Parametric Time Broadcast Protocols

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VMCAI'19

Motivation

Our framework allows to model **Networks** of **identical** systems.

$\Rightarrow$ *Communication protocols*

- Expressive (Timing features)
- Abstract (Parameters)

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Goal: Parametric model checking

Real motivation: Models mixing 2 types of parameters

Outline

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(Parametric) Timed Automata

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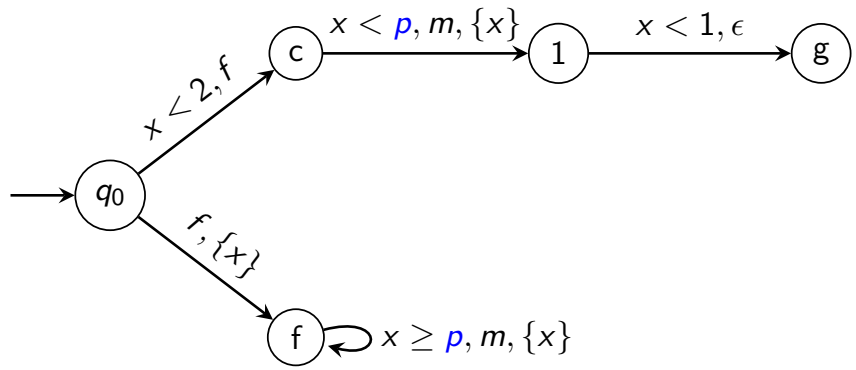
- Clocks
- Guarded transitions
- (Invariants)
- Timing Parameters p



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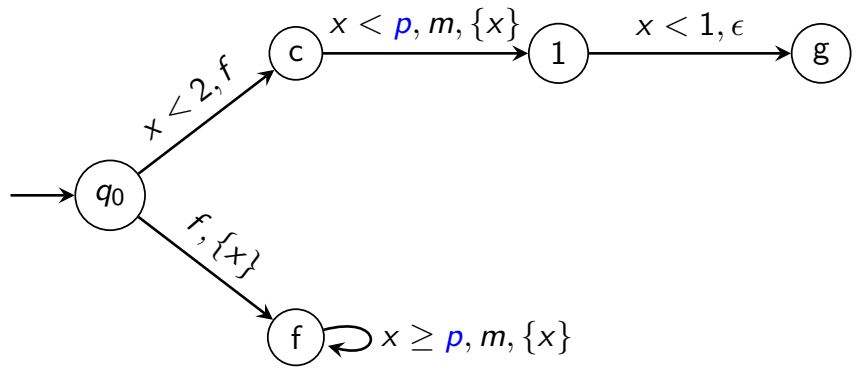

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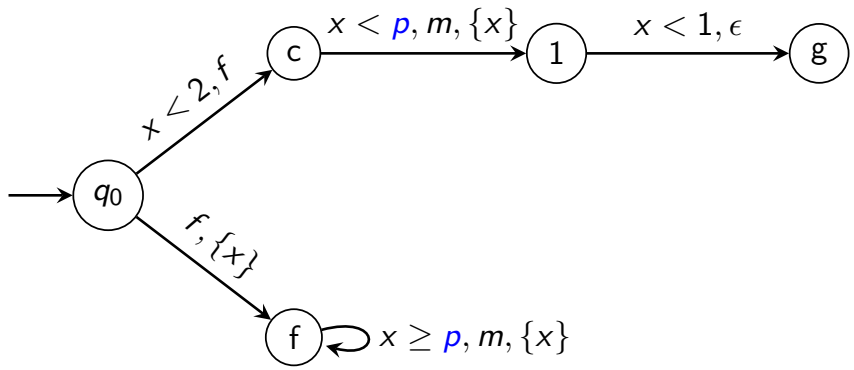


Does there exist a value of p such that g is reachable?

