Partitions of direct products of complete graphs into independent dominating sets

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Let $G = (V, E)$ be an undirected finite simple graph without loops. A set $S \subseteq V$ is called a dominating set if for every vertex $v \in V \setminus S$ there exists a vertex $u \in S$ such that $u$ is adjacent to $v$. A set $S \subseteq V$ is called independent if no two vertices in $S$ are adjacent. A set $S \subseteq V$ is called an independent dominating set of $G$ if it is both independent and dominating set of $G$. A partition of the vertex set $V$ into independent dominating sets is called an idomatic partition of $G$. Clearly, an idomatic partition of a graph $G$ represents a proper coloring of the vertices of $G$. The maximum order of an idomatic partition of $G$ is called the idomatic number $id(G)$ and this parameter was introduced by Cockayne and Hedetniemi in 1977. Notice that not every graph has an idomatic partition. For example, $C_5$ has no idomatic partition. The direct product $G \times H$ of two graphs $G$ and $H$ is defined by $V(G \times H) = V(G) \times V(H)$, and where two vertices $(u_1, u_2), (v_1, v_2)$ are joined by an edge in $E(G \times H)$ if $(u_1, v_1) \in E(G)$ and $(u_2, v_2) \in E(H)$.

In this talk, we give a full characterization of the idomatic partitions of the direct product of three complete graphs by using an elementary algebraic approach. This partially answer a question of Dunbar et al. 2000.