Asymptotic Density of Properties in Cellular Automata

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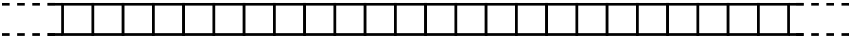

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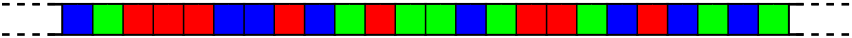

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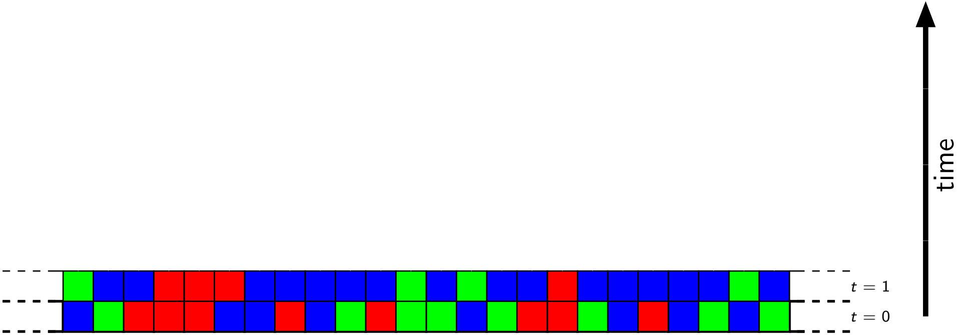

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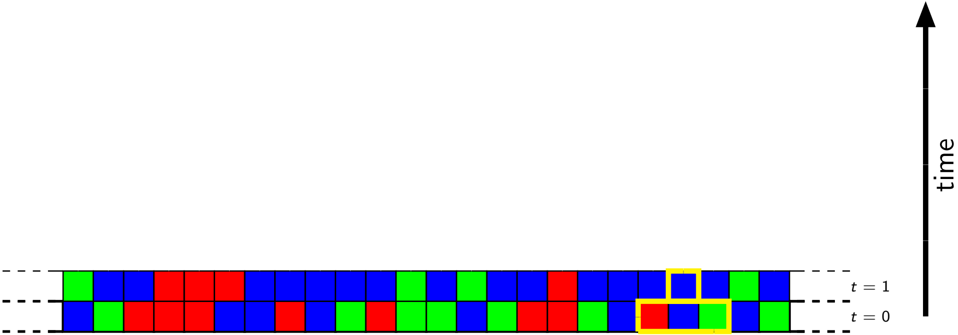

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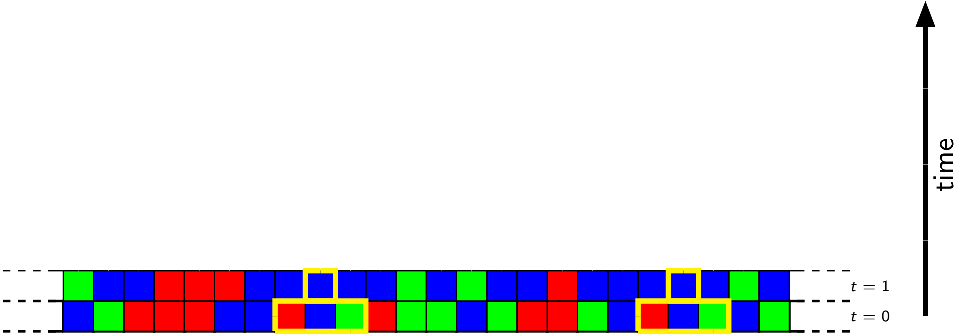

