costs must be sustained to adapt the stock of capital (Mussa, 1977). We use here the formulation adopted in Delli Gatti et al. (2000)

$$C_{it}^{K} = \frac{\gamma}{2}\frac{(K_{it} - K_{it-1})^2}{\bar{K}_{t-1}} \quad (5)$$

where $\bar{K}$ is the average capital of the economy. This introduces a first mean field interaction in the model.

The economic result. using equations (1)-(5) the economic result is

$$\pi_{it} = \nu_{it}K_{it} - w_{it}K_{it} - \frac{\gamma}{2}\frac{(K_{it} - K_{it-1})^2}{\bar{K}_{t-1}} + u_{it}K_{it}$$

note that because of the Leontievian assumption, ν_{it} is also the labor productivity. It is natural to think that the wage is related to the (latest known) average labor productivity. From this base we use the following assumption: $w_{it} = \bar{\nu}_{t-1}$ that introduces a second mean field interaction being $\bar{\nu}_{t-1}$ the average productivity of the period before. Under this assumption we can write

$$\pi_{it} = (\nu_{it} - \bar{\nu}_{t-1})K_{it} - \frac{\gamma}{2}\frac{(K_{it} - K_{it-1})^2}{\bar{K}_{t-1}} + u_{it}K_{it} \quad (6)$$

In order to simplify, we avoid discussing the effects of changing the capital level on the economic result. A discussion of these aspects would reveal that the investment involves the movement of the debt stock and affect minimally the economic result. This effect does not modify the behavior of the system and can be eliminated under a further simplifying assumption.

The evolution of Capital

To choose the optimal level of capital the firm maximizes the economic result function, but with two changes. First of all, at the time of the choice, firms don't know the realization of the random variable so that it is replaced with the average value. This allows us to omit the term $u_{it}K_{it}$ being the mean of u equal to zero. Secondly we assume that firms don't know also the average capital and replace it with their own level of capital. Consequently, the objective function to maximize is

$$\pi_{it} = (\nu_{it} - \bar{\nu}_{t-1})K_{it} - \frac{\gamma}{2}\frac{(K_{it} - K_{it-1})^2}{K_{it-1}}$$

Maximizing with respect to K_{it} we have the first element we need, that is, the dynamics of the capital

$$K_{it} = \frac{\nu_{it} - \bar{\nu}_{t-1}}{\gamma}K_{it-1} + K_{it-1} \quad (7)$$

The dynamics of the productivity

The second element we need is the dynamics of the productivity ν_{it}. This variable moves if the firm funds Research and Development activities.

Investment in Research and Development activities. At the end of the period the economic result is realized. It can be positive (profit) or negative (loss).

When a profit is realized the firm has to decide how to use it. We assume that it can be used other than to increase the equity base, to finance Research and Development (R&D) activities. In particular R&D investments are assumed to be

$$R\&D_{it} = \pi_{it}\eta_{it}$$

where η_{it} is the share of profit dedicated to R&D. We assume that this share is an increasing function of the financial soundness of the firm represented by the equity ratio as follows:

$$\eta_{it} = \begin{cases} a_{it-1} & \text{if } \pi_{it} > 0 \\ 0 & \text{if } \pi_{it} \leq 0 \end{cases}$$

From these considerations we can also recover the dynamics of the equity base:

$$A_{it} = A_{it-1} + (1 - \eta_{it})\pi_{it} \quad (8)$$

The dynamics of the productivity. The outcome of the R&D investment is stochastic and the probability of success increases with the amount of funds dedicated to these activities. We formulate this probability as

$$pr = \frac{1}{1 + e^{-b(R\&D_{it} - c)}}$$

where b and c are parameters.

If a firm obtains a success from its R&D activities, its productivity increases by a constant amount β, so that the dynamic of the productivity is

$$\begin{cases} \nu_{it+1} = \nu_{it} + \beta & \text{with probability } pr \\ \nu_{it+1} = \nu_{it} & \text{with probability } 1 - pr \end{cases} \quad (9)$$

SIMULATIONS

We simulate the model using object oriented programming languages. In a first implementation the objective-C version of the SWARM library is used. The validity of the results is checked coding a second time the same model using the RePast java library. We run a large number of experiments to check how the model reacts to changes in the initial conditions, the size of the system (that is the number of firms) and the parameters. Among them, the parameter γ has a very important role. For high values of this parameter the system displays a limit cycle, while cycles disappear for low values. In between, there is a non negligible region where the system gives rise to complex systems dynamics.

