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LS2N Laboratoire des Sciences du Numérique de Nantes

CSL 2026

Paris, France

26th February 2026

Parametric disjunctive timed networks

Étienne André, Swen Jacobs, Engel Lefaucheux

LS2N, Nantes Université, Bretagne, France
CISPA Helmholtz Center for Information Security, Germany
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(in one slide)

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Concurrent multi-agent real-time systems

Complex systems involving

- Multiple agents
- Concurrency
- Real-time constraints

Hard!

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Complex systems involving

- Multiple agents
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- Real-time constraints

Hard!

+ uncertainty

? unknown number of agents

? timed uncertainty (delays known with finite precision, or unknown)

Even harder!

Model checking in a nutshell

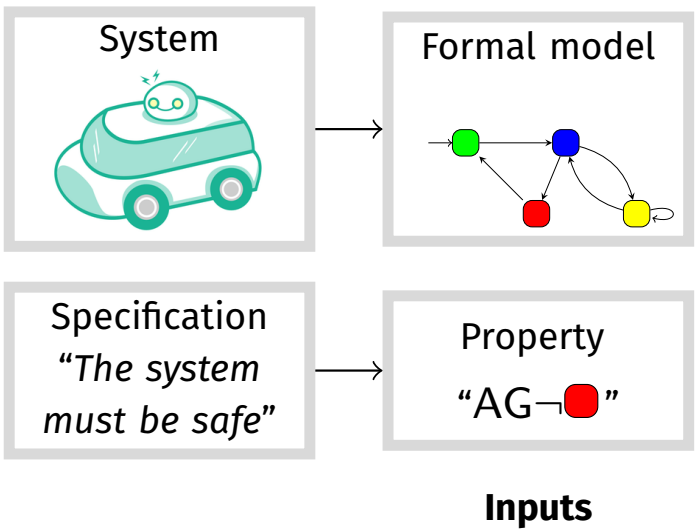
- Principle of model checking

System

Specification
"The system must be safe"

Model checking in a nutshell

- Principle of model checking



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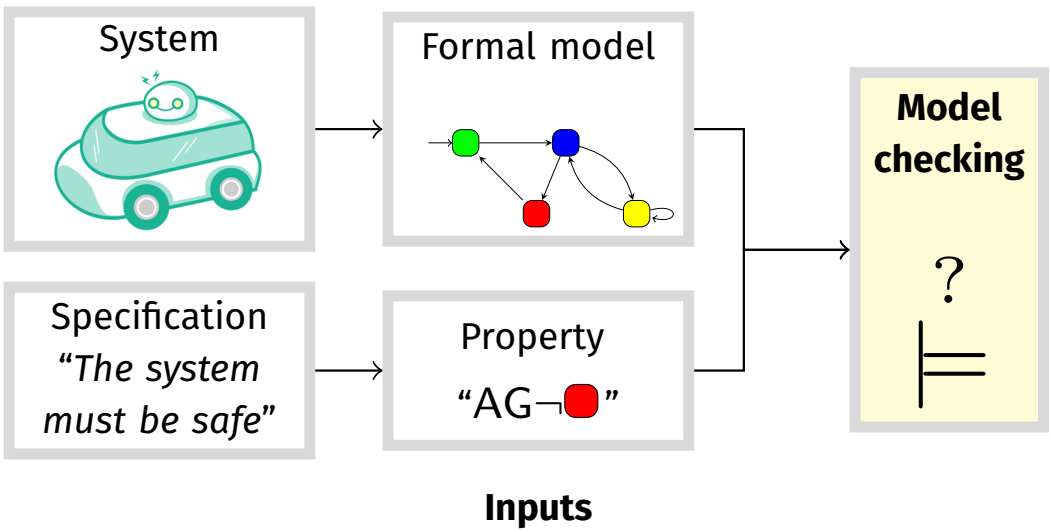

- Question: does the model of the system satisfy the property?

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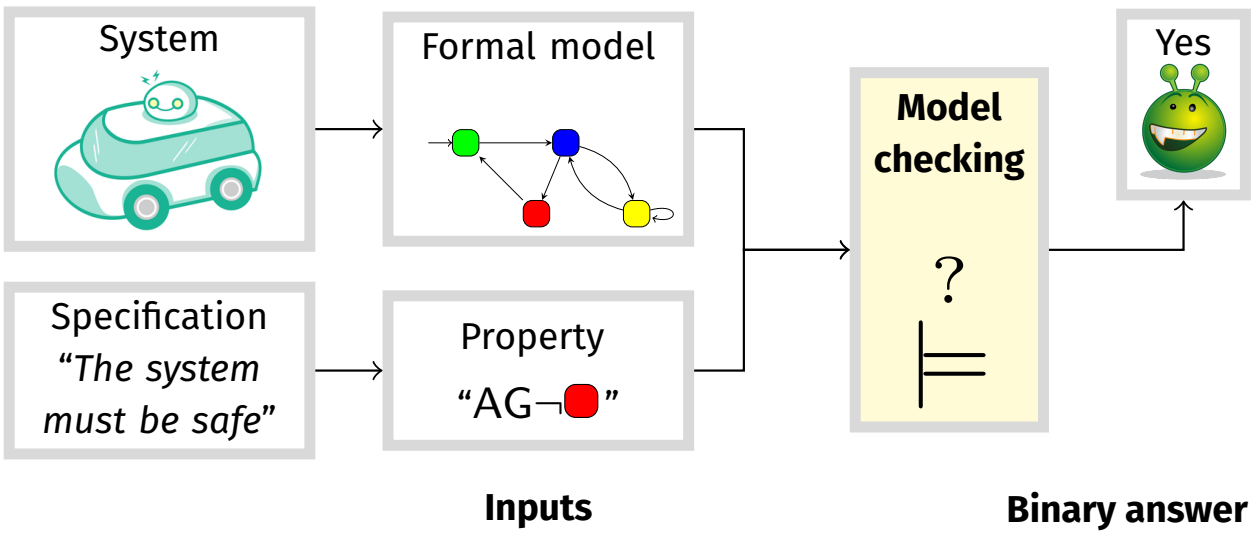

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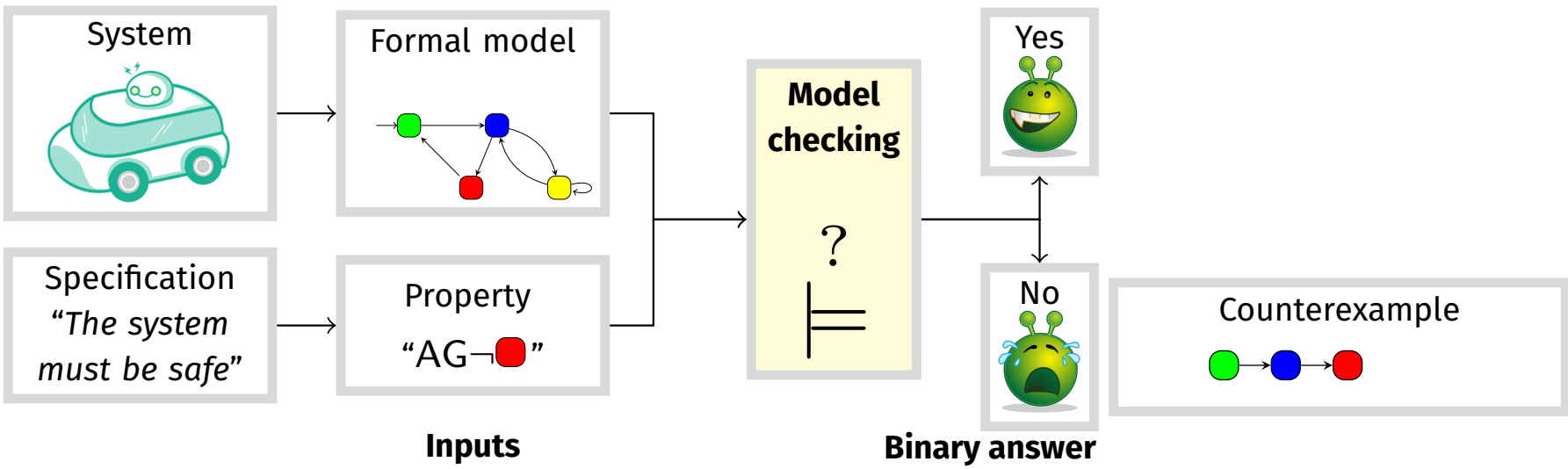

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Outline

Timed automata

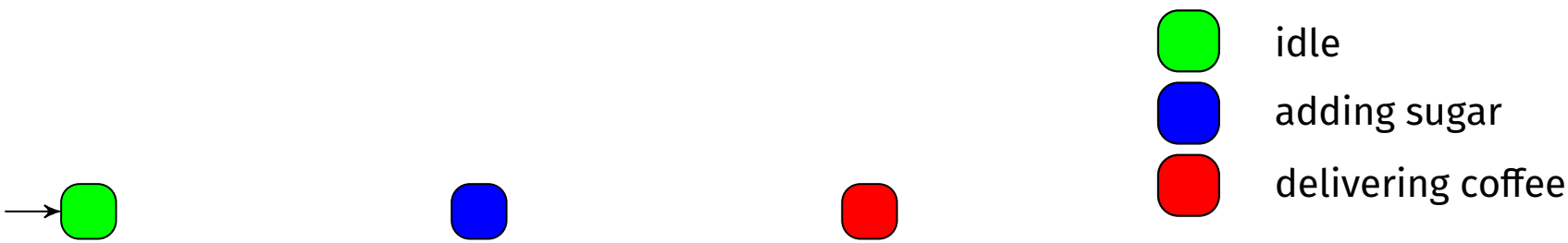
Extension of finite-state automata to concurrency [AD94]