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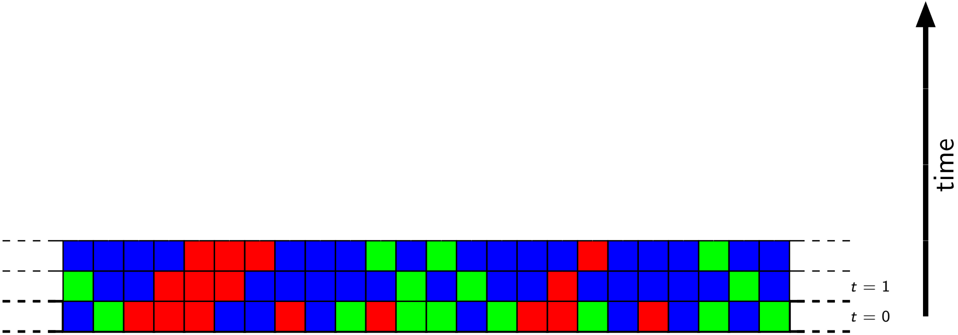

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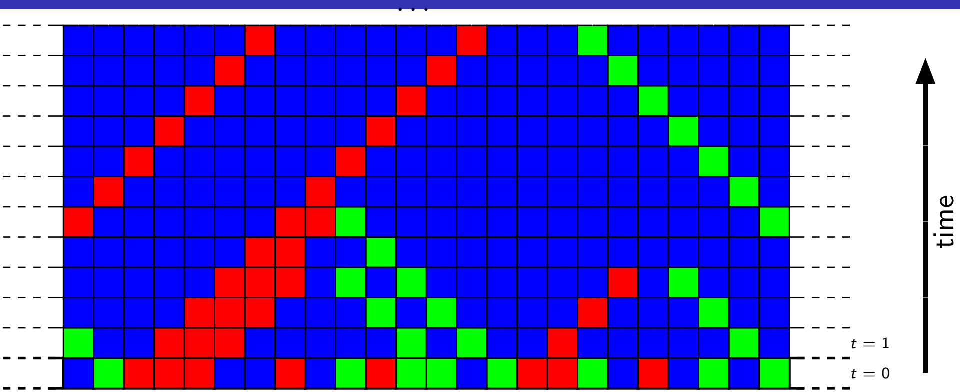

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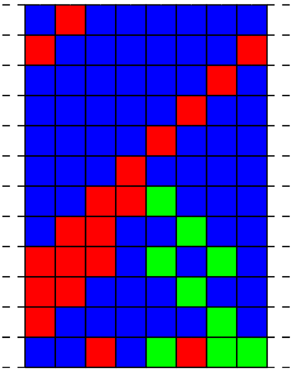

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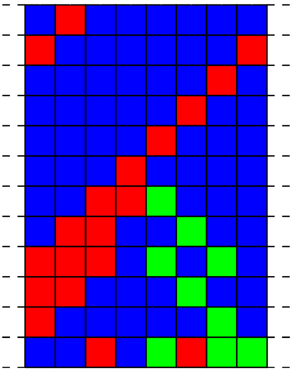

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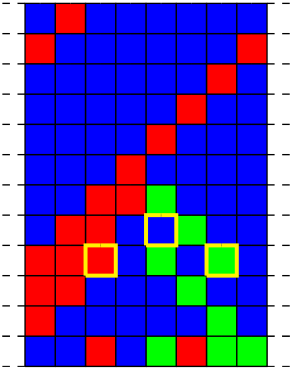

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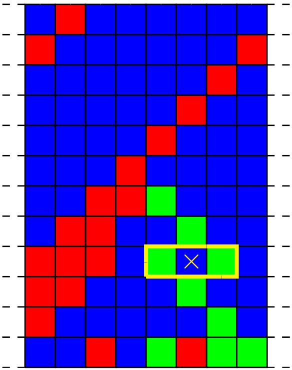

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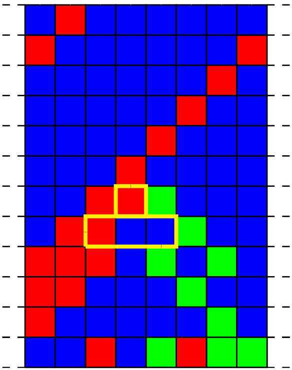