We describe here in details one of these experiments where we set $\gamma = 1.5$. The comments below serve also to better explain how the model works. At the beginning the code creates 100000 identical firms giving them the following initial conditions: $K_{i0} = 100$, $A_{i0} = 30$, $\nu_{i0} = 0.1$. The parameters b, c and β are set to 3, 2 and 0.01, respectively.
The algorithm goes through the following steps:

```
1 reset values to firms that meet
                      the exit conditions
2 update the capital
3 update equity ratio
4 update the profit using the random
                                 variable
5 update investments in R&D
6 update productivity
7 update equity base
8 collect data
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Because some steps are technical, we discuss the flow of events in a logical order, this implies that the order reported above will not be respected on some occasions. First of all firms decide their new capital level (step 2) using equation (7): firms with a productivity higher than the average increase capital while the others reduce it. Firms employ the new stock of capital in the production and realize a production equal to $\nu_{it} K_{it}$. Once the production is realized it is sold on the market. The average revenues from sales is equal to production, but some firms realize a higher revenue and some others a lower one due to contingent situations represented by the random variable u that is supposed to be uniform with bounds -0.1 and 0.1. Now, with revenues in their hands the entrepreneurs have to pay their costs: wages and adjustment costs. Here two situations are possible. In the first one, the revenue from sales is higher than costs and consequently a profit is realized. In the second, the revenue from sales is lower than costs and the firm suffers a loss. This is the content of equation (6) implemented in step 4. In step 5 firms with a profit spend a share equal to their equity ratio (that is calculated in step 3) of profit in R&D, while firms that suffer a loss make no expenditures in R&D. After this computation we know for each firm how much they spend in R&D. This allows us to update the productivity using equation (9) in step 6. Moreover, knowing R&D expenditures allows us to determine how the economic result affects the equity base. We do this coding equation (8) in step 7. Finally we record data (step 8) and a new iteration is about to start. At the beginning of the new iteration (step 1), we check the value of the equity base computed in step 7. If it is negative the variables of the firm are initialized with the following values $K_{it} = K_{i0} = 100$, $A_{it} = A_{i0} = 30$, $\nu_{it} = \bar{\nu}_{t-1}$. This can happen to firms that suffer a loss in step 4 of the previous iteration. The fact that they suffer a loss means that they are not able to cover costs with the revenues from sales. At this point they must resort to their internal funds represented by the equity base. In some cases even the equity base is not sufficient to provide the additional needed funds and the firm must exit the market. In addition to this exit mechanism we add also a threshold to the size of the firm, that is, firms with a low level of capital ($K_{it-1} < 20$) but with a positive equity base are also replaced. However, this second exit mechanism is present to catch exceptions and does not affect the simulation results. Finally, note that resetting the variables of the firms when they meet these conditions is the same as assumuming a one-to-one replacement situation and the number of firms is constant to 100000 during the whole simulation.
