Combinatorics and Arithmetic for Physics

IHÉS, 24-25 October 2018

1 Speakers

Speaker	Affiliation	Title
N. Behr	Paris VII	Operational Methods in the Study of Sobolev-Jacobi Polynomials
A. Bostan	INRIA	Transcendence in the enumeration of lattice walks
M. Bozejko	Wroclav	Positive definite functions on permutation (Coxeter) groups with applications to generalized CCR relations and operator spaces
P. Cartier	IHES	A Combinatorial Presentation of Various Galois Theories
G.H.E. Duchamp	P XIII + IHP	Combinatorics of characters, Schützenberger's calculus and continuation of Li
D. Grigoryev	CNRS-Lille 1	Tropical combinatorial Nullstellensatz and fewnomials
D. Gurevich	Valenciennes Univ.	Symmetric polynomials in quantum algebras
N. Iyudu	IHES	Sklyanin algebras via Groebner bases and finiteness conditions for potential algebras
R. Kerner	LPTMC	Quarks as a combinatorial puzzle: a new approach to quantum chromodynamics
G. Koshevoy	Poncelet Lab & HSE	Geometric Kashiwara star duality and DT transformations
H.N. Minh	Lille 2 & LIPN	A family of Eulerian functions involved in the regularization of divergent polyzetas
K.A. Penson	LPTMC, Paris 6	Integer ratios of factorials as Hausdorff moments versus algebraicity
P. Vanhove	CEA/Saclay	Mirror symmetry and Feynman integrals

2 Program

Time	Speaker	Activity	Remarks
Wednesday 24/10			
09:30-10:00	-	Accueil	
10:00-10:50	Kerner	Talk	
10:50-11:10	-	Pause	
11:10-12:00	Gurevich	Talk	
12:00-12h50	Bostan	Talk	
12:50-14:30	-	Lunch/Free time	
14:30-15:20	Bozejko	Talk	
15:20-15:40	-	Pause	
15:40-16h30	Penson	Talk	
16:30-17:20	Behr	Talk	
Thursday 25/10			
09:30-10:00	-	Accueil	
10:00-10:50	Koshevoy	Talk	
10:50-11:10	-	Pause	
11:10-12:00	Duchamp	Talk	
12:00-12h50	Minh	Talk	
12:50-14:00	-	Lunch/Free time	
14:00-14:50	Vanhove	Talk	
14:50-15h40	Grigoryev	Talk	
15:40-16:00	-	Pause	
16:00-16h50	Iyudu	Talk	
16:50-17:40	Cartier	Talk	

3 Abstracts

Nicolas Behr

Speaker: Nicolas Behr, IRIF Université Paris Diderot
Title: Operational Methods in the Study of Sobolev-Jacobi Polynomials
Abstract:
Inspired by ideas from umbral calculus and based on the two types of integrals occurring in the defining equations for the gamma and the reciprocal gamma functions, respectively, we develop a multi-variate version of the so-called umbral image technique. Besides providing a class of new formulae for generalized hypergeometric functions and an implementation of series manipulations for computing lacunary generating functions, our main application of these techniques is the study of Sobolev-Jacobi polynomials. Motivated by applications to theoretical chemistry, we moreover present a deep link between generalized normal-ordering techniques introduced by Gurappa and Panigrahi, two-variable Hermite polynomials and our integral-based series transforms.
This is joint work with G. Dattoli (ENEA Frascati), G.H.E. Duchamp (Paris 13), Silvia Licciardi (ENEA Frascati) and K.A. Penson (Paris 6).

Alin Bostan

Speaker:
Title: Transcendence in the enumeration of lattice walks
Abstract: Classifying lattice walks confined to the quarter plane is an important problem in enumerative combinatorics. A key role in the classification is played by an associated group of birational transformations, whose finiteness turns out to be equivalent to the differential finiteness of the corresponding generating function. We will give an overview of recent results on structural properties and explicit formulas for those generating functions, with an emphasis on their transcendence.

Marek Bozejko

Speaker: Marek Bozejko,
Title: Positive definite functions on permutation (Coxeter) groups with applications to generalized CCR-relations and operator spaces
Abstract:
In my talk we will consider the following topics:

1. Lengths functions $L_i, i = 1, 2$ related to numbers of inversions and connected components on the permutation(Coxeter) groups (W, S).

2. Positive definite functions of the Poisson type $P_i(x) = exp(-L_i(x)), x \in W$.

3. Generalized CCR relations related to the Weyl groups of type A, B, D and new type II factorial von Neumann algebras.

4. Riesz product on (W, S) and operator spaces of row and columns related to arbitrary Coxeter groups (W,S).

References:

1. M. Bozejko, S. Gal, W. Mlotkowski. Positive definite functions on Coxeter groups with applications to operator spaces and noncommutative probability, Comm. Math. Phys., 361 (2018), no. 2, 583604 .