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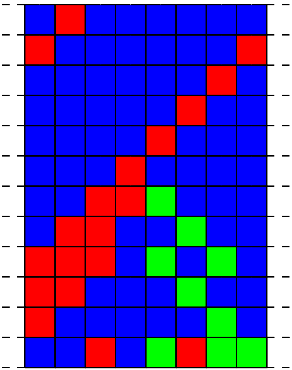

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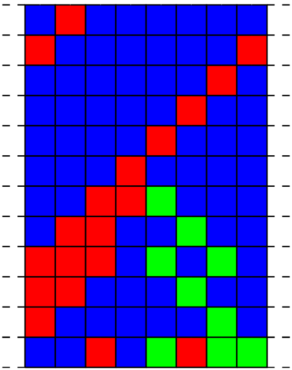

- **It defines a global behaviour**
  - for *configurations* $x \in Q^{\mathbb{Z}}$
  - the *global rule*: $F : Q^{\mathbb{Z}} \to Q^{\mathbb{Z}}$ is defined locally: $F(x)_z = \delta(x_{z+\nu_1}, x_{z+\nu_2}, \ldots, x_{z+\nu_k})$

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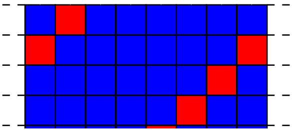

Simple (finite) description $\leftrightarrow$ **Complex global behaviour**

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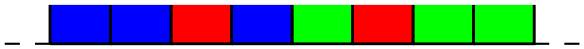

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Some examples (1/2)

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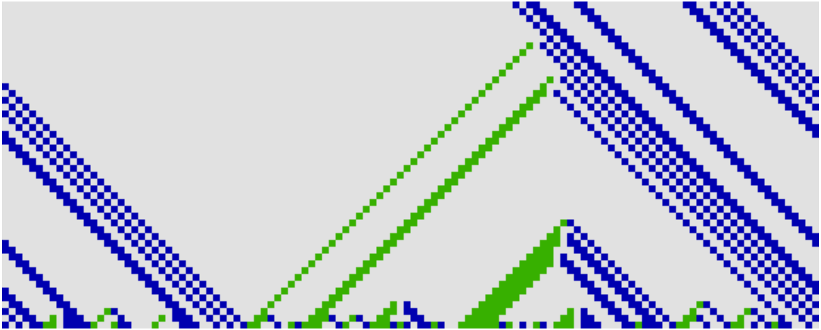
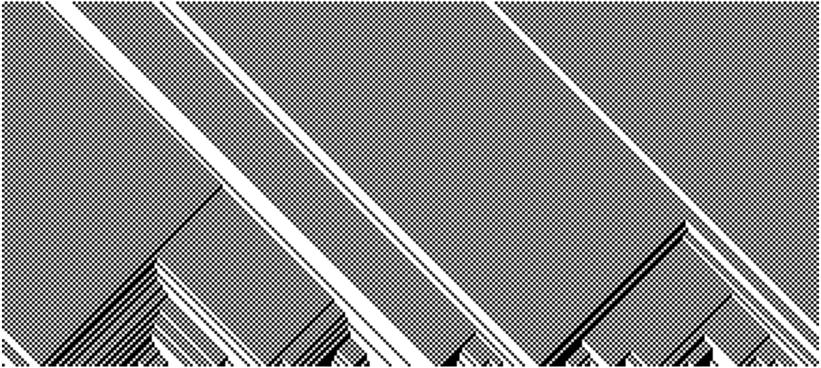
- JustGliders is $(\{L,\emptyset,R\},\{-1,0,1\},\delta_{JG})$ with δ_{JG} s.t. L moves left, R moves right, and they disappear if they collide :



Some examples (2/2)

- 184 is $(\{0,1\},\{-1,0,1\},\delta_{184})$ with $\delta_{184} : \begin{cases} 10? \mapsto 1 \\ ?10 \mapsto 0 \\ ?11 \mapsto 1 \\ 00? \mapsto 0 \end{cases}$



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Here we don't focus on particular CA :

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Definition (Nilpotency)

$$\mathcal{A} \in \textbf{Nil} \quad \stackrel{def}{\Leftrightarrow} \quad \Omega_{\mathcal{A}} = \{c\}$$

"The CA always converges to this single configuration."