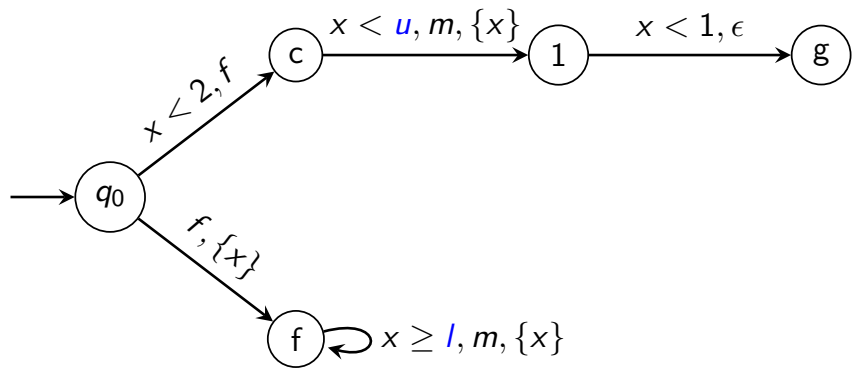
Undecidable in the general setting (1 clock compared to 1 parameter)

(L/U)-PTA

L/U-PTA

L/U-PTA are PTA where parameters are partitioned into

- Lower-bound parameters (L)
- Upper-bound parameters (U)



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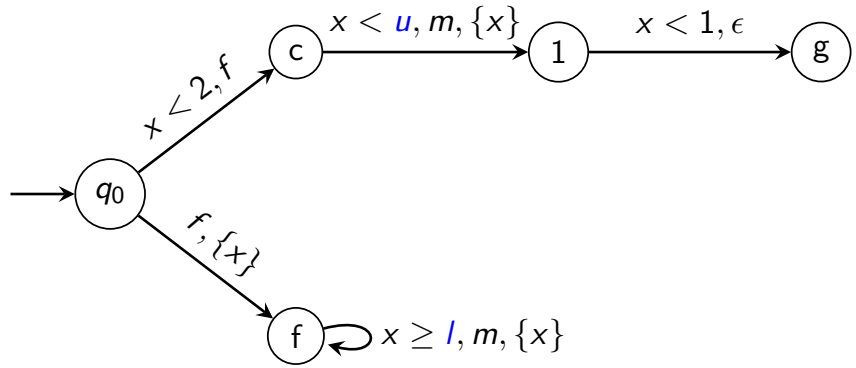
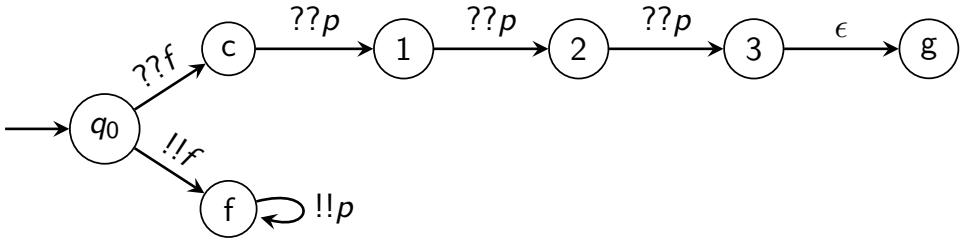


In this context, some reachability problems become decidable.

Broadcast Protocol Networks



- Parametric number of identical processes n
- Communicating
- Arbitrary communication topologies:
  - Reconfigurable: Non-det Receiver set
  - Clique: All processes receive
  - ...

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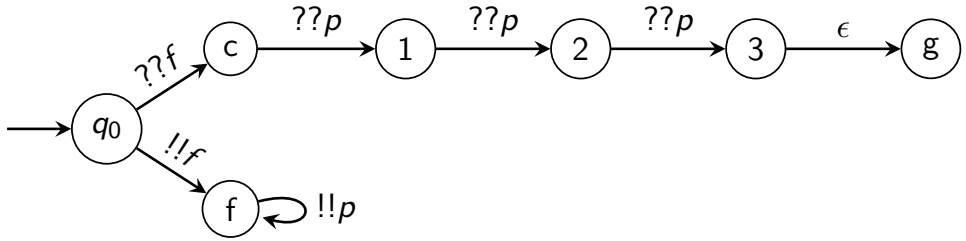