2. Bozejko Marek, Lytvynov Eugene, Rodionova Irina. An extended anyon Fock space and noncommutative Meixner-type orthogonal polynomials in infinite dimensions. Russian Mathematical Surveys 70 :5,857-899, (2017).

3. M. Bozejko, Ejsmont W., Hasebe T.: Fock space associated to Coxeter groups of type B. J. Funct. Anal. 269(6), 17691795 (2015)

4. M. Bozejko, Ejsmont W., Hasebe T.: Noncommutative probability of type D. Int. J. Math. 28(2), 1750010, 30 (2017)

5. M. Bozejko, Januszkiewicz T., Spatzier R.J.: Infinite Coxeter groups do not have Kazhdan's property. J. Oper. Theory 19(1), 6367 (1988).

6. Belinschi S.T., Bozejko M., Lehner F., Speicher R.: The normal distribution is free infinitely divisible. Adv. Math. 226(4), 36773698 (2011).

Pierre Cartier

Speaker:
Title:
Abstract:

Gérard H. E. Duchamp

Speaker: Gérard H. E. Duchamp (IHP & Paris XIII, France)
Title: Combinatorics of characters, Schützenberger's calculus and continuation of Li
Abstract: We start from a new territory, that of (noncommutative) formal power series, to encode polylogarithms and harmonic sums. In this talk, we introduce the object(s), notations and calculus devoted to this very particular Sweedler's dual. In passing, we pay a small tribute to Marcel Paul Schützenberger.
References:

1. G. H. E. Duchamp and C. Tollu, *Sweedler's duals and Schtzenberger's calculus*, In K. Ebrahimi-Fard, M. Marcolli and W. van Suijlekom (eds), Combinatorics and Physics, p. 67 - 78, Amer. Math. Soc. (Contemporary Mathematics, vol. 539), 2011.
arXiv:0712.0125v3 [math.CO]

2. Ngo Quoc Hoan, G.H.E. Duchamp, V. Hoang Ngoc Minh, *Kleene stars of the plane, polylogarithms and symmetries*, to appear in TCS.
arXiv:1602.02801 [math.CO]

Dimitri Grigoryev

Speaker: Dimitri Grigoryev (Lille)
Title: Tropical combinatorial Nullstellensatz and fewnomials
Abstract:
We give tropical analogues of

1. the combinatorial Nullstellensatz due to N. Alon and of Risler-Ronga conjecture;
2. Schwartz-Zippel lemma providing an exact bound on the number of points in a finite grid at which a polynomial of a fixed degree can vanish;
3. a universal testing set for sparse polynomials (for classical polynomials constructed by Grigoriev-Karpinski and Ben-Or-Tiwari);
4. Shub-Smale τ-conjecture.

These results were obtained jointly with V. Podolskii.

Dmitry Gurevich

Speaker: Dmitry Gurevich (Valenciennes University)
Title: Symmetric polynomials in quantum algebras
Abstract:
I'll introduce some new quantum algebras which are called Generalized Yangians. Their definition is based on the notion of compatible R-matrices. In these algebras quantum analogs of some symmetric polynomials (elementary ones, power sums) are well-defined. These quantum symmetric polynomials generate commutative subalgebras called Bethe. Also, I plan to exhibit some quantum analogs of the classical identities (Cayley-Hamilton, Newton).

Natalia Iyudu

Speaker: Natalia Iyudu
Title: Sklyanin algebras via Groebner bases and finiteness conditions for potential algebras
Abstract:
I will discuss how some questions on Sklyanin algebras can be solved using combinatorial techniques, namely, the theory of Groebner bases (rewriting techniques in the ideals of associative algebras). We calculate the Poincaré series, prove Koszulity, PBW, Calabi-Yau, etc., depending on the parameters of the Sklyanin algebras. There was a gap in the Artin-Schelter classification of algebras of global dimension 3, where Koszulity and the Poincaré series for Sklyanin algebras were proved only generically. It was filled in the Grothendieck Festschrift paper of Artin, Tate and Van den Bergh,

using the geometry of elliptic curves. Our point is that we recover these results by purely algebraic, combinatorial means.
We use similar methods for other potential algebras as well, including homology of moduli of pointed curves given by Keel relations, and contraction algebras arising in noncommutative resolution of singularities.

Richard Kerner

Speaker: Richard Kerner (LPTMC)
Title: Quarks as a combinatorial puzzle: a new approach to quantum chromodynamics.
Abstract:
Quarks cannot propagate outside the nucleons or mesons, but inside they seem to behave at high energies as almost free particles. An alternative approach to color dynamics is proposed. It is based on the observation that the $Z_2 \times Z_3 = Z_6$ cyclic group generated by the sixth root of unity can be put into one-to-one correspondence with three colors and three anti-colors. If we identify 0 as "colorless", or 'white", there are two ternary and three binary combinations of roots yielding zero. Combining spin with color, and including particle-antiparticle symmetry, we arrive at 12-component objects, satisfying generalized Dirac equation whose solutions cannot propagate alone due to the complex wave vectors, but can form propagating combinations via ternary or binary products. Relativistic invariance realized via non-standard complex realizations of the Lorentz group is also discussed.