Intrinsic simulation (1/2)

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Mazoyer, Delorme, Rapaport, Ollinger, Theyssier (1998-2010)

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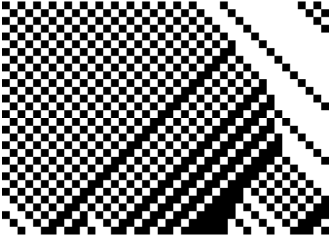

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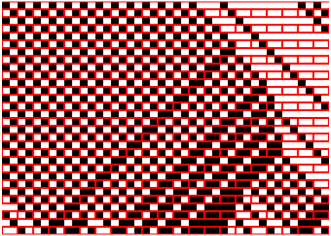

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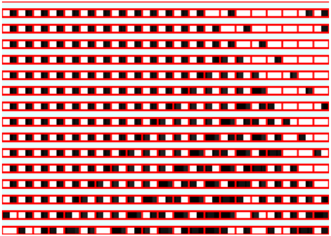

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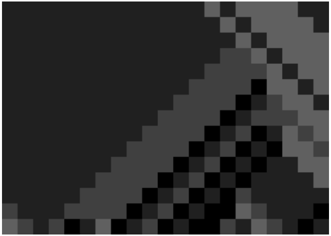
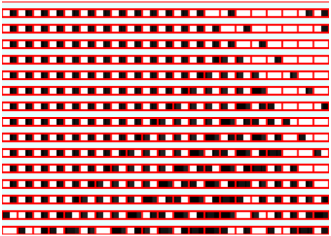
 $\leftrightarrow$ 

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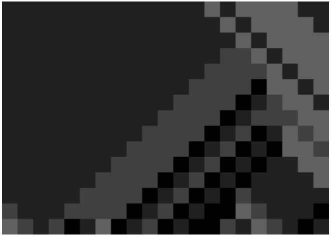
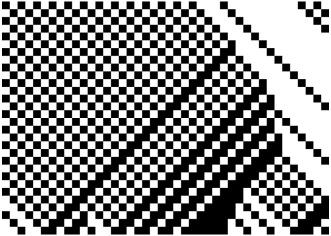
 ↔ 

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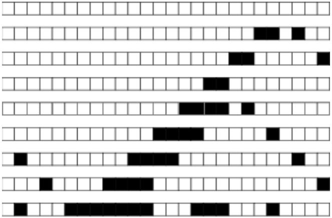

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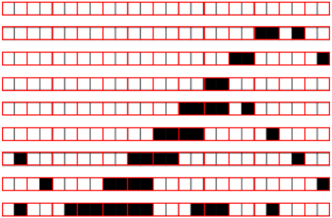

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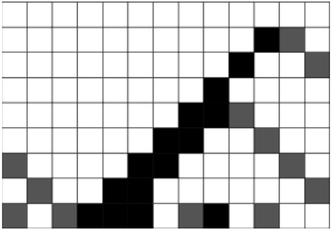

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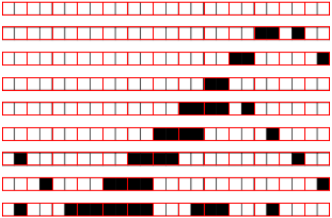

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$\mathcal{U} \in \textbf{Univ} \quad \overset{def}{\Leftrightarrow} \quad \forall \mathcal{A},\ \mathcal{A} \preccurlyeq_{\sqsubseteq} \mathcal{U}$

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Theorem (N. Ollinger – 2003)

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Remarks :

- Central notion in CA litterature,
- Stronger than Turing universality in CA,
- Elements of **Univ** are maximal elements in the preorder induced by $\preccurlyeq_{\sqsubseteq}$.