- ☺ multiple agents (multiple timed automata in parallel)
- ☺ concurrency
- ☺ synchronisation
- ☺ real-time constraints (clocks)
- ☺ a number of decidable problems

[AD94] Rajeev Alur and David L. Dill. "A theory of timed automata". In: *Theoretical Computer Science* 126.2 (Apr. 1994), pp. 183–235

Timed automaton (TA)

- Finite-state automaton (sets of locations)



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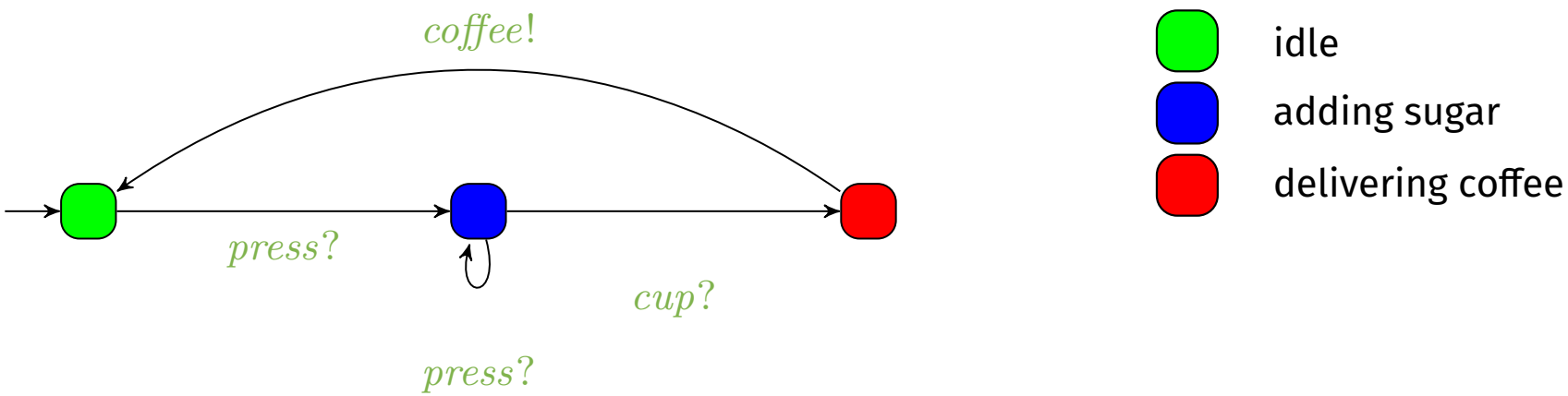
Timed automaton (TA)

- Finite-state automaton (sets of locations and actions)



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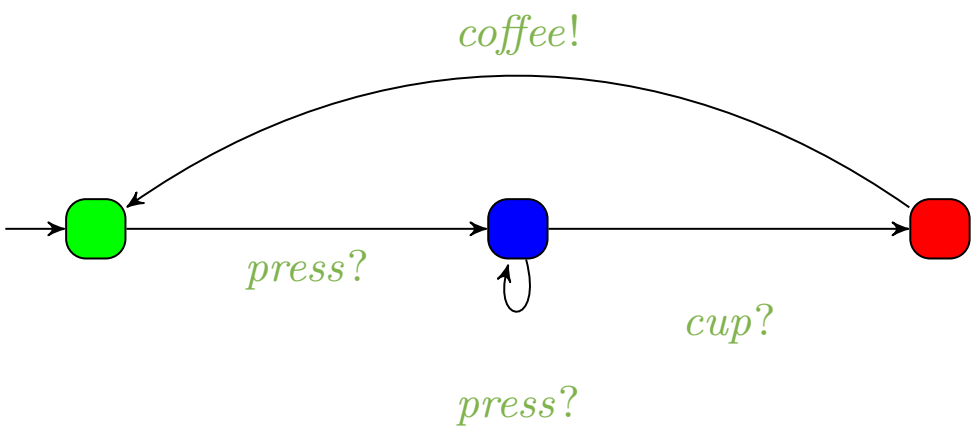
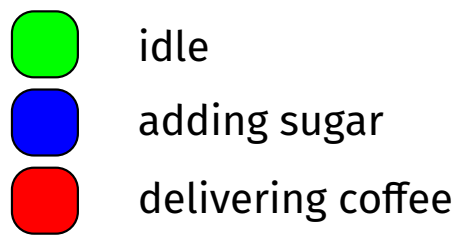
Timed automaton (TA)

- Finite-state automaton (sets of locations and actions) augmented with a set X of clocks [AD94]
  - Real-valued variables evolving linearly at the same rate





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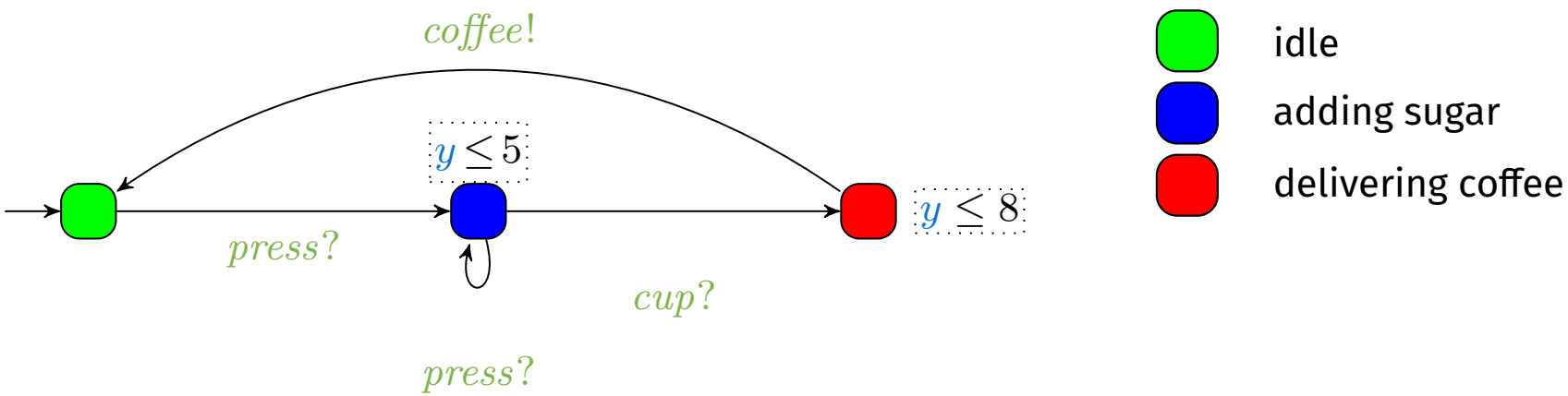

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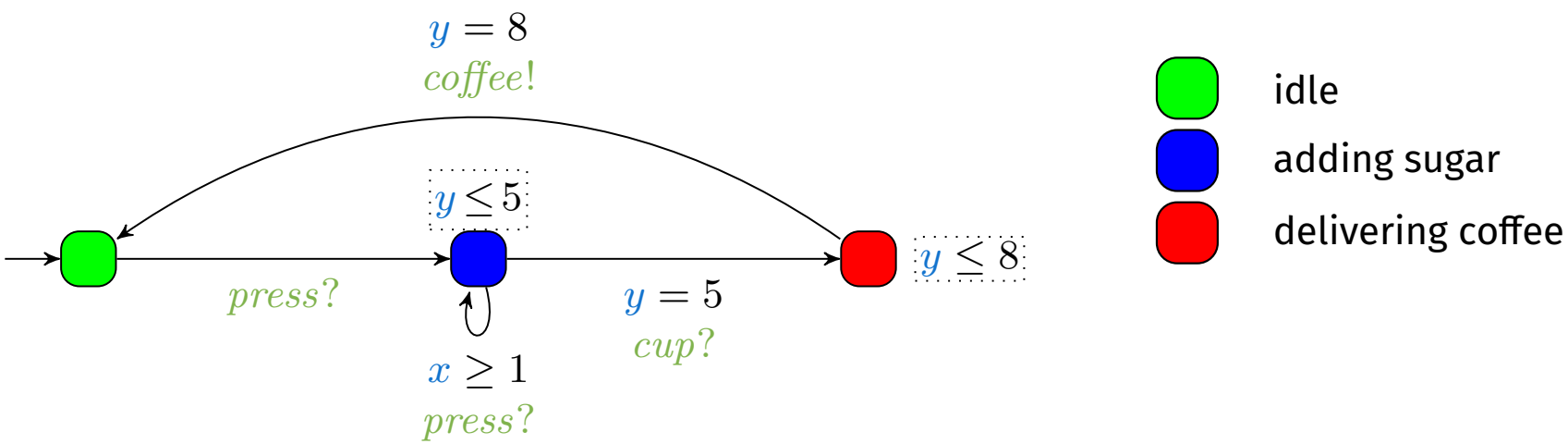

