Exact and explicit forms and combinatorial content of Lévy stable distributions

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We briefly recall the orgin, history and some physical applications of Lévy stable probability distributions. We report on recent findings of exact and explicit expressions for one-sided $(0 < \alpha < 1)$ and two-sided $(1 < \alpha \leq 2)$ Lévy stable densities $g_{\alpha}(x)$, of index α for all $\alpha = l/k$, with k and l positive integers. We shall exemplify analytically and graphically several examples of known and infinite ensemble of new formulae for such distributions. We observe that in one-sided case $(0 < l/k < 1) g_{l/k}(x)$ is a solution of the Stieltjes moment problem with negative moments being integer combinatorial sequences of factorial type. This last property, when seen as a conventional Stieltjes moment problem, can be solved with the use of inverse Mellin transform. In this way we derive an explicit formulae for $g_{l/k}(x)$ in terms of Meijer G functions. The problem of non-uniqueness of so obtained solutions is discussed.