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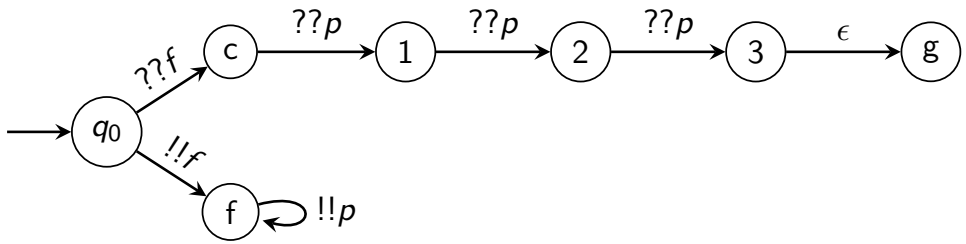

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Undecidable in general, but not for cliques

Outline

Parametric Time Broadcast Protocol Networks

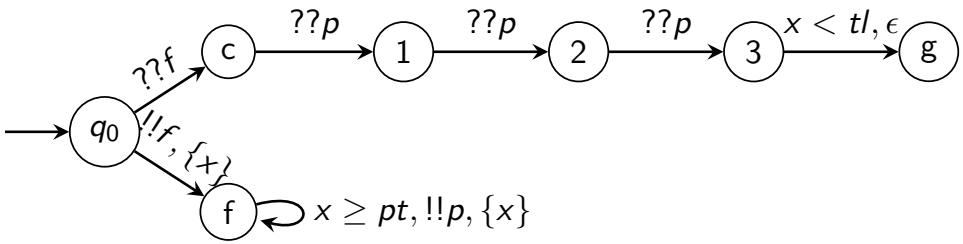
Definition (PTBP Network)

A **Parametric Time Broadcast Protocol Network** is a *Broadcast Protocol Network*, with a *parametric number* of identical processes, which are all modeled using a *Parametric Timed Automaton*.

- Two types of parameters:
  - Timing
  - Number of processes
- Transitions are of three types:
  - internal: ϵ
  - emissions: $!!m$
  - receptions: $??m$

In the following, we assume that all timing parameters are **bounded**.

PTBP Network - Example



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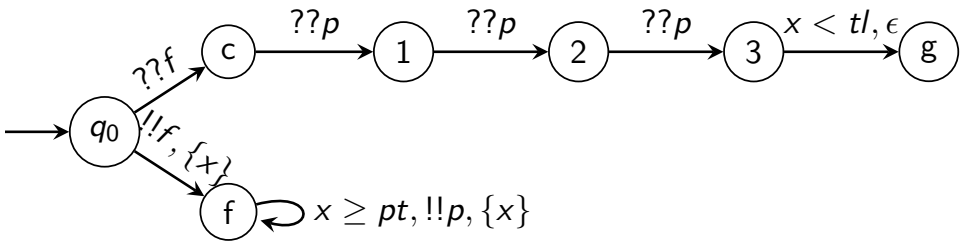