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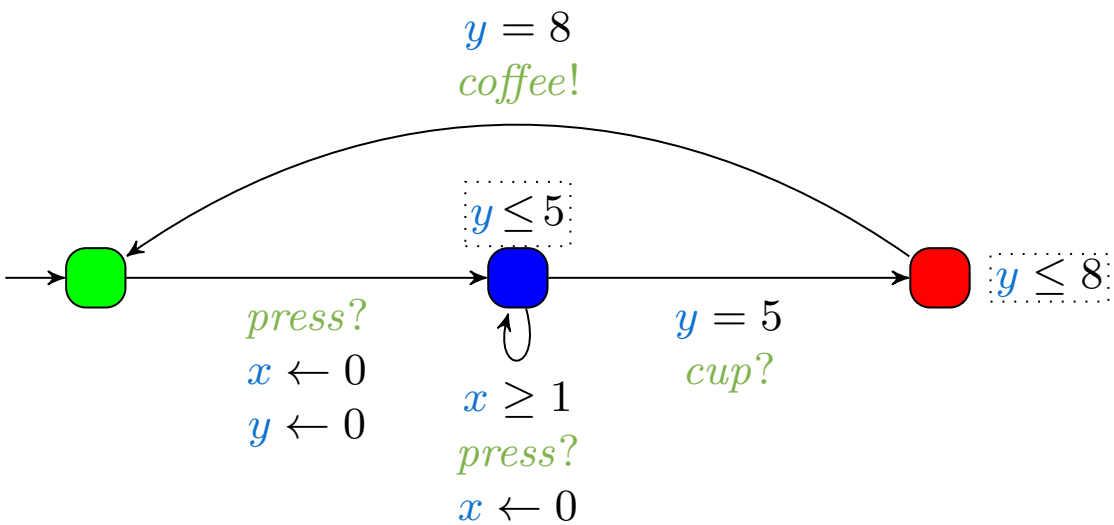
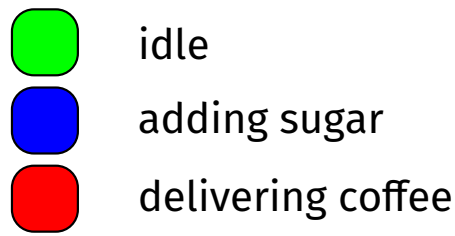
  - Clock reset: some of the clocks can be set to 0 along transitions





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Timed automata: advantages and drawbacks

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- ☹ requires infinite precision w.r.t. time

Outline

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Need for timed uncertainty

- Challenge 1: systems incompletely specified
  - Some delays may not be known yet, or may change
- Challenge 2: Robustness [Mar11]
  - What happens if 8 is implemented with 7.99?
- Challenge 3: Optimization of timing constants
- Challenge 4: Avoiding numerous verifications
  - If one of the timing delays of the model changes, should I model check again the whole system?

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  - If one of the timing delays of the model changes, should I model check again the whole system?
- A solution: Parametric analysis
  - Consider that timing constants are unknown (parameters)
  - Find good values for the parameters s.t. the system behaves well

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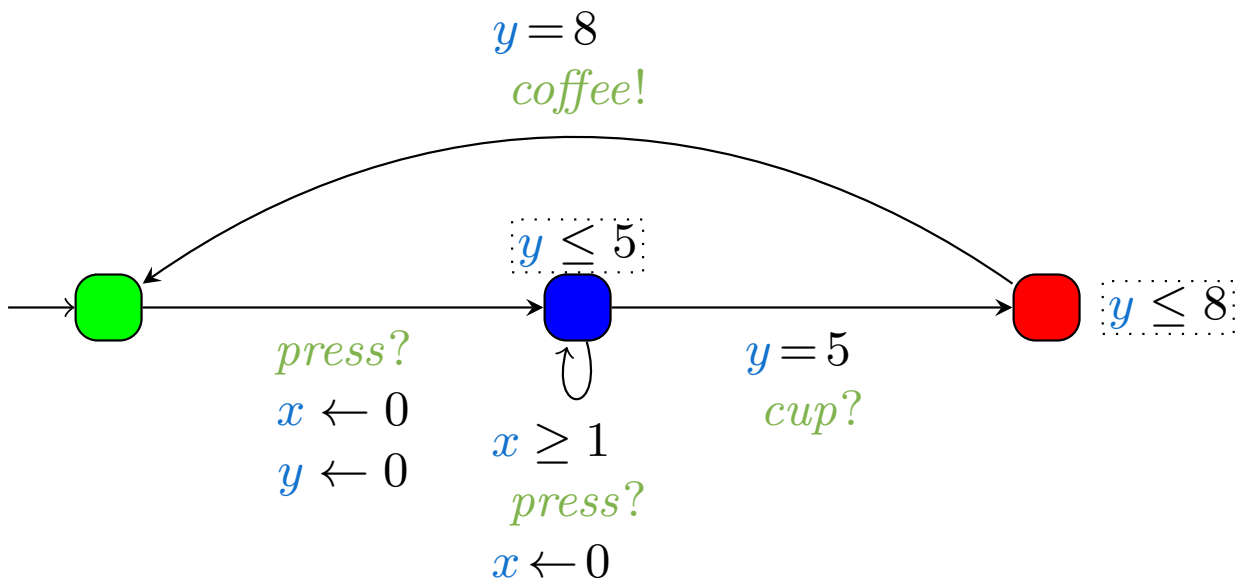
Parametric Timed Automaton (PTA)

- Timed automaton (sets of locations, actions and clocks)



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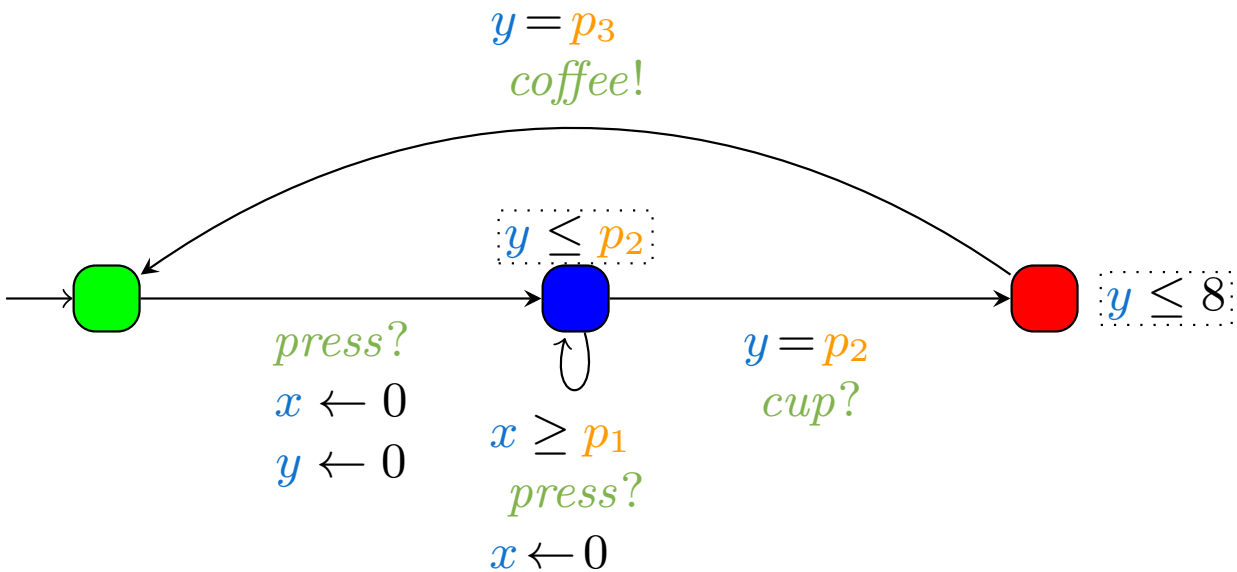

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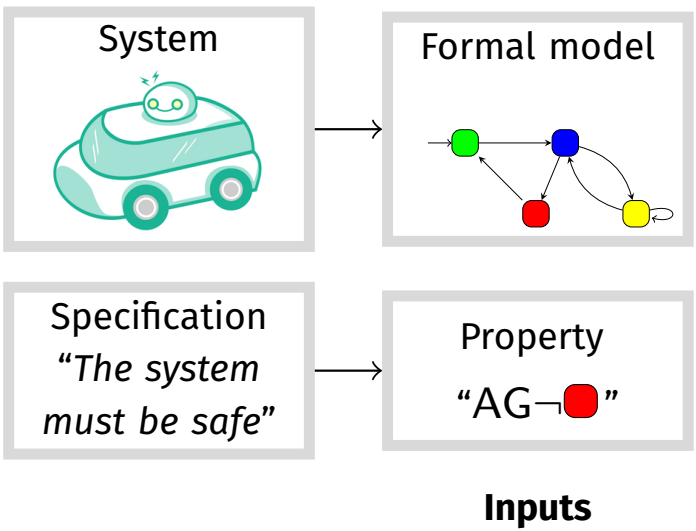

