

Sokendai Lectures Tokyo, Japan



物理情報システムのための形式手法

Timed model checking – Part 2

Timed automata

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Beyond finite state automata

Finite State Automata give a simple syntax and a formal semantics to model **qualitative** aspects of systems

- Executions, sequence of actions
- Modular definitions (parallelism)
- Powerful checking (reachability, safety, liveness...)

But what about quantitative aspects:

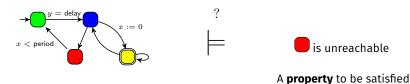
- Time ("the airbag always eventually inflates, but maybe 10 seconds after the crash")
- Temperature ("the alarm always eventually ring, but maybe when the temperature is above 75 degrees")

Partie 2: Timed model checking - Plan

- 1 Timed automata
- 2 Specifying with timed temporal logics
- 3 Specifying with observers
- 4 Decidability
- 5 Timed automata in practice
- 6 Beyond timed automata...

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Model checking timed concurrent systems



A timed model of the system

■ Question: does the model of the system **satisfy** the property?



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Formalisms

Many formalisms were proposed to model and verify timed systems

■ time(d) Petri nets [Merlin, 1974]

■ timed automata [Alur and Dill, 1994]

■ timed process algebras [Sun et al., 2009b]

etc.

We use here timed automata

See [Bérard et al., 2005, Srba, 2008, Bérard et al., 2013] for a comparison between timed Petri nets and timed automata

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Formal definition of timed automata

Definition (Timed automaton)

A **timed automaton (TA)** ${\mathcal A}$ is a 7-tuple of the form ${\mathcal A}=(L,\Sigma,\ell_0,L_F,X,I,E)$, where

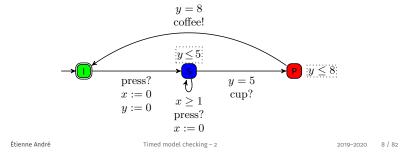
- \blacksquare L is a finite set of locations,
- lacksquare $\ell_0 \in L$ is the initial location,
- \blacksquare $L_F \subseteq L$ is the set of accepting (or final) locations,
- \blacksquare Σ is a finite set of actions,
- \blacksquare X is a set of clocks.
- lacksquare I is the invariant, assigning to every $\ell \in L$ a clock constraint $I(\ell)$, and
- E is a step (or "transition") relation consisting of elements of the form $e=(\ell,g,a,R,\ell')$, also denoted by $\ell \overset{g,a,R}{\longrightarrow} \ell'$, where $\ell,\ell' \in L$, $a \in \Sigma$, $R \subseteq X$ is a set of clock variables to be reset by the step, and g (the step guard) is a clock constraint.

Timed automaton (TA)

- Finite state automaton (sets of locations) and actions) augmented with a set *x* of clocks [Alur and Dill, 1994]
 - Real-valued variables evolving linearly at the same rate
 - Can be compared to integer constants in invariants and guards

Features

- Location invariant: property to be verified to stay at a location
- Transition guard: property to be verified to enable a transition
- Clock reset: some of the clocks can be set to 0 at each transition



Clock constraints

Definition (clock constraint)

A **clock constraint** is a conjunction of atomic constraints

What is an atomic constraint?

Various definitions in the literature:

- lacksquare Originally [Alur and Dill, 1994]: $x\in [c_1,c_2]$ with $c_1\in \mathbb{N}$ and $c_2\in \mathbb{N}\cup \{\infty\}$
- lacktriangle Comparing clock values (diagonal constraints) $x_1-x_2\bowtie c$
 - $\quad\blacksquare\;\bowtie\in\{<,\leq,=,\geq,>\}$

For now, we assume the following syntax:

