On Certain Statistics of Random Weighted Partitions of Large Integers

Ljuben Mutafchiev
American University in Bulgaria, 2700 Blagoevgrad, Bulgaria
and Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences
ljuben@aubg.bg
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Abstract

A weighted partition of the positive integer \( n \) is a multiset of size \( n \) whose decomposition into a union of disjoint components (parts) satisfies the following condition: for a given sequence of non-negative numbers \( \{ b_k \}_{k \geq 1} \), a part of size \( k \) appears in exactly one of \( b_k \) possible types. Assuming that a weighted partition of \( n \) is selected uniformly at random from the set of all such partitions, we study the limiting distributions of the largest part size \( X_n \) and of the number of parts \( \xi_n \) as \( n \to \infty \). Under certain fairly general assumptions on the Dirichlet generating series \( D(s) = \sum_{k=1}^{\infty} b_k k^{-s}, s = \sigma + iy \), G. Meinardus, Math. Z. 59(1954), 388-398, has obtained the asymptotic of the total number of weighted partitions of \( n \). We assume that Meinardus conditions hold and prove that \( X_n \) and \( \xi_n \), appropriately normalized, converge weakly to non-degenerate probability distributions.