Gleb Koshevoy

Speaker: Gleb Koshevoy (Poncelet Lab., Moscow)
Title: Geometric Kashiwara star duality and DT transformations
Abstract:

Hoang Ngoc Minh

Speaker: Hoang Ngoc Minh (Lille II & LIPN)
Title: A family of Eulerian functions involved in regularization of divergent polyzetas
Abstract:

Karol Penson

Speaker: Karol Penson (LPTMC & Paris 6)
Title: Integer ratios of factorials as Hausdorff moments versus algebraicity
Abstract:
Consider two series of positive integers:

$$a = a_1, a_2, \ldots, a_K \text{ and } b = b_1, b_2, \ldots, b_K, b_{K+1},$$

with $\sum_{i=1}^{K} a_i = \sum_{i=1}^{K+1} b_i$; $K = 1, 2, ...$ We form the following ratios of factorials

$$u_n(a,b) = \frac{(a_1.n)!(a_2.n)!\ldots(a_K.n)!}{(b_1.n)!(b_2.n)!\ldots(b_{K+1}.n)!} \text{ for } n = 0, 1\ldots \quad (1)$$

It turns out that, for many choices of a and b, the ratios $u_n(a,b)$ in (1) are themselves integers. In these cases we conceive $u_n(a,b)$ as nth moments of the weight functions $W(a,b,x)$ in the Hausdorff moment problem $u_n(a,b) = \int_0^{R(a,b)} x^n.W(a,b,x)dx$, where $R(a,b)$ is the upper edge of the support $[0, R(a,b)]$. We solve exactly and explicitly the above Hausdorff moment problem via the inverse Mellin transform method thus providing the analytic forms of $R(a,b)$ as well as of $W(a,b,x)$ in terms of Meijer G-functions and generalized hypergeometric functions. We prove formally the positivity of the weights $W(a,b,x)$ which are all U-shaped and singular at both edges of the support; as such they are generalizations of the arcsin distributions. We discuss a potential link between the proven algebraicity of the ordinary generating functions of $u_n(a,b)$ and a possible algebraicity of corresponding weights $W(a,b,x)$.
(Joint work with G. H. E. Duchamp, N. Behr and G. Koshevoy)

Pierre Vanhove

Speaker: P. Vanhove (CEA/Saclay)
Title: Mirror symmetry and Feynman integrals
Abstract:
We will describe the connection between Feynman integrals and period integrals of motivic cohomology. We will detail the toric approach to the evaluation of the Feynman integral. We formulate the conjecture that the all loop sunset Feynman integral in two spacetime dimensions is given by the genus zero Gromow-Witten local prepotential. We prove this conjecture for the case of the two-loop graph. By considering the limiting mixed Hodge structure of the Batyrev dual A-model, we arrive at an expression for the two-loop sunset Feynman integral in terms of the local Gromov-Witten prepotential of the del Pezzo surface of degree 6.

4 Participants

- Hélène Airault (Univ. de Picardie, Lamfa, CNRS 7352)
- Daniel Barsky (LAGA, Paris XIII)
- Nicolas Behr (IRIF Université Paris Diderot)
- Joseph Bengeloun (LIPN, Paris XIII)
- Alin Bostan (INRIA Saclay)
- Marek Bozejko (University of Wroclaw)
- Pierre Cartier (IHES)
- Gérard H. E. Duchamp (IHP & Paris 13)
- Justine Falque (Orsay Paris XI)
- Nihar Gargava (EPFL, Lausanne)
- Dimitri Grigoryev (CNRS-Lille 1)
- Dmitry Gurevich (Université de Valenciennes)
- Natalia Iyudu (University of Edinburgh & IHES)
- Richard Kerner (LPTMC)
- Gleb Koshevoy (Poncelet Lab., Moscow)
- Christian Lavault (LIPN, Paris XIII)
- Jerzy Lukierski (LPTMC Paris VI)
- Jean-Marie Maillard (LPTMC Paris VI)
- Hoàng Ngoc Minh (Lille 2 & LIPN)
- Karol Penson (LPTMC & Paris 6)
- Alban Quadrat (Inria Lille - Nord Europe)
- Vincent Rivasseau (Orsay Paris XI)
- Nicolas Thiéry (Orsay Paris XI)
- Christophe Tollu (LIPN, Paris XIII)
- Pierre Vanhove (CEA/Saclay)