Internship proposal:
λ-theories and computable trees

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Keywords. λ-calculus, denotational semantics, computable trees, λ-theory, relational graph models

Potential collaborations. Giulio Manzonnetto (LIPN) or Thomas Seiller (LIPN)

Prerequisite. High capacity of abstraction. Basics in computability theory. Advance level on denotational semantics and lambda calculus.

Principle

A λ-theory is a congruence of λ-terms modulo β-reduction, i.e., it is an equivalent relation $\equiv$ between λ-terms such that $((\lambda x.M)N) \equiv M[N/x]$ and such that, whenever $M \equiv M'$, then $\lambda x.M \equiv \lambda x.M'$, $M\ N \equiv M'N$ and $N\ M \equiv N\ M'$.

A suffix-prefix-closed sets of computable trees is a set S of (infinite) computable trees of unbounded arrity such that $t \in S$ iff t have a son $s \in S$.

We conjecture that an important subclass of λ-theories (those BT-sensible λ-theories respecting the ω-rule) corresponds exactly to suffix-prefix-closed sets of computable trees, with a (contravariant) conservation of the order.

More specifically, we conjecture that both of these class are equivalent to an intermediate one: the class of relational graph models quotiented by their induced interpretation in the lambda calculus. We know how to construct a relational graph models out of a suffix-prefix-closed set, but haven't prove yet that the induced λ-theories are different, neither that we cover all those that we expect.

The objective of the internship is to show the easier direction : that the induced λ-theories are all different. The other direction can be attempted afterwards or can be the starting point of a thesis.

References

Most of the technical content can be found a submitted article on relational graph models.