

## PUZZLE 1

# MATHEMATICAL ENCOUNTERS

ABSTRACT. We present some mathematical and combinatorial objects that can be encountered in real life. In the first section, we list a few unusual definitions and illustrative descriptions. In the second section, we give more well-known examples of the same objects. The third section remains a mystery.

### 1. DEFINITIONS

**Definition 1.** Un, dos, tres, quatre, cinc, sis, set,...

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**Definition 2.** Crucial advocate of a cause.

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**Definition 3.** Filmed non-human actors.

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**Definition 4.**



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**Definition 5.**

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**Definition 6.**

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**Definition 7.**

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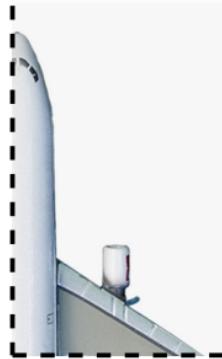
**Definition 8.** Autopilot services in a car, useful if you have a narrow garage.

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**Definition 9.** A plan or idea that will likely not work.

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**Definition 10.**



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**Definition 11.** Look on your face that shows how logical you are.

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**Definition 12.** Someone from Warsaw who is not very smart.

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**Definition 13.**



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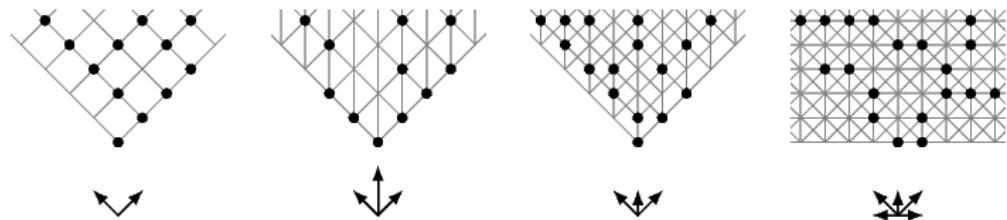
**Definition 14.** Personality of someone who is always completely optimistic.

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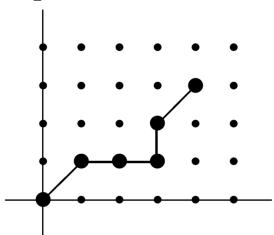
## 2. EXAMPLES

**Example A.**  $\{(x, y) \in \mathbb{Z}^2, x \geq 0 \text{ and } y \geq 0\}$

**Example B.**



**Example C.**



**Example D.**

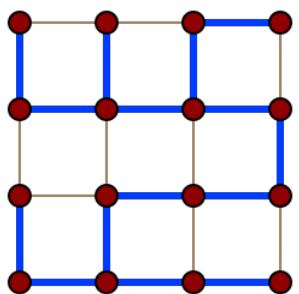
$n = 2 : 11 \quad 12 \quad 21$

$n = 3 : 111 \quad 112 \quad 121 \quad 211 \quad 113 \quad 131 \quad 311 \quad 122 \quad 212 \quad 221 \quad 123 \quad 132 \quad 213 \quad 231 \quad 312 \quad 321$

**Example E.**

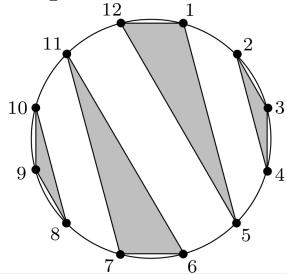
	1	2	3	4
1	+	+	+	/
2	/	+	/	/
3	/	/	/	/
4	/	/		

**Example F.**



**Example G.** 1, 1, 2, 5, 14, 42, ...

**Example H.**



**Example I.** For  $p(z) = \frac{z+2}{(8-z)(5-z)^2}$ , 8 is the only one.

**Example J.** Property of a sequence that satisfies  $a_i^2 \geq a_{i-1}a_{i+1}$  for all  $i$ .

**Example K.** A property of a matrix whose minors are all nonnegative.

$$\text{Example L. } \frac{x^4 - 2x^3 + x^2 + 1}{x^2 + 7x - 28}$$

**Example M.** For generalized ballot sequences in two dimensions, its value is always  $-3/2$ .

**Example N.**

8	7	5	2	1
5	4	2		
4	3	1		
2	1			

### 3. CONCLUSION

The study of the matrix  $M$  below is left to the reader as a trivial exercise.

$$M = \begin{pmatrix} n & o & w & m & a & y & b & e & t & r & y & t & o & w \\ o & r & k & o & u & t & t & h & e & d & e & f & i & n \\ i & t & i & o & n & s & a & n & d & a & l & s & o & a \\ l & l & t & h & e & e & x & a & m & p & l & e & s & . \\ r & e & l & a & t & e & t & h & e & m & t & o & g & e \\ t & h & e & r & n & i & c & e & l & y & . & t & h & e \\ n & o & b & t & a & i & n & / & s & e & l & e & c & t \\ h & e & r & e & y & o & u & r & 1 & 4 & c & o & r & r \\ e & c & t & e & n & t & r & i & e & s & . & t & h & e \\ d & e & f & i & n & i & t & i & o & n & s & c & o & r \\ r & e & s & p & o & n & d & t & o & r & o & w & s & , \\ w & h & e & r & e & a & s & c & o & l & u & m & n & s \\ a & r & e & o & u & r & e & x & a & m & p & l & e & s \\ . & g & e & t & l & i & n & k & a & n & d & w & i & n \end{pmatrix}$$

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Sources for the images:

- Definition 5: Andrew Bossi on Wikipedia
- Definition 13: [Graphics RF](#) on [Vecteezy](#)
- Example B: Axel Bacher
- Example C: Christian Krattenthaler
- Example E: Cameron Marcott
- Example F: David Eppstein on Wikipedia
- Example H: Drew Armstrong