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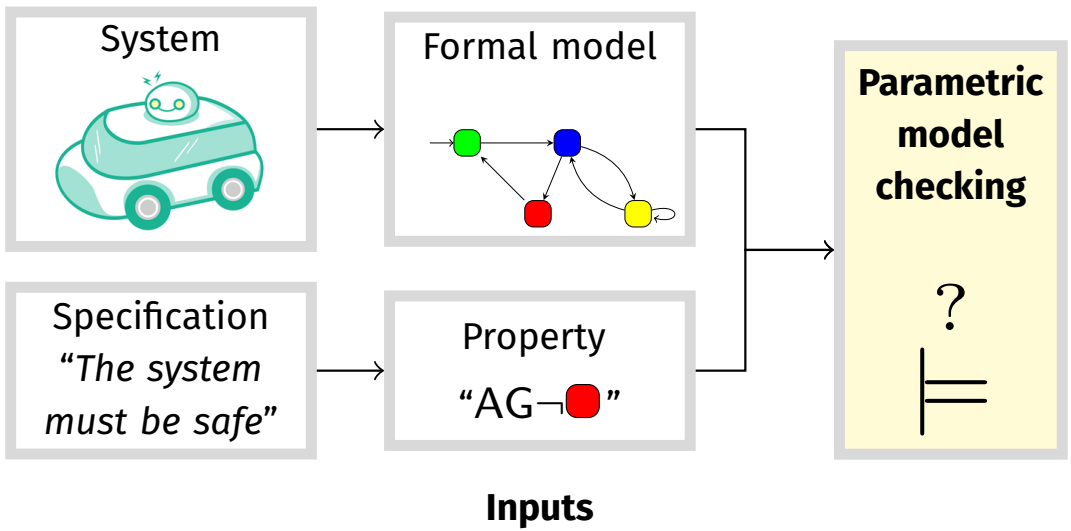

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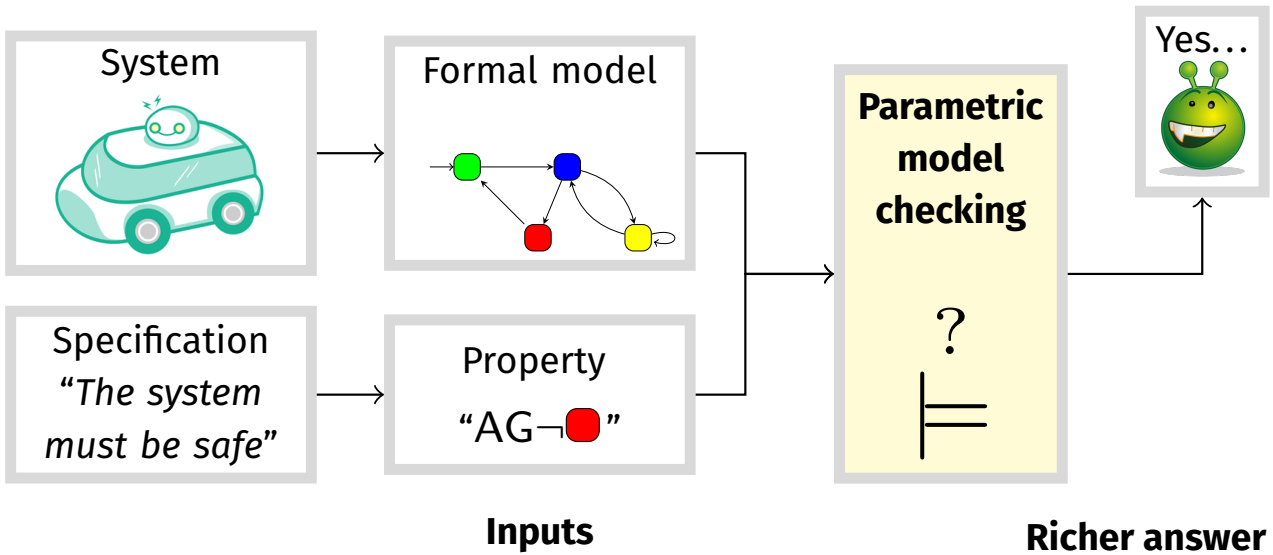

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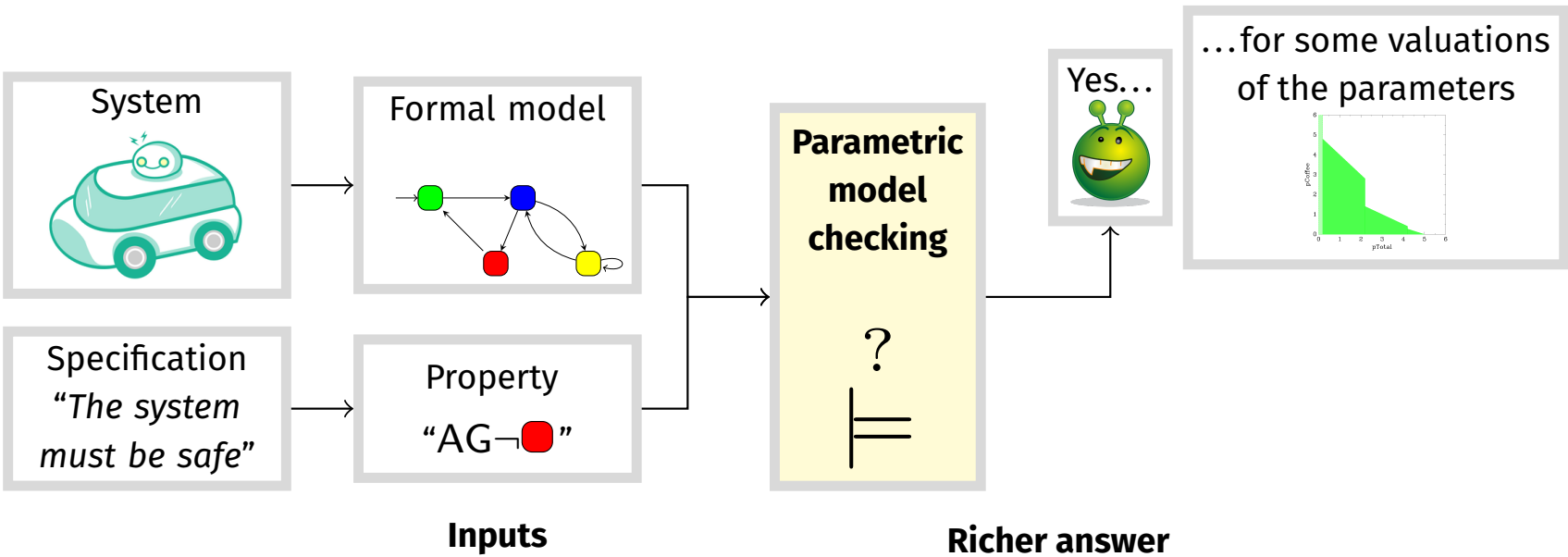

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Need for uncertainty in the number of participants

The number of system agents may be unknown

- networks
- communication protocols
- leader election

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Natural reachability questions include

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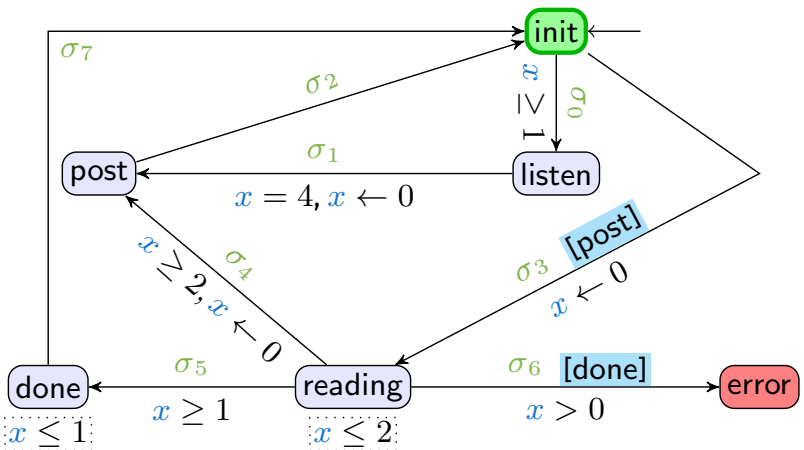
Considering networks of timed automata of arbitrary size:

☹ Many undecidable problems

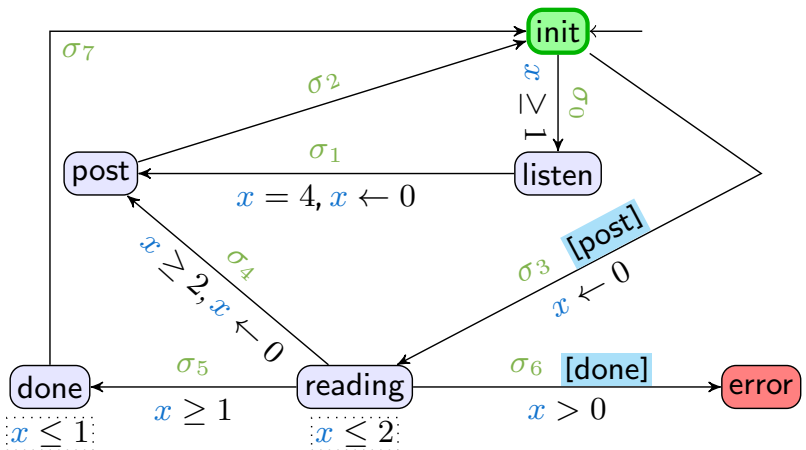
Disjunctive timed networks (DTNs)