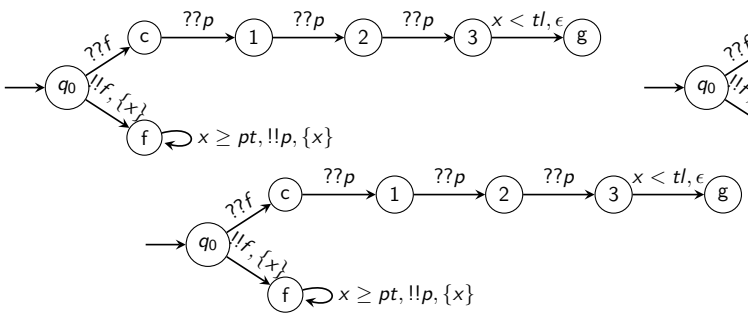

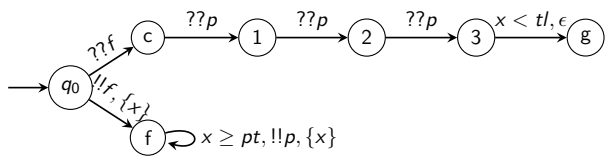

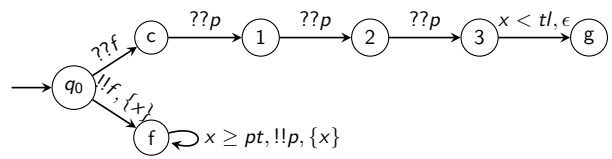

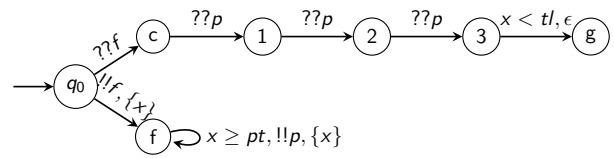

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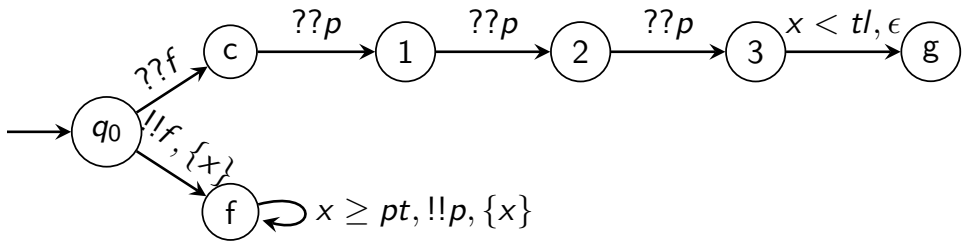

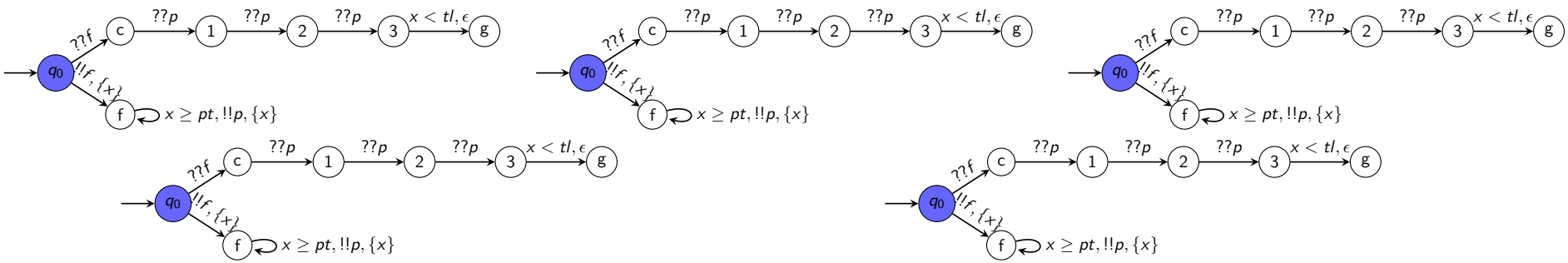

$$\begin{pmatrix} q_0, 0 \\ q_0, 0 \\ q_0, 0 \\ q_0, 0 \\ q_0, 0 \end{pmatrix}$$

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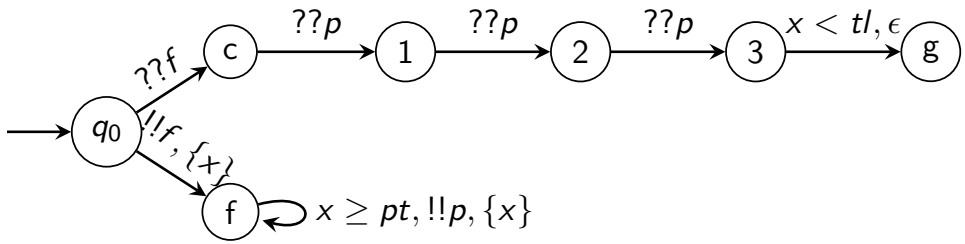