 $\blacksquare x \bowtie c$, with $x \in X$ and $c \in \mathbb{N}$

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Exercise 1

Draw the TA $\mathcal{A}=(L,\Sigma,l_1,\{l_2\},X,I,E)$ such that

- $L = \{l_1, l_2, l_3, l_4\},\$
- $\Sigma = \{a_1, a_2, a_3\},\$
- $X = \{x_1, x_2\},$
- $I(l_1) = x_1 \le 3$, and $I(l_3) = x_2 \ge 2$,
- $$\begin{split} \blacksquare & E = \{(l_1, x_1 \geq 2, a_1, \{x_1\}, l_2), \\ & (l_1, x_2 \leq 1, a_2, \emptyset, l_3), \\ & (l_2, x_2 = 1, a_3, \{x_2\}, l_2), \\ & (l_2, \mathsf{true}, a_1, \emptyset, l_3), \\ & (l_3, \mathsf{true}, a_2, \{x_1, x_2\}, l_4), \\ & (l_4, x_2 > 2, a_3, \emptyset, l_3)\} \end{split}$$

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Exercise 2

Give the formal TA corresponding to the timed coffee machine.

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Parallel composition of timed automata (1/2)

Just as finite-state automata, timed automata can be composed through **parallel composition** using synchronization actions

$$\mathcal{A}_1 = (L_1, \Sigma_1, (\ell_0)_1, (L_F)_1, X_1, I_1, E_1)
\mathcal{A}_2 = (L_2, \Sigma_2, (\ell_0)_2, (L_F)_2, X_2, I_2, E_2)$$

Then we define $\mathcal{A}_1 \parallel \mathcal{A}_2$ as

Parallel composition of timed automata (2/2)

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Concrete runs of timed automata

- Concrete state of a TA: pair (ℓ, w) , where
 - \blacksquare ℓ is a location,
 - \blacksquare w is a valuation of each clock

Example: $\left(\bigcirc, \begin{pmatrix} x=1.2 \\ y=3.7 \end{pmatrix} \right)$

■ Concrete run: alternating sequence of concrete states and actions or time elapse

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Timed transition systems

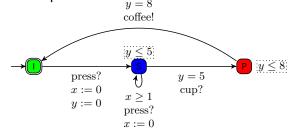
Definition (Timed transition system)

A **timed transition system** (TTS) is a tuple $\mathcal{TTS} = (S, \Sigma, S_0, S_F,
ightarrow)$, where

- \blacksquare S is a set of states;
- lacksquare Σ is an alphabet of events;
- lacksquare $S_0\subseteq S$ is a set of initial states;
- lacksquare $S_F\subseteq S$ is a set of final (or accepting) states; and,
- lacksquare \to : $S imes (\Sigma \cup \mathbb{R}_{\geq 0}) o 2^S$ is a transition relation.

We write $s_1 \stackrel{a}{\longrightarrow} s_2$ when $(s_1, a, s_2) \in \rightarrow$.

Example of concrete runs



- Possible concrete runs for the coffee machine
 - Coffee with no sugar
 - Coffee with 2 doses of sugar

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Concrete semantics of timed automata: definition

Definition (Concrete semantics of a TA)

Given a TA $\mathcal{A}=(\Sigma,L,\ell_0,L_F,X,I,E)$, the concrete semantics of \mathcal{A} is given by the timed transition system $(S,E,S_0,S_F,\rightarrow)$, with

$$\blacksquare \ S = \{(\ell, w) \in L \times \mathbb{R}_{\geq 0}^{|X|} \mid \qquad ,$$

$$lacksquare S_0 = \qquad \qquad ext{(with $\vec{0} \models I(\ell_0)$), and}$$

■
$$S_F = \{(\ell, w) \in$$

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- lacksquare ightarrow consists of the discrete and (continuous) delay transition relations:
 - discrete transitions: $(\ell, w) \xrightarrow{e} (\ell', w')$, if $(\ell, w), (\ell', w') \in S$, there exists $e = (\ell, g, a, R, \ell') \in E$, w' = w[R], and $w \models g$.