Network of identical timed automata (“processes”) communicating via location guards [SS20]

- “a process can take a transition guarded by a location $[\ell]$ if at least one other process currently occupies ℓ”



$x = 0$



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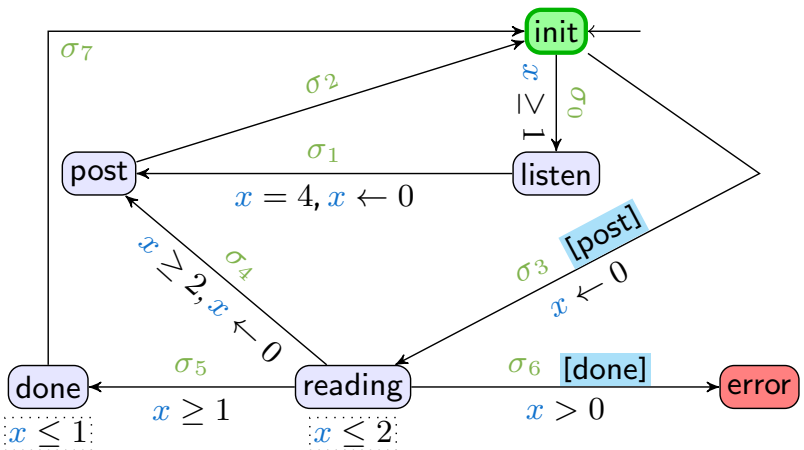

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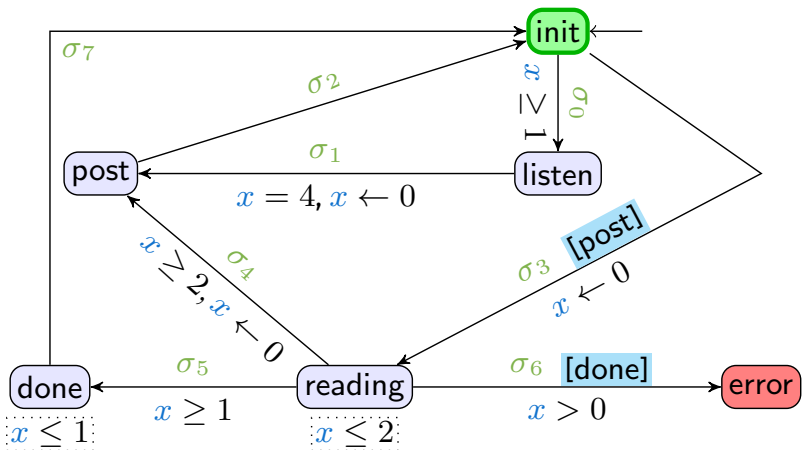

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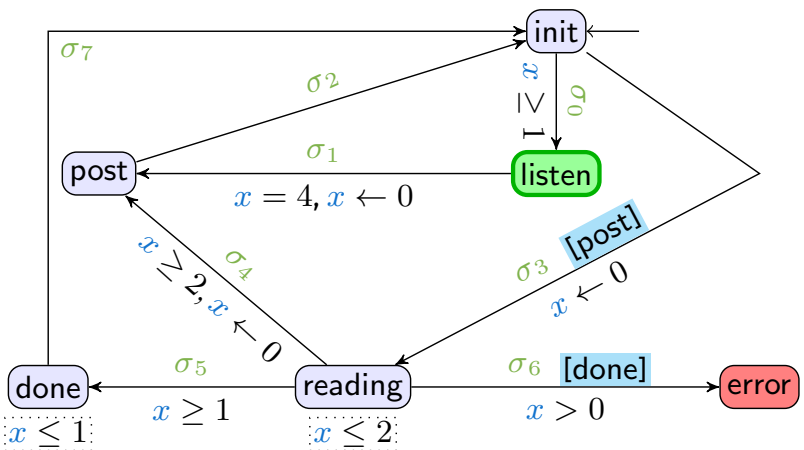

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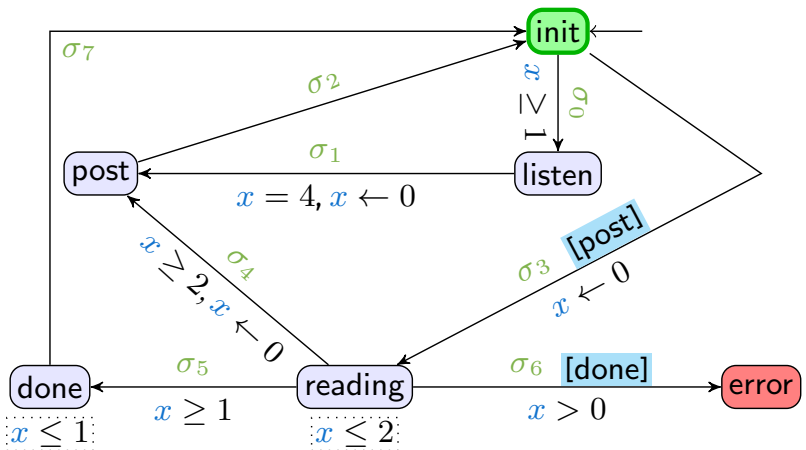

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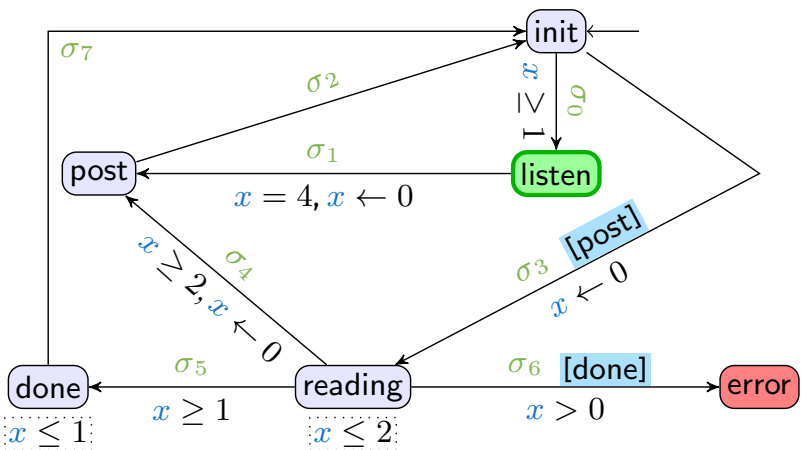

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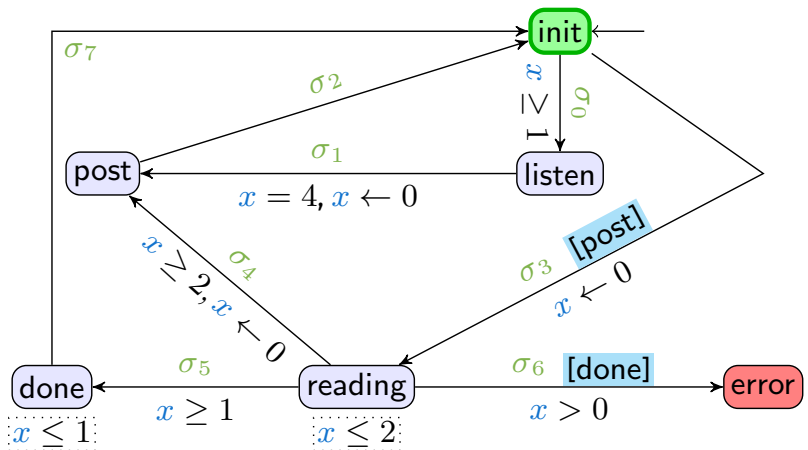

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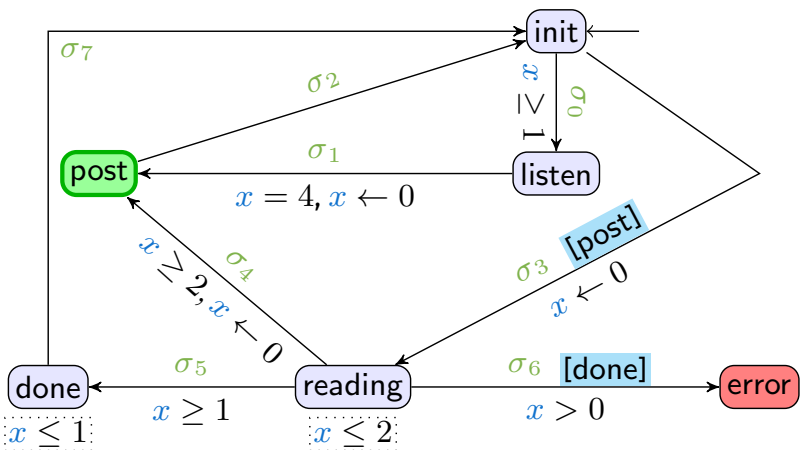