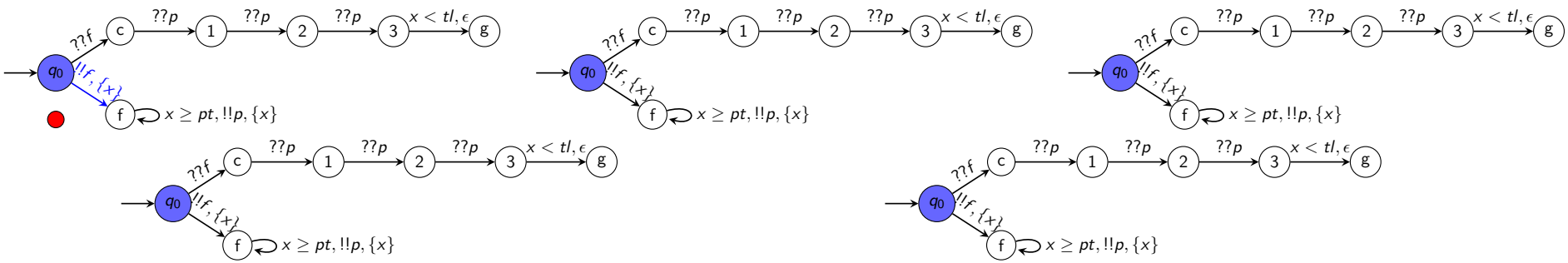

$$\begin{pmatrix} q_0, 0 \\ q_0, 0 \\ q_0, 0 \\ q_0, 0 \\ q_0, 0 \end{pmatrix} \xrightarrow{0.1, 1, f, \emptyset}$$

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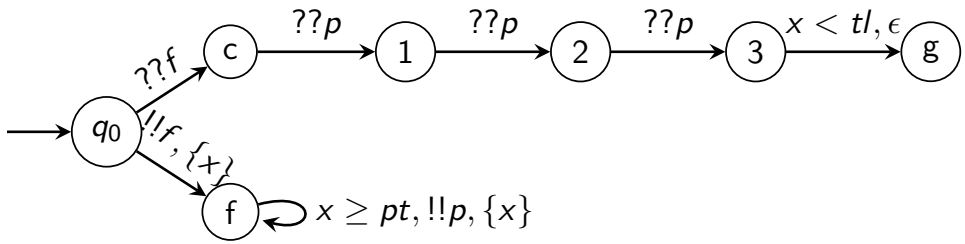

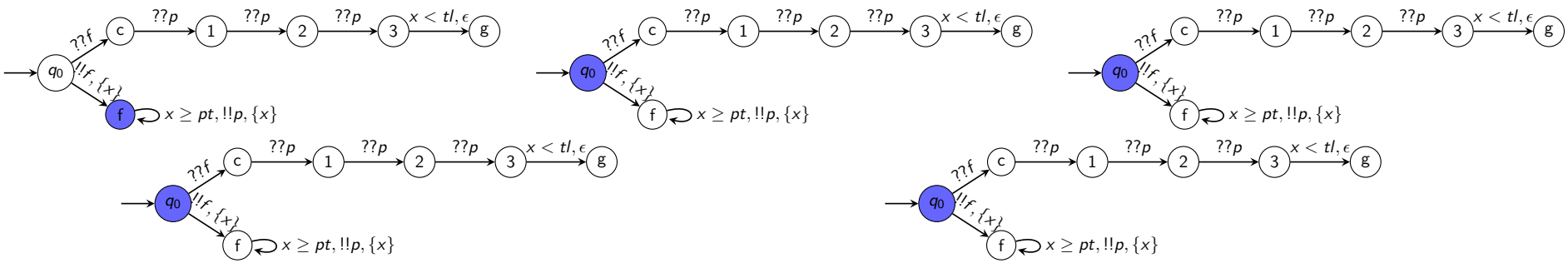

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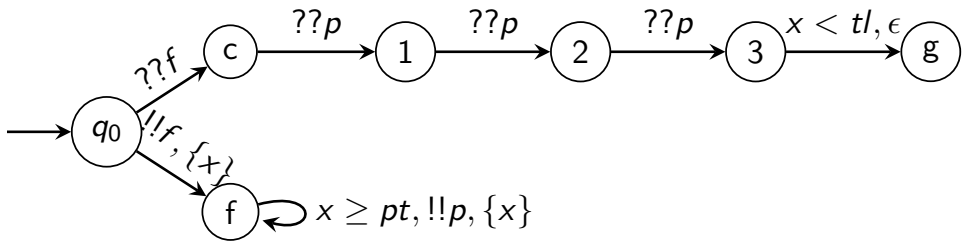