Notation:

$$w[R](x) = \begin{cases} & \text{if } x \in R \\ & \text{otherwise} \end{cases}$$

Concrete semantics of timed automata: definition (cont.)

We write $(\ell,w) \stackrel{(d,e)}{\mapsto} (\ell',w')$ or $((\ell,w),(d,e),(\ell',w')) \in \mapsto$ for a combination of a delay and discrete transitions if

$$\exists w'' : (\ell, w) \xrightarrow{d} (\ell, w'') \xrightarrow{e} (\ell', w')$$

Some remarks on the semantics of timed automata:

- Is TTS finite?
- Is TTS finitely branching?

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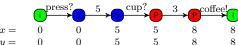
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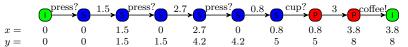
Timed words: exercise

Give the (formal) run and the associated timed words associated with the two example runs of the coffee machine:

■ Coffee with no sugar



■ Coffee with 2 doses of sugar



Timed words

Definition (timed word)

A **timed word** over an alphabet of actions Σ is a possibly infinite sequence of the form $(a_0, d_0)(a_1, d_1) \cdots$ such that, for all integer $i \geq 0$, $a_i \in \Sigma$ and $d_i \leq d_{i+1}$.

Definition (timed word associated with a concrete run)

Given a concrete run ρ $(l_0,w_0)(d_0,e_0)(l_1,w_1)\cdots(d_i,e_i)(l_i,w_i)\cdots$, the **timed** word associated with ρ is

$$(\mathsf{Act}(e_0), d_0)(\mathsf{Act}(e_1),$$

Notation: $Act(e_i)$ denotes the action of edge e_i

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Timed language

Definition (timed language)

Given a TA \mathcal{A} , the **timed language** of \mathcal{A} is the set of timed words associated with the runs of \mathcal{A} ending in a location

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Timed language: Example 1

Give the timed language of the following automaton

a, b



x := 0

[Alur and Dill, 1994]

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Timed language: Example 2

Give the timed language of the following automaton

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Timed language: Example 3

Give the timed language of the coffee machine

Accepting locations?

Timed automata may or may not be equipped with accepting locations

Often, timed automata with no accepting locations are called timed safety automata [Henzinger et al., 1994]

In that case the timed language can be defined as:

- All possible timed words read by the automaton
- All possible maximal timed words read by the automaton
 - Maximal: infinite or that cannot be extended
- All possible infinite timed words read by the automaton

Theorem

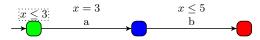
The expressive power of timed safety automata is strictly less than timed automata with accepting locations [Henzinger et al., 1995]

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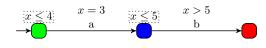
Deadlocks and timelocks

Timed automata can be subject to two annoying behaviors:

- **Deadlock**: similar to finite state automata
 - Can be a problem of



- Timelock: coming from the timed nature of TAs
 - Can



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The Zeno problem (2/2)

Problem (Zeno runs)

An infinite number of actions in finite time is impossible in practice

■ Processors have finite precision

Zeno runs must be **pruned** when performing model checking

Some solutions:

■ Transform the TA (with an additional clock)

[Tripakis, 1999, Tripakis et al., 2005, Bowman and Gómez, 2006, Gómez and Bowman, 2007]

■ Transform the zone graph

[Herbreteau et al., 2012]

■ Consider a different but closely related formalism

[Sun et al., 2013]

■ Transform the TA on-the-fly

[Wang et al., 2015]

The Zeno problem (1/2)

Definition (Zeno run)

A run is Zeno if it contains an **infinite** number of actions in **finite** time.