The results are showed in the following graphs and commented on in next section.

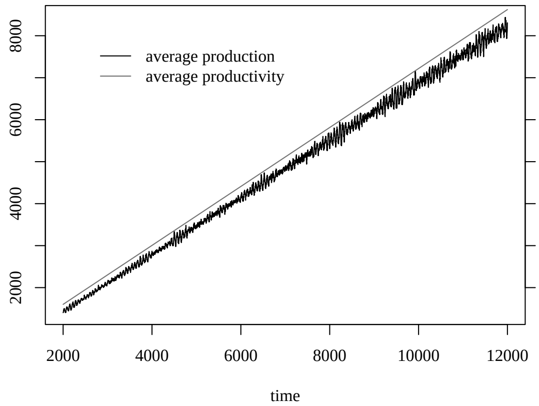

Figure 1: Average production (left axis and black line) and average productivity (right axis and gray line). 2000 time steps have been discarded to eliminate the transient state

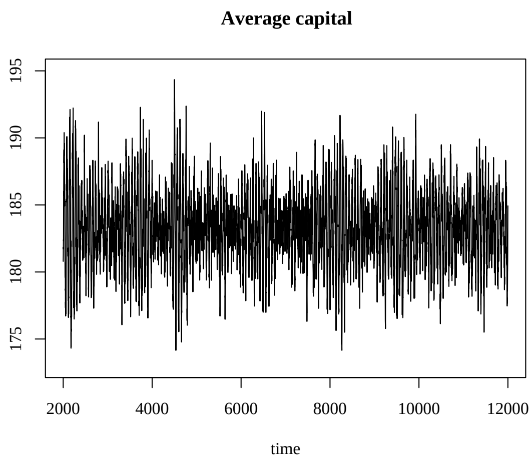

Figure 2: Average Capital. 2000 time steps have been discarded to eliminate the transient state

DISCUSSION

Although the reported graphs could contain interesting features from the economic point of view, the focus of the discussion will be mainly on the type of dynamics a system like this can generate.

Figures 1 and 2 show the dynamics of the variables involved in the production function (production, capital and productivity) for 10000 simulation time steps. We don't show the initial 2000 time steps to avoid the transient state (it occurs in decrising oscillations). In the graphs the average values are reported. Regarding production and capital, one might be interested in the aggregate values. They can be obtained by multiplying those reported in the graphs for the number of firms in the economy that in our case is fixed and equal to 100000. Consequently, the qualitative behaviors of the aggregate production and capital are exactly the same as those reported in the graphs. First of all, from figure 1 it is evident that the increasing trend in production is due to the increasing productivity. Secondly, it is also evident that production is much more volatile than productivity. Having the production function in mind it is straightforward that the volatility of production depends on that of capital; figure 2 confirms this deduction. It is also easy to see that the volatility is not constant, but changes with time in an irregular way. This pushes us toward a more accurate investigation. Figures 1 and 2 report too much data to get an insight into the nature of the volatility by visual inspection.

Figures 3, 4 and 5 shed some light on the phenomenon. In these graphs three contiguous sub-periods with different volatilities are shown.

Figure 3 shows that, in the time span 7750-8750, we are not dealing with stochastic volatility but with a more structured behavior: limit cycles. This is surprising because it is the result of an agent based model with idiosyncratic shocks. As discussed above, obtaining a limit cycle in a dynamical system in not hard, but dynamical systems contemplate the presence of a very low number of equations. In the present model the cyclical behavior is obtained averaging a large number of stochastic equations (one for each firm). From a probability theory point of view, what is reported in the figure is an average of a large number of identical stochastic processes. Figure 3 suggests that the law of large numbers, according to which one expects a very smooth behavior of the average value, does not hold at list