Subfamilies of CA (example 1)

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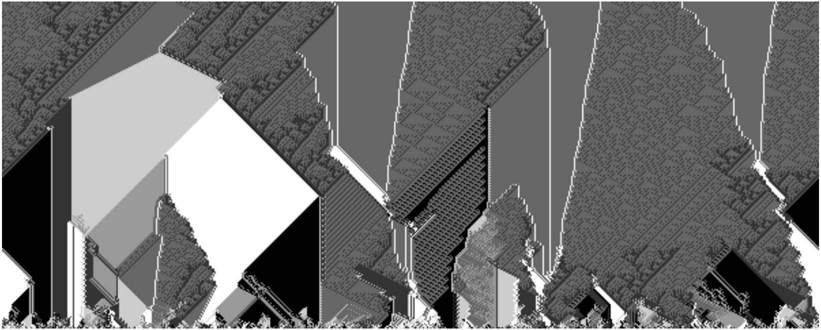
- **Captive CA**

Definition (Captive CA)

$$\mathcal{A} \in \mathcal{K} \quad \overset{def}{\Leftrightarrow} \quad \forall x_1, x_2, \ldots, x_k \in Q, \; \delta_{\mathcal{A}}(x_1, x_2, \ldots, x_k) \in \{x_1, x_2, \ldots, x_k\}$$

- Introduced by G. Theyssier (2004),
- under some conditions most captive CA are universal (2005).



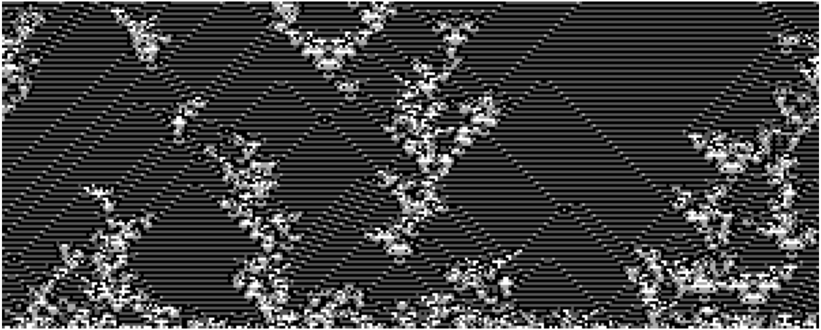
Subfamilies of CA (example 2)

- **Multiset CA**

Definition (Multiset CA)

$\mathcal{A} \in \mathcal{MS} \quad \overset{def}{\Leftrightarrow} \quad$ *for all permutation* $\pi : \{1, \dots k\} \to \{1, \dots k\}$, $\delta_{\mathcal{A}}(x_1, x_2, \dots, x_k) = \delta_{\mathcal{A}}(x_{\pi(1)}, x_{\pi(2)}, \dots, x_{\pi(k)})$

- Captures the idea of *isotropy*.
- Other interesting properties (*rescalings*...).



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- **Our contribution :**
  - a *unified framework* to study density among CA or subfamilies,
  - various results.

Objects and properties

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We consider the set **CA** of triplets (Q_n, V_k, δ) for $n, k \in \mathbb{N}$, with

- $Q_n = \{0, 1, \ldots, n-1\}$
- V_k *centered and connected* neighbourhood of size k
- δ any function $(Q_n)^k \to Q_n$

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- and consider the proportions

$$D_{n,k}(\mathcal{C}, \mathcal{P}) \stackrel{\text{def}}{=} \frac{\#\left(\mathcal{C}_{n,k} \cap \mathcal{P}\right)}{\#\left(\mathcal{C}_{n,k}\right)}$$

$\mathcal{C}_{n,k}$ elements of size (n, k) of the family $\mathcal{C}$,
$\mathcal{P}$ a property.