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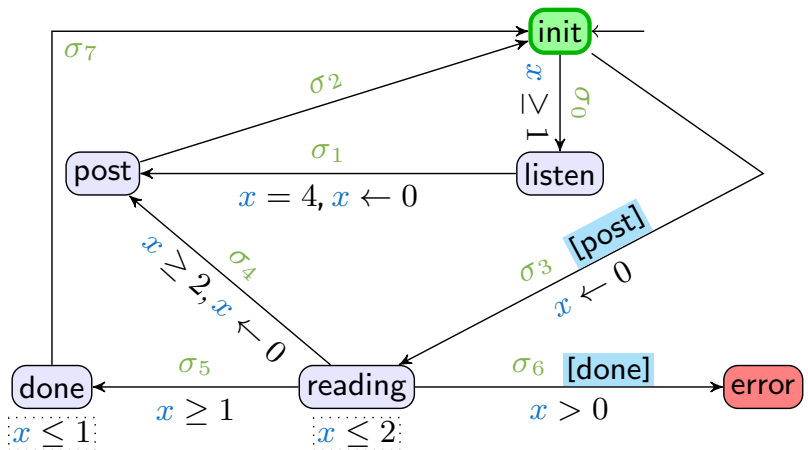

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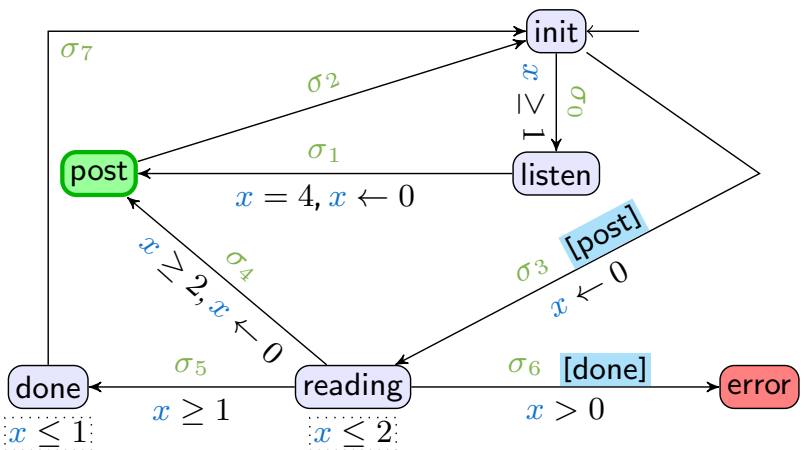

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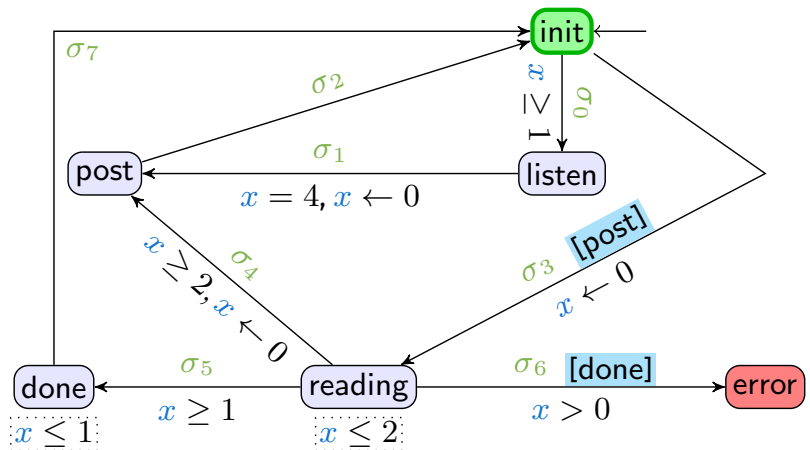

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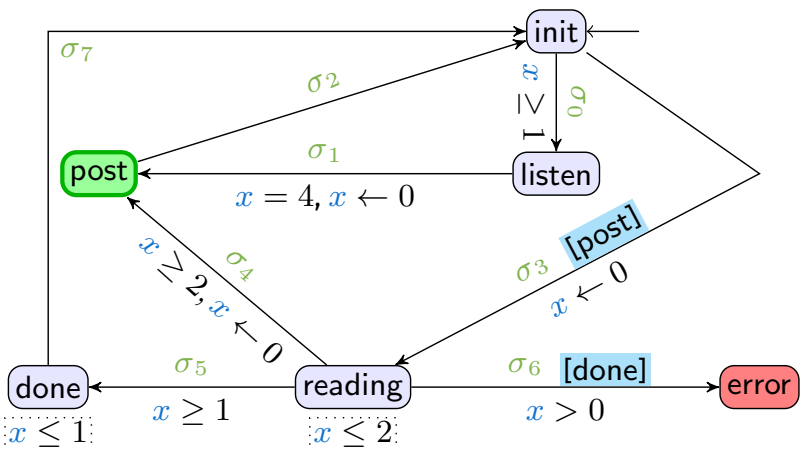

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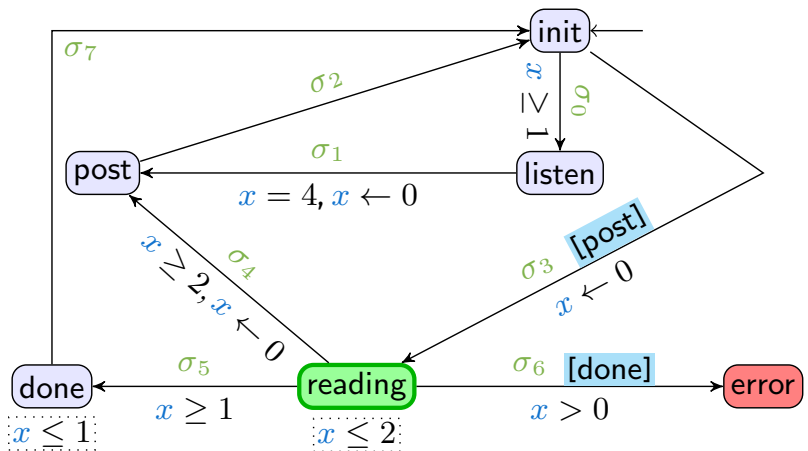

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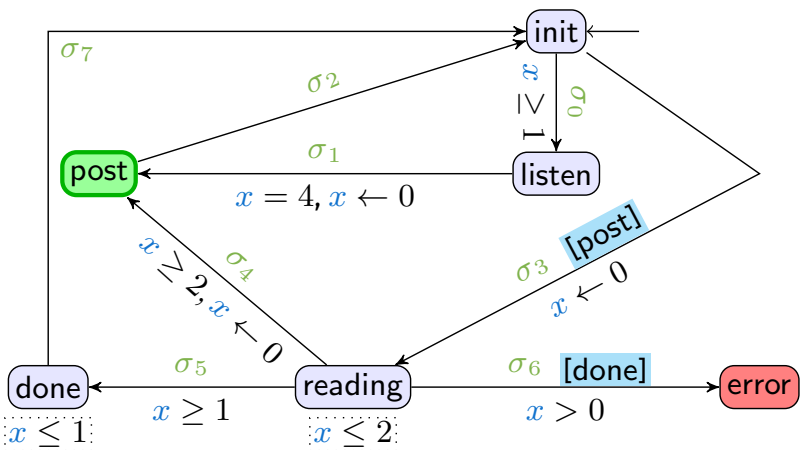

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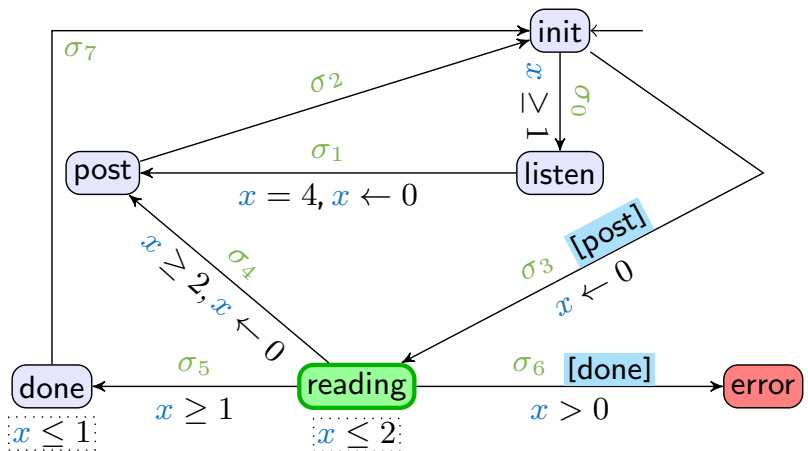