in the reported periods. Furthermore, we cannot maintain that this is a feature of the individual behavior preserved at the aggregate level. The uncorrelated idiosyncratic shock present in the model differentiates firms' decisions and, from this point of view, the law of large numbers should apply. A cyclical behavior of the average requires that the various components of the system act in a strong correlated way that, in the absence of a representative agent, could be possible if a coordination mechanism were contemplated. But here we have no coordination device, here each entrepreneur decides alone using its private information (capital and productivity) and the average level of the productivity. Our final conjecture is that in an agent based model the presence of a replacement process and that of mean field interactions (Aoki, 1996) can give rise to a considerable degree of self-organization.

A second observation comes from comparing Figures 3, 4,

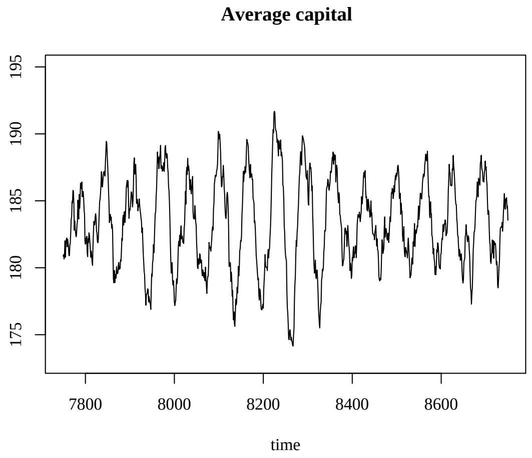

Figure 3: Average Capital from time 7750 to 8750

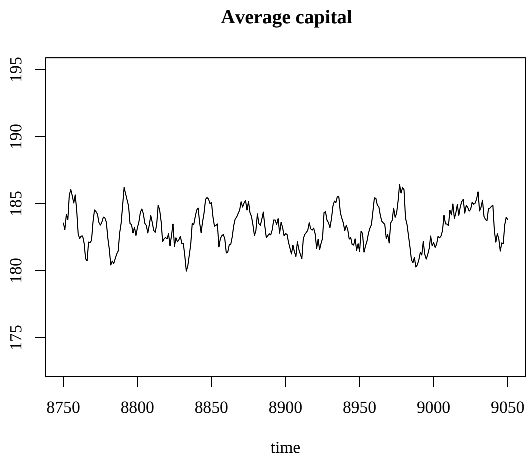

Figure 4: Average Capital from time 8751 to 9050

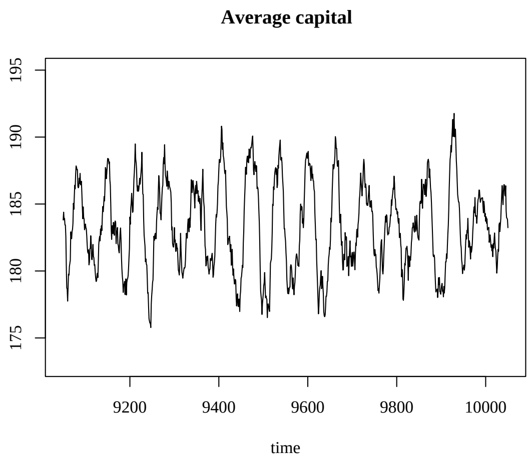

Figure 5: Average Capital from time 9051 to 10050

and 5: the system seems to be able to change attractor in time. The dynamic presented in Figure 4 is quite different from the ones visible in Figures 3 and 5 although they were obtained with the same parameters. In Figure 4 the law of large numbers seems to have a stronger effect than in the other two graphs, that is, the degree of self-organization changes with time. At the actual state of the investigation this phenomenon seems to be a deep emergent property of the system. Indeed one candidate for the explanation could be the average productivity (because it is not fluctuating around a constant value), but looking at its smooth behavior, it seems hard to give it the responsibility to change the system behavior from a limit cycle to (something similar to) an equilibrium and then back to a limit cycle.

The changes in the attractor are also showed in Figure 6 where average values for very short time spans (they are subperiods of Figure 3 and 4) are showed in the equity ratio-capital phase space. It is evident how the economic dynamics can commute between simple (as in the time span 8850-8920) to more structured (as in the time span 8220-8290) attractors. This Figure is also interesting from the economic point of view. Indeed, as discussed in the introduction, GS's type models prove that there is a relationship between the aggregate production and the financial soundness of the economy. Indeed the Figure shows that this relationship exists and is strong in some time spans. Furthermore, looking at the black line in Figure 6, it could be maintained that production and financial fragility move as described by Minsky (1982) in his financial fragility theory of macroeconomic fluctuations. On the other hand, this behavior is not always so strong to be detected as the gray line of the Figure shows.

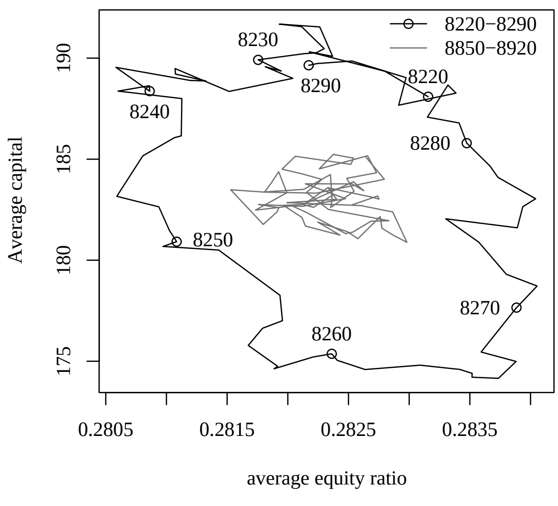

Figure 6: Different attractors in two different time spans

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BIOGRAPHY

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