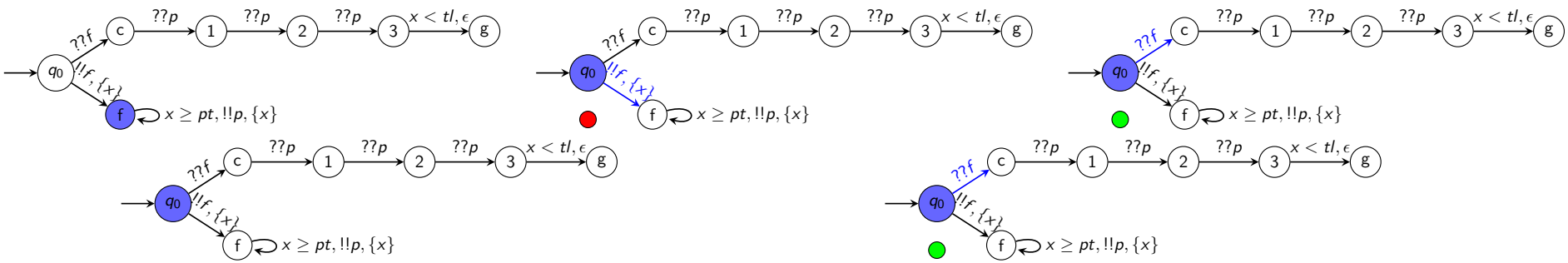

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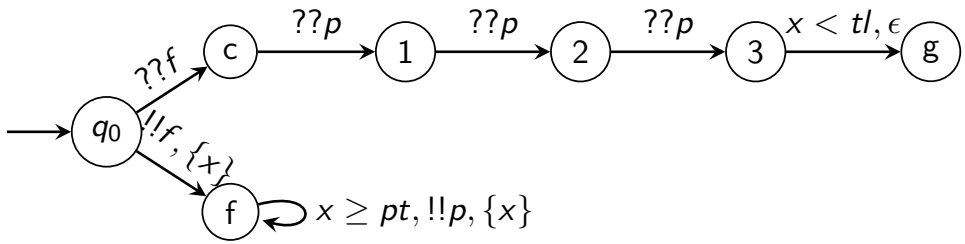

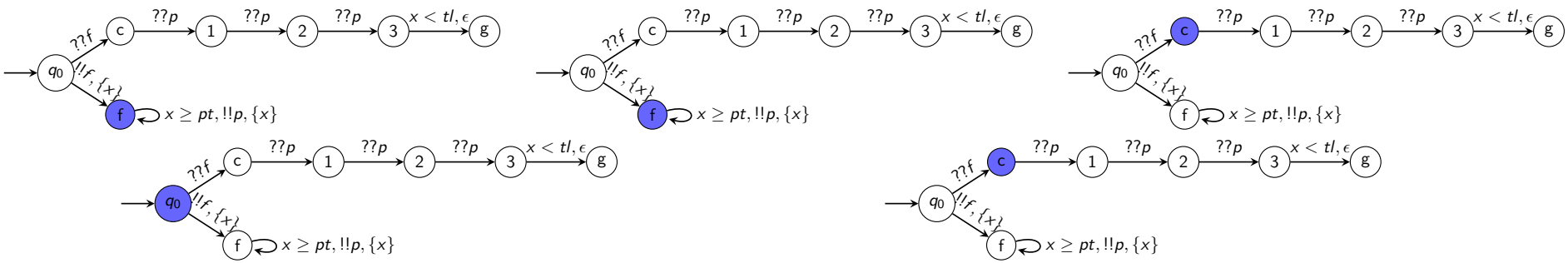

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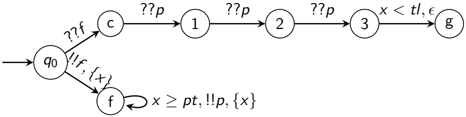
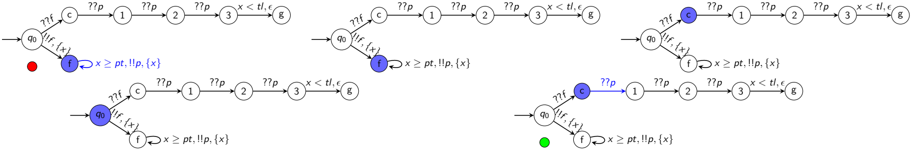


- $pt \geq 5.4$



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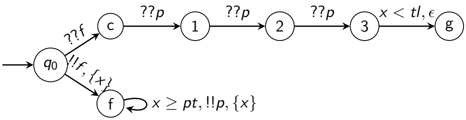

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