■ Example of TA containing at least one Zeno run

■ Example of TA containing only non-Zeno runs

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Example: Railroad gate controller [Alur et al., 1993b]

Design three timed automata in parallel:

- The **train**: once it is approaching (action approach), it will come in (action in) after at least 5 time units, then go out (action out) and finally exit (action exit) after at most 6 time units
- The **gate**: upon reception of a lower signal, starts to lower; once it is down, and upon reception of a raise signal, the gate raises again; the time to lower and to raise the gate is an interval [1,3]
- The **controller**: once a train approaches (action approach), it triggers the lower signal within [2,3] time units; then, once the train exits (action exit), it triggers the raise signal again within [2,4] time units

All TAs are cyclic, i. e., repeat the same behavior forever.

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Example: Railroad gate controller (train)

Example: Railroad gate controller (gate)

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Example: Railroad gate controller (controller)

Example: A hardware gate

Q

The output Q reacts to the change of the input I (actions I^\uparrow and I^\downarrow) after a delay [5,9]

[Chevallier et al., 2009]

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Example: A nuclear power plant

Example: A nuclear power plant (solution)

Design a PTA modeling a nuclear power plant:

- At first, the plant is in normal mode.
- Suddenly, it may start to heat (action startHeating).
- \blacksquare At that point, a timer is set; after p_2 time units, the timer will trigger an alarm (action alarm).
- \blacksquare Then, p_3 time units later, a watering system (action watering) starts.
- This watering system lasts for at most p_4 time units, after which the plant is cool again (action cool) and goes back to the normal mode.
- However, p_1 time units after the plant starts to heat, the plant may explode at any time (action boom)—unless of course it is cool again.

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Example: A real-time system

Design a (network of) timed automata modeling the following components:

- a periodic task T_1 of period 5 with offset 2, best and worst case execution times in [3,4]
- 2 a sporadic task T_2 of minimum interarrival time 20, best and worst case execution times in [1,2]
- a non-preemptive scheduler with fixed priority

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Example: A real-time system (solution)

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Timed temporal logics

- Specify properties on the order **and the delays** between events
- No X operator because

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Semantics of TCTL: discrete vs. continuous

Two semantics:



■ Discrete (point-wise) semantics: timed words (0,0)(0,2.046)(0,3.3)(0,6.9)

Are they equivalent?

TCTL (Timed CTL) [Alur et al., 1993a]

TCTL expresses formulas on the **order** and the **time** between the **future** events **for some or for all paths**, using a set of atomic propositions AP

■ Timed extension of CTL

Quantifiers over paths:

$$\varphi ::= p \in AP \mid \neg p \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathsf{E}\psi \mid \mathsf{A}\psi$$

Quantifiers over states:

$$\psi ::= \varphi \mathsf{U}_I \varphi$$

I is an interval of the form [a,b], [a,b), (a,b), (a,b), $[a,\infty)$, or (a,∞) , where $a, b \in \mathbb{N}$

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Continuous semantics of TCTL

iff p holds at the current position $s \models p$

iff p does not hold at the current position $s \models \neg p$

iff $s \models \varphi \land s \models \psi$ $s \models \varphi \wedge \psi$

 $s \models \varphi \lor \psi$ iff $s \models \varphi \lor s \models \psi$

 $s \models \mathsf{E} \psi \mathsf{U}_I \varphi$ iff there exists a future path and $t \in I$ for which ψ holds until t and φ holds at t

 $s \models \mathsf{A}\psi\mathsf{U}_I\varphi$ iff for all future paths, there exists $t \in I$ for which ψ holds until t and φ holds at t

Informal description of the U (the rest is similar):

$$s \models \mathsf{E} \psi \mathsf{U}_I \varphi \quad \text{iff} \quad \text{there exists } n > 0 \text{ such that } \varphi \text{ holds from point } n$$
 (with the time of point n within I) and for each $0 < m < n$, ψ holds at point m

Note: strict version of the U, considered in [Bouyer et al., 2017] (not necessarily standard)

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Semantics of TCTL: discrete vs. continuous (example)

Exhibit a word and a TCTL formula for which:

- 1 the formula holds under the continuous but not the discrete semantics
- 2 the formula holds under the discrete but not the continuous semantics