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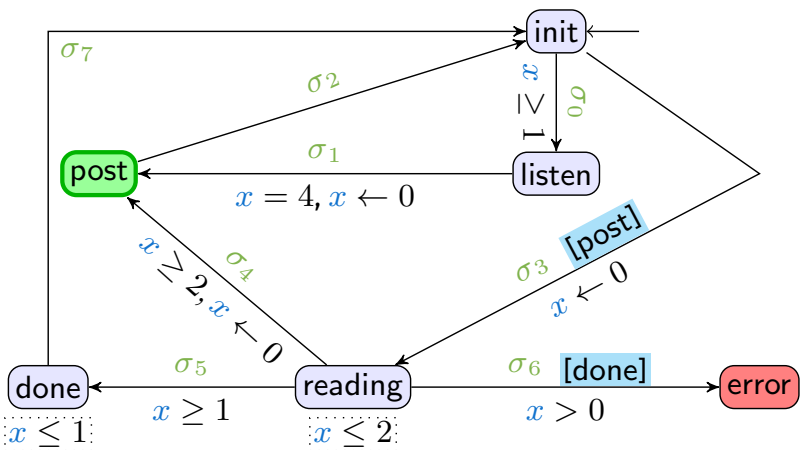
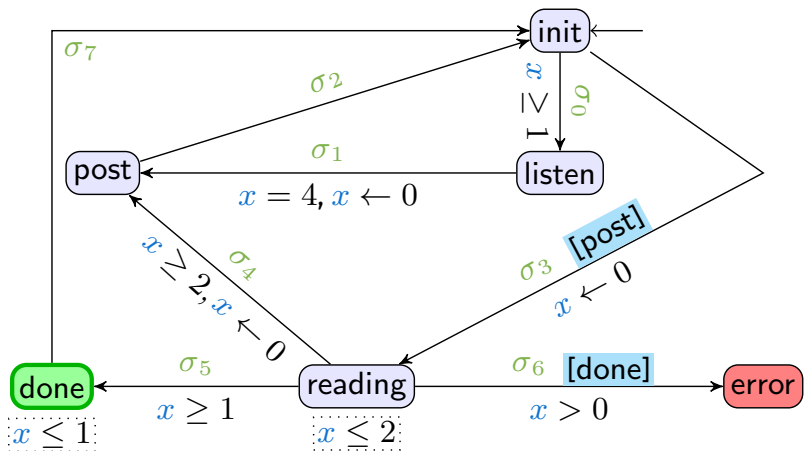


$x = 1.8$



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[SS20] Luca Spalazzi and Francesco Spegni. “Parameterized model checking of networks of timed automata with Boolean guards”. In: *Theoretical Computer Science* 813 (2020), pp. 248–269

Disjunctive timed networks (DTNs)

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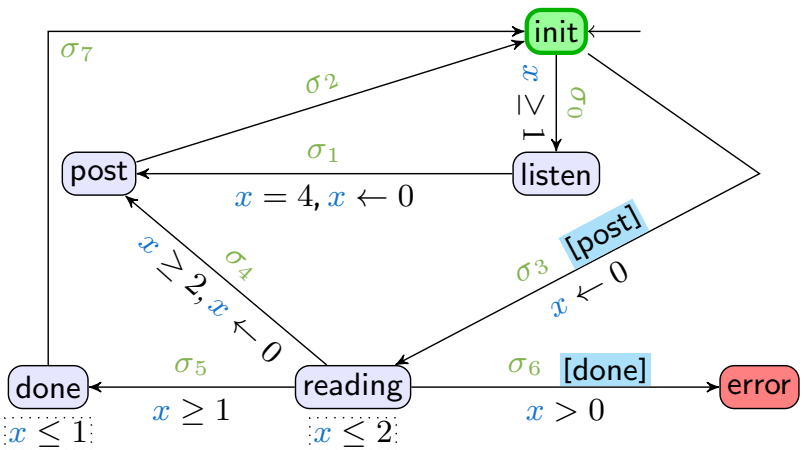

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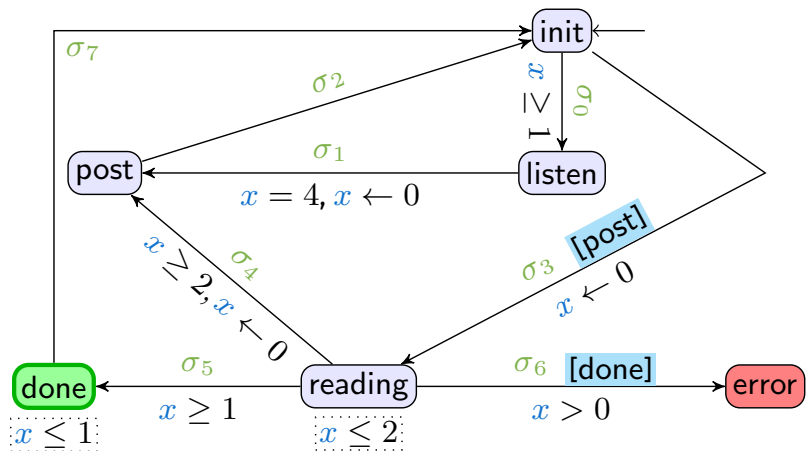

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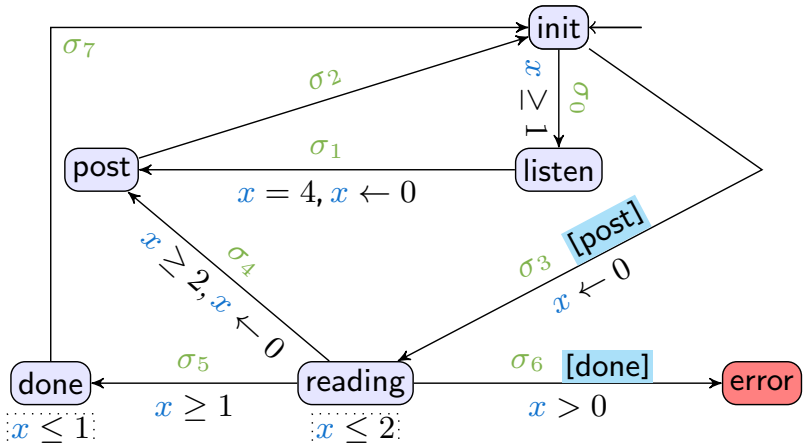
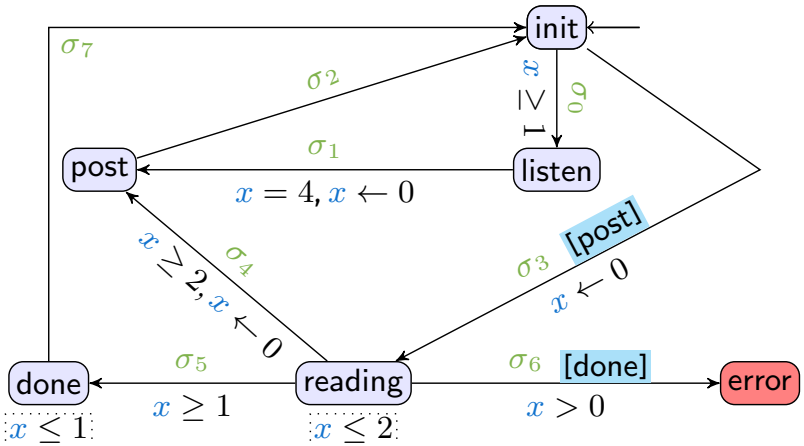
Disjunctive timed networks: problems



Example of questions

- Is location “error” reachable by one process for some number of processes?
  - 1 process:

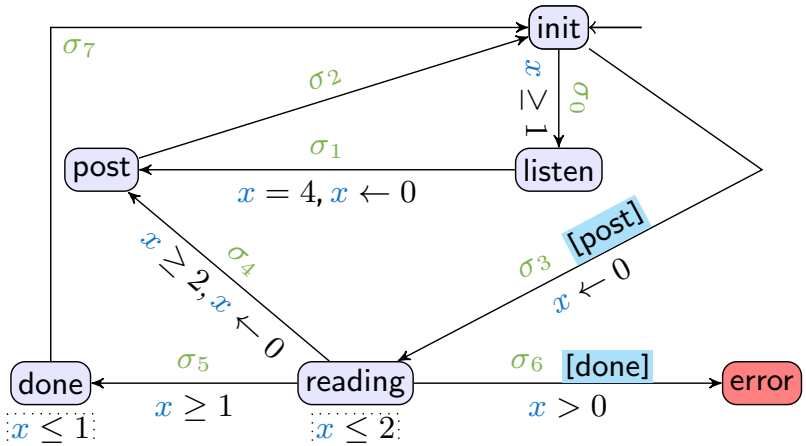
Disjunctive timed networks: problems



Example of questions

- Is location “error” reachable by one process for some number of processes?
  - 1 process: ×
  - 2 processes:

Disjunctive timed networks: problems



Example of questions

- Is location “error” reachable by one process for some number of processes?
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  - 2 processes: ×
  - 3 processes:

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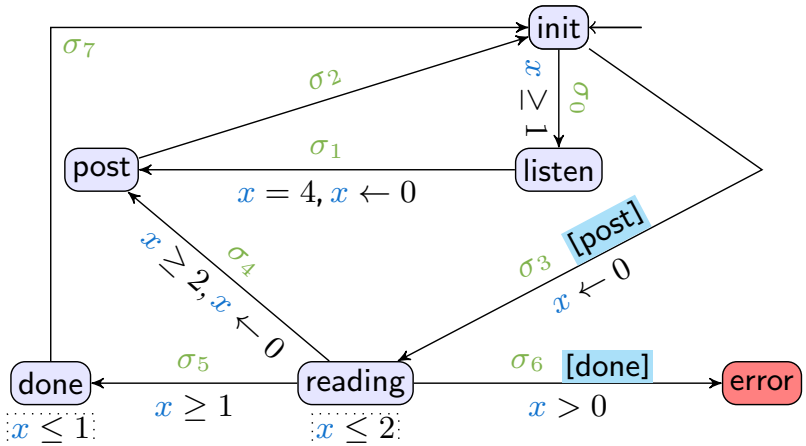