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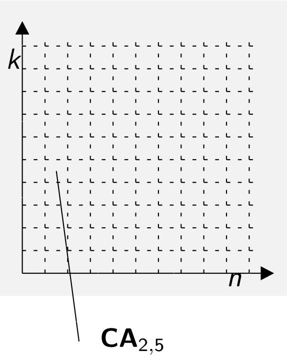

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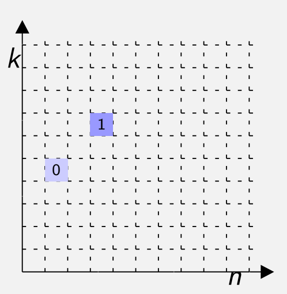

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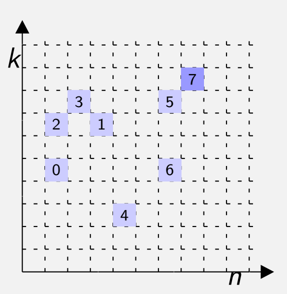

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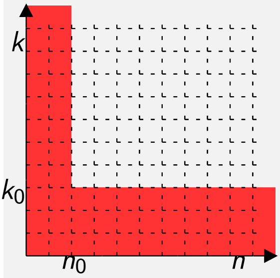

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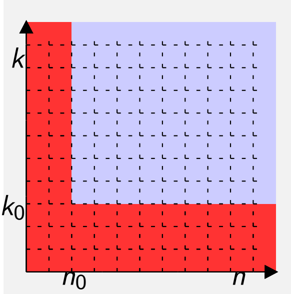

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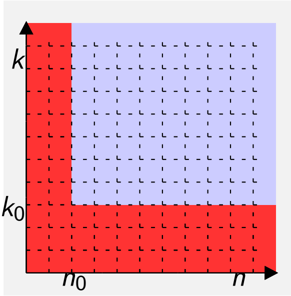

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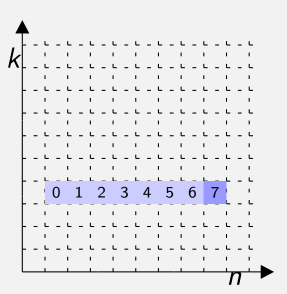

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$\mathbb{N}_x \overset{def}{=} \mathbb{N} \setminus \{0, \ldots, x-1\}$

- every possible size (with surjective path)
- or particular paths
  e.g. if $\rho_n = \pi_1 \circ \rho$ or $\rho_k = \pi_2 \circ \rho$ is upperbounded)

Density of properties

Definition (Density of $\mathcal{P}$ among $\mathcal{C}$ following ρ :)

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Density is path-independent in the surjective case.

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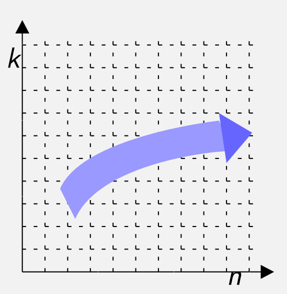

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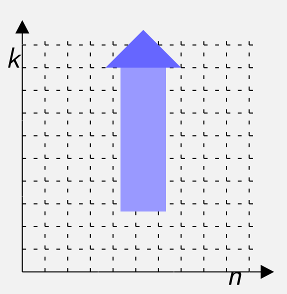

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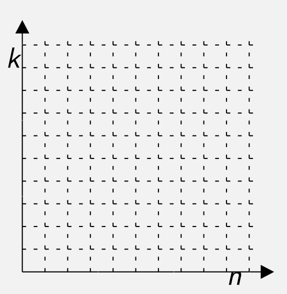

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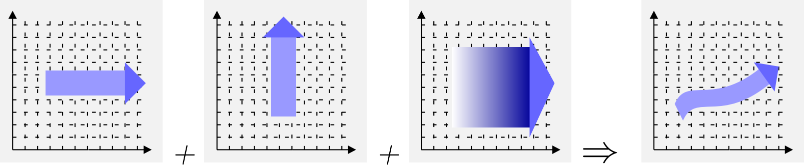

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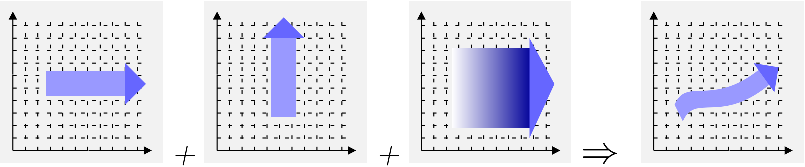

$+$ specific combinatorial arguments for *each* case.

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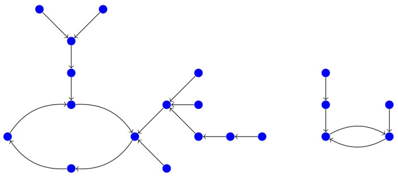

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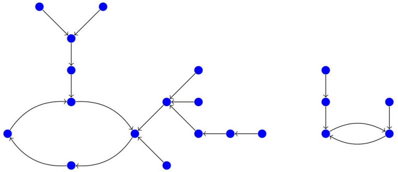

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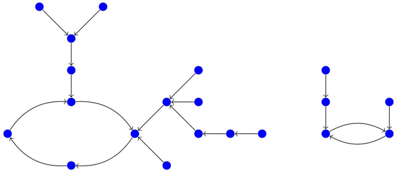

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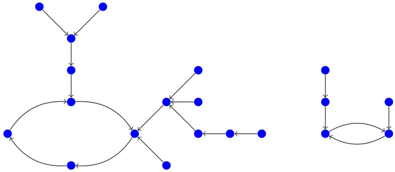

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- "trees are asympotically negligible among functionnal graphs"…

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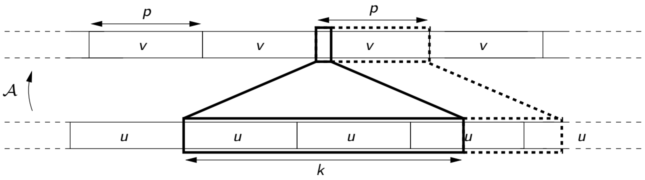

- Transitions $u^* \mapsto x$ are *constrained*,
- Combining those constraints makes it possible to conclude..

Link with Kolmogorov Complexity

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- Gives a procedure to prove negligeability:
  "Describe shortly CA from $\mathcal{P}$."

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- Which takes a total number of $(1+m).\lceil log(m)\rceil + \lceil m^k.log(m)\rceil + \lceil (n^k - m^k).log(n)\rceil$ bits

CA having a sub-automaton

Proposition

The set of CA having a non-trivial sub-automaton is negligible among any $(1,3)$*-path.*

- To describe a CA $\mathcal{A}$ of size (n,k) having a sub-automaton $\mathcal{B}$ of size (m,k), $1 < m < n$, it is sufficient to describe :
  1. the size m $log(n)$ bits
  2. the states of the sub-automaton $m.log(n)$ bits
  3. the transition rule of $\mathcal{B}$ $m^k.log(m)$ bits
  4. the remaining transitions $(n^k - m^k).log(n)$ bits
- Which takes a total number of $(1+m).\lceil log(m)\rceil + \lceil m^k.log(m)\rceil + \lceil (n^k - m^k).log(n)\rceil$ bits
- The gain tends to infinity (...).

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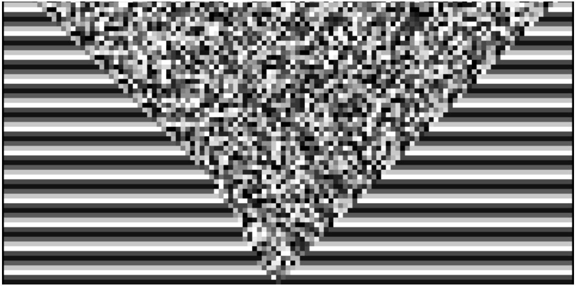

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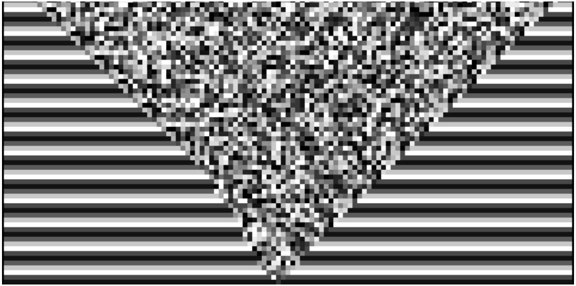

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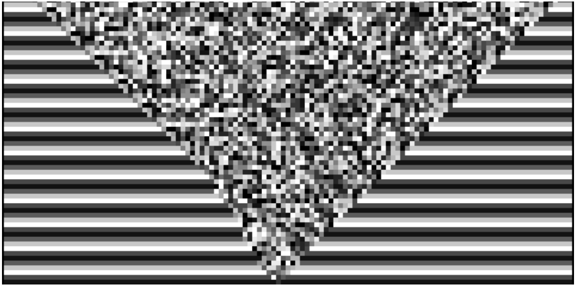

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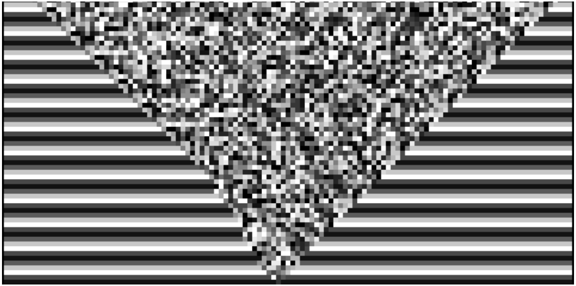

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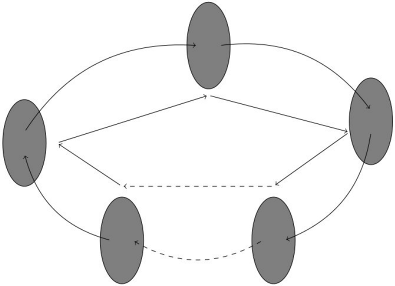

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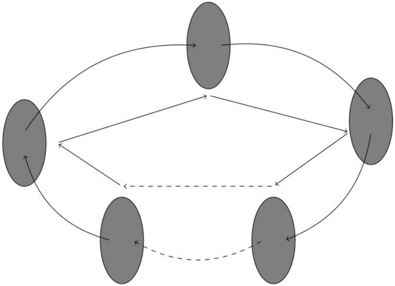

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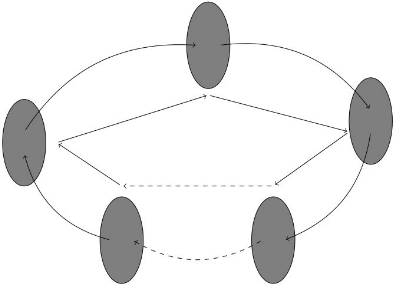

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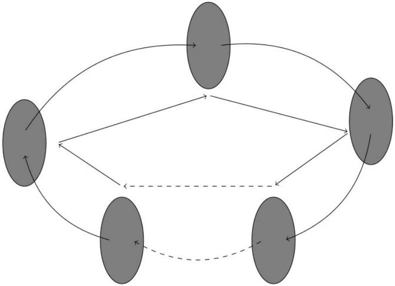

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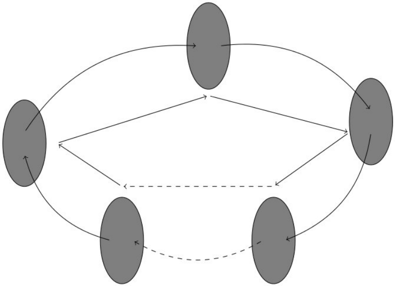

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- In *random functional graphs*, the number of cycles is increasing with the number of states.

Summary and main results about density among CA

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NB: *2 classes out of 4 from Kurka's classification are negligible.*

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Two necessary steps for each family :

- Point out a universal CA in $\mathcal{C}$,
- Find possible simulation subshifts,
  - in increasing number along the considered paths,
  - on which the *simulating probability* is not too small,
  - which are *independents*.

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- Universality is not as *algorithmic* as we thought before.

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    - Propagation of information $\overset{?}{\rightsquigarrow}$ *sensitivity*,

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- **In both cases, precise the information :**
  - *Convergence speed* of limit densities,
  - Precise *finite proportions*.