An example TA:

A concrete run ω (continuous semantics):

Equivalent run ρ in the discrete semantics:

TCTL: Examples

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- "Whatever happens, the plane will never crash in the next 10 minutes"
- "I may get a job within one year"
- "I am sure to get a job within one year"
- "Whenever a fire breaks, it is sure that the alarm will start ringing at least 5 seconds and at most 10 seconds later"
- "Whatever happens, I will love you for 2 years after we marry"

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TCTL: Examples (coffee machine)

- "Whenever the button is pressed, a coffee is necessarily eventually delivered within 10 units of time."
- "It must never happen that the button can be pressed twice within 1 unit of time."
- "It must never happen that the button can be pressed twice within a time strictly less than 1 unit of time."

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Observers for timed automata

Observers (both untimed and timed) can be used for timed automata Just as for FA:

- A TA observer is an automaton that **observes** the system behavior
- It synchronizes with other automata's actions
- It can **read** the clocks of the system, and/or feature its own clock(s)
- It must be non-blocking
 - Pay attention to timelocks or deadlocks!
- Its location(s) give an indication on the system property

Then verifying the property reduces to a reachability condition on the observer (in parallel with the system)

The expressive power of observers for timed automata has been studied in [Aceto et al., 1998, Aceto et al., 2003]

Other timed temporal logics

■ MTL: linear time

[Koymans, 1990]

Can be seen as a timed extension of LTL (just as TCTL is a timed extension of CTL)

■ Variant: MITL

[Alur et al., 1996]

Variant of MTL disallowing punctuality

■ STL: to reason about **signals**

[Maler and Nickovic, 2004]

etc.

See, e.g., [Bouyer et al., 2017] for a partial survey

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Exercise: An observer for the coffee machine

Design an observer for the coffee machine verifying that it must never happen that the button can be pressed twice within a time strictly less than 1 unit of time.

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What is decidability?

Definition

A decision problem is **decidable** if one can design an algorithm that, for any input of the problem, can answer yes or no (in a finite time, with a finite memory).

"given three integers, is one of them the product of the other two?"

"given a context-free grammar, does it generate all strings?"

"given a Turing machine, will it eventually halt?"

"given a timed automaton, does there exist a run from the initial state to a given location ℓ ?"

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Problem: an infinite concrete semantics

- Time is **dense**: transitions can be taken anytime
 - Infinite number of timed runs
 - **Infinite** number of states
 - **Infinitely** branching structure
 - Model checking needs a **finite** structure!

Why studying decidability?

If a decision problem is undecidable, it is hopeless to look for algorithms yielding exact solutions (because that is **impossible**)

However, one can:

- design **semi-algorithms**: if the algorithm halts, then its result is correct
- design algorithms yielding over- or under-approximations

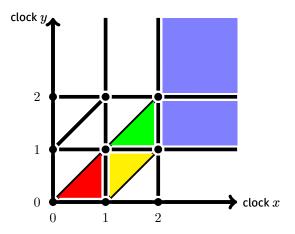
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Dense time

- A first remark: Some runs are equivalent
 - \blacksquare Taking the press? action at t=1.5 or t=1.57 is equivalent w.r.t. the possible actions
- Idea: reason with abstractions
 - Region automaton [Alur and Dill, 1994], and zone automaton
 - Example: in location _____, all clock values in the following zone are equivalent $y \leq 5 \land y x \geq 4$
 - This abstraction is **finite**

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Regions

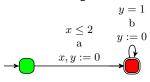


Inspired by a similar $mathbb{MT}_{E}X$ illustration by Patricia Bouyer

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Region graph construction: exercise

Construct the region graph of the following TA:



Region graph construction

Two successors:

- time-elapsing
- clock reset

(see white board for the graph construction)

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On the region graph finiteness

Is the region graph of TAs finite?