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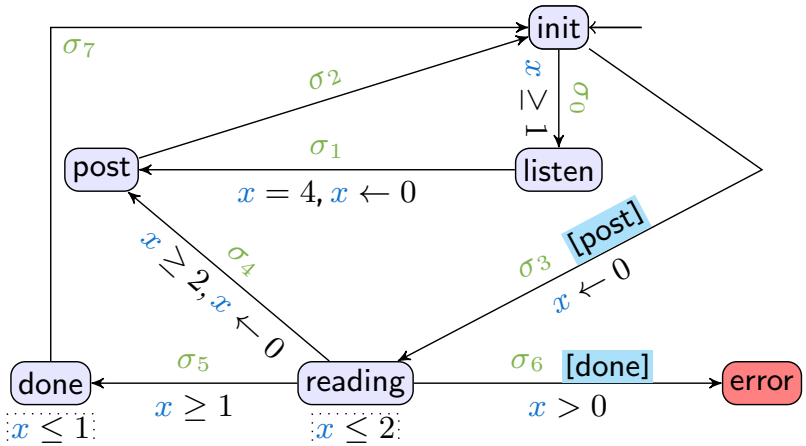

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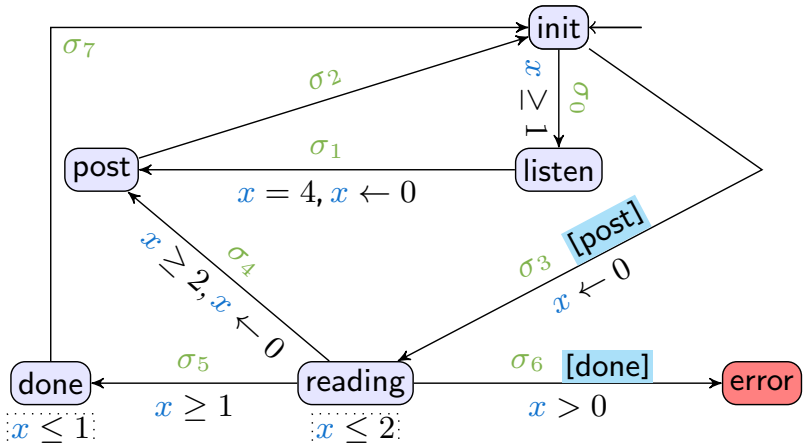

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  - ≥ 3 processes: √

- Is location “error” reachable by all processes for some number of processes?
  - ×

Disjunctive timed networks: decidability

Invariants	Local properties	Global properties
Without	$\surd$ [SS20][And+24]	$(?)$
With	$\surd$ [And+25]	$(?)$

Informal definitions:

Local properties “reachability for one process”

Global properties “reachability for all processes”

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Disjunctive timed networks: advantages and drawbacks

- ☺ multiple agents
- ☺ concurrency
- ☺ synchronisation
- ☺ real-time constraints (clocks)
- ☺ some decidable problems

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- ☺ arbitrary number of agents
- ☹ requires infinite precision w.r.t. time

Outline

Objective: combine both types of uncertainty

Consider an unbounded number of agents under uncertainty over time

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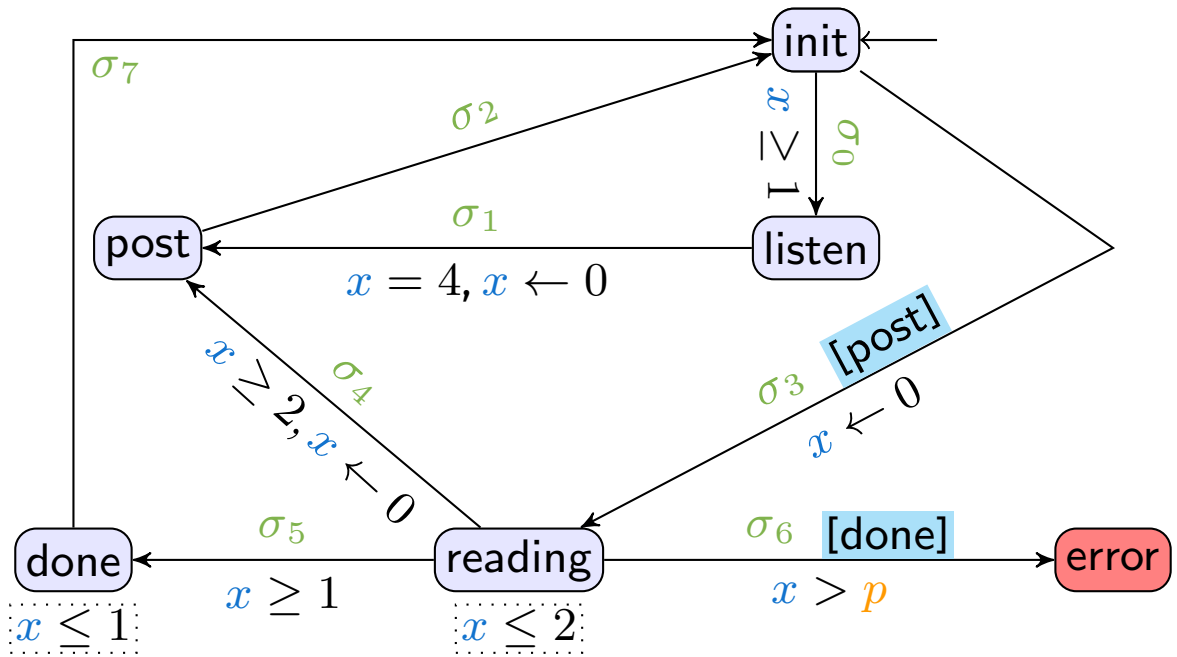
Consider an unbounded number of agents under uncertainty over time

- We introduce parametric disjunctive timed networks (PDTNs)



Challenge

Many existing undecidability results when considering only one of our two types of uncertainty in isolation

- ☹ single parametric timed automaton with ≥ 3 clocks
- ☹ single parametric timed automaton with 1 clock compared to 1 parameter (and 3 other clocks)
- ☹ networks of arbitrary numbers of non-parametric timed automata with other synchronisation mechanisms than location guards

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Challenge: exhibit decidability results for subclasses of PDTNs

Main existential problem: "does there exist a number of processes and a valuation of the timing parameters such that one process can reach a given location?"

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Useful notably for safety

Outline

Our paper in one slide

Clocks	Integer-valued parameters	Invariants	Local properties	Global properties
1	arbitrary	**Without**	√	
		With		
arbitrary	1-param. fully-parametric	**Without**		
		With		
arbitrary	arbitrary, L/U	**Without**		
		With		

☺ Main decidability result: formula in Presburger arithmetic enhanced with the divisibility operand, written using affine parametric semi-linear sets

Our paper in one slide

Clocks	Integer-valued parameters	Invariants	Local properties	Global properties
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		With	$\times$	
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arbitrary	arbitrary, L/U	**Without**		
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- ☺ Main decidability result: formula in Presburger arithmetic enhanced with the divisibility operand, written using affine parametric semi-linear sets
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  - either invariants to avoid processes “left behind”

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- ☺ Also decidable: single parameter, no constant (apart from 0) in the PTAs

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		With	× ☻	×
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		With	√	
arbitrary	arbitrary, L/U	**Without**	√	
		With	√	

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  - monotonicity: boils down to non-parametric DTNs

Our paper in one slide

Clocks	Integer-valued parameters	Invariants	Local properties	Global properties
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		With	$\times$ ☻	$\times$
arbitrary	1-param. fully-parametric	**Without**	$\sqrt{}$	?
		With	$\sqrt{}$	?
arbitrary	arbitrary, L/U	**Without**	$\sqrt{}$	?
		With	$\sqrt{}$	?

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  - Do all processes reach a given location for at least one valuation of the parameters and at least one number of processes?

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Additional slides

Parameterized verification

Undecidable in general [AK86]

- Even in very restricted settings (for safety properties of finite-state processes with rather weak communication primitives, such as token-passing or transition guards) [Suz88][EK00]

[AK86] Krzysztof R. Apt and Dexter Kozen. "Limits for Automatic Verification of Finite-State Concurrent Systems". In: *Information Processing Letters* 22.6 (1986), pp. 307–309

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Parameterized timed verification

Several lines of research studying networks with a parametric number of timed components, including

- [AJ03]
- [AAC12]
- [BF13]
- [Ami+15]
- [Abd+16]
- [And+19]

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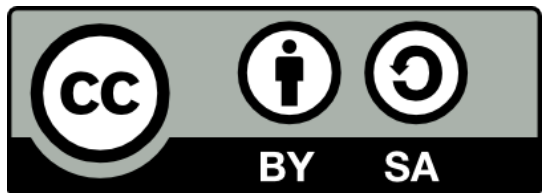

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