Partitions of direct products of complete graphs into independent dominating sets

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Let $G = (V, E)$ be an undirected finite simple graph without loops. A set $S \subseteq V$ is called a *dominating set* if for every vertex $v \in V \setminus S$ there exists a vertex $u \in S$ such that u is adjacent to v. A set $S \subseteq V$ is called *independent* if no two vertices in S are adjacent. A set $S \subseteq V$ is called an *independent dominating set* of G if it is both independent and dominating set of G. A partition of the vertex set V into independent dominating sets is called an *idomatic partition* of G. Clearly, an idomatic partition of a graph G represents a proper coloring of the vertices of G. The maximum order of an idomatic partition of G is called the *idomatic number* $id(G)$ and this parameter was introduced by Cockayne and Hedetniemi in 1977. Notice that not every graph has an idomatic partition. For example, C_5 has no idomatic partition. The *direct product* $G \times H$ of two graphs G and H is defined by $V(G \times H) = V(G) \times V(H)$, and where two vertices $(u_1, u_2), (v_1, v_2)$ are joined by an edge in $E(G \times H)$ if $\{u_1, v_1\} \in E(G)$ and $\{u_2, v_2\} \in E(H)$.

In this talk, we give a full characterization of the idomatic partitions of the direct product of three complete graphs by using an elementary algebraic approach. This partially answer a question of Dunbar et al. 2000.