 \blacksquare Example with two clocks x, y:

Solution: k-extrapolation

 \blacksquare Idea: "all integer (resp. rational) clock valuations above the greatest constant k of the TA are equivalent" [Alur and Dill, 1994]

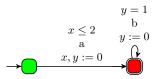
With this additional technicality, there is a **finite number** of regions in a TA

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Extrapolation: illustration

Extrapolation: exercise

Construct the region graph (with the k-extrapolation) of the following TA:



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Zone construction for timed automata

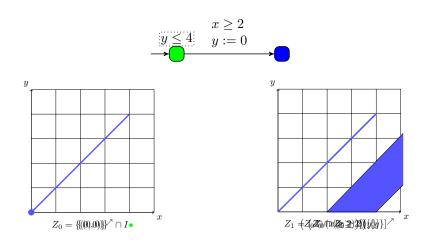
- **Objective**: group all concrete states reachable by the same sequence of discrete actions
- lacksquare Symbolic state: a location ℓ and a (infinite) set of states Z
- lacktriangle For timed automata, Z can be represented by a **convex polyhedron** with a special form called **zone**, with constraints

$$-d_{0i} \le x_i \le d_{i0}$$
 and $x_i - x_j \le d_{ij}$

■ Computation of successive reachable symbolic states can be performed symbolically with polyhedral operations: for edge $e=(\ell,a,g,R,\ell')$:

$$\mathrm{Succ}\big((\ell,Z),e\big) = \Big(\ell', \big((Z\cap g)[R]\cap I(\ell')\big)^{\nearrow}\cap I(\ell')\Big)$$

Zone construction for timed automata: Example

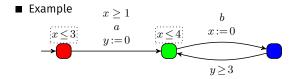


TikZ animation based on a LTpX code by Didier Lime

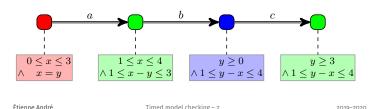
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Zone graph of timed automata

- **Abstract state** of a TA: pair (ℓ, C) , where
 - \blacksquare ℓ is a location, and C is a constraint on the clocks ("zone")
- **Abstract run**: alternating sequence of **abstract states** and actions



■ Possible abstract run from the zone graph of this TA



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More on zones

- Symbolic states can be efficiently computed using Difference Bound Matrices (DBMs)
- \blacksquare isReachable can be applied to the abstract semantics of timed automata (the underlying finite transition system)
- The zone graph is theoretically larger than the region graph but practically smaller
 - On-the-fly construction
 - Various optimization techniques

On the zone graph finiteness

Is the zone graph of TAs finite?

■ Example:

Solution: k-extrapolation

 \blacksquare Idea: "all clock valuations above the greatest constant k of the TA are equivalent" [Bengtsson and Yi, 2003]

■ Can we do more efficient?

■ L/U-abstractions [Behrmann et al., 2006] ■ Lazy abstractions [Herbreteau et al., 2013]

With this additional technicality, there is a **finite number** of reachable zones in a

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Decision problems for timed automata

The finiteness of the region automaton allows us to check properties

© **Reachability** of a location (PSPACE-complete)

[Alur and Dill, 1994]

② Liveness (Büchi conditions)

TCTL model-checking

[Alur and Dill, 1994]

Some problems impossible to check using the zone graph (but still decidable)

on-Zenoness emptiness check

[Gómez and Bowman, 2007]

Some undecidable problems

universality of the timed language

[Alur and Dill, 1994]

(2) timed language inclusion

[Alur and Dill, 1994]

■ Some decidable subclasses

[Alur and Dill, 1994, Ouaknine and Worrell, 2003, Ouaknine and Worrell, 2004] [Abdulla et al., 2008, Bertrand et al., 2011]

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Syntactic variants of timed automata

Variants of the syntax with consequences on the decidability

■ Can we use **diagonal** constraints ("x - y")? [Bouyer, 2003]

■ Can we **reset** clocks to constants $\neq 0$? [Bouyer et al., 2004]

■ Can we reset clocks to other clocks? [Bouyer et al., 2004]

■ Can we reset clocks to unknown constants? [André et al., 2019]

■ Can we **stop** the elapsing of some clocks? [Cassez and Larsen, 2000]

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Software supporting timed automata

Timed automata have been successfully used since the 1990s

Tools for modeling and verifying models specified using TA

■ HYTECH (also hybrid, parametric timed automata) [Henzinger et al., 1997]

■ KRONOS [Yovine, 1997]

■ TREX (also parametric timed automata) [Annichini et al., 2001]

UPPAAL [Larsen et al., 1997]

■ ROMÉO (parametric time Petri nets) [Lime et al., 2009]

PAT (also other formalisms) [Sun et al., 2009a]

■ IMITATOR (also parametric timed automata) [André et al., 2012]

Further challenges

Controller synthesis

[Sankur et al., 2013, Bacci et al., 2018]

■ Game theory

■ Timed language inclusion (using TA as a specification language)

■ Decidable subclasses [Ouaknine and Worrell, 2003, Ouaknine and Worrell, 2004]

■ Practical algorithms [Wang et al., 2017]

■ Robustness [De Wulf et al., 2004, Bouyer et al., 2013, Bacci et al., 2018]

■ Distributed algorithms [Laarman et al., 2013, Zhang et al., 2016]

Still a very active research field!

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Some case studies and application domains

■ Scheduling and real-time systems

[Fehnker, 1999, Abdeddaïm and Maler, 2001, Adbeddaïm et al., 2006, Abdeddaïm and Masson, 2012]

■ Protocols

■ Bounded retransmission protocol [D'Argenio et al., 1997]

■ Audio-video protocol [Havelund et al., 1997]

■ Fast Reservation Protocol [Tripakis and Yovine, 1998]

■ IEEE 1394a root contention protocol [Simons and Stoelinga, 2001]

■ Hardware circuits

[Bozga et al., 2002, Chevallier et al., 2009]

■ **Health** and biology [Schivo et al., 2014]

■ Monitoring [Waga et al., 2016, Waga et al., 2018]

■ Survey on the industrial use of UPPAAL [Larsen et al., 2018]

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What's beyond timed automata...?

■ Stopping clocks: **stopwatch automata**

[Cassez and Larsen, 2000]

- ② Undecidable
- ① Interesting application domains

■ Adding costs: energy

[Behrmann et al., 2001, Alur et al., 2004]

■ Enriching TA with tasks

[Fersman et al., 2007]

Adding unknown parameters

[Alur et al., 1993b]

■ Allowing non-linear clocks: **hybrid** automata

[Henzinger, 1996, Asarin et al., 2012]

■ Adding probabilities

[Kwiatkowska et al., 2002]

Statistical model checking

[Legay et al., 2010]

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Source and references

Towards a parametrization...

- Challenge 1: systems incompletely specified
 - Some delays may not be known yet, or may change

■ Challenge 2: Robustness

[Markey, 2011]

- \blacksquare What happens if 8 is implemented with 7.99?
- Can I **really** get a coffee with 5 doses of sugar?
- Challenge 3: **Optimization of timing constants**
 - Up to which value of the delay between two actions press? can I still order a coffee with 3 doses of sugar?
- Challenge 4: Avoiding numerous verifications
 - If one of the timing delays of the model changes, should I model check again the whole system?
- A solution: Parametric analysis
 - Consider that timing constants are unknown (parameters)
 - Find **good values** for the parameters s.t. the system behaves well

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General references

■ Timed Automata: Semantics, Algorithms and Tools [Bengtsson and Yi, 2003]

■ Systems and Software Verification [Bérard et al., 2001]

■ Principles of Model Checking [Baier and Katoen, 2008]

■ Timed temporal logics [Bouyer et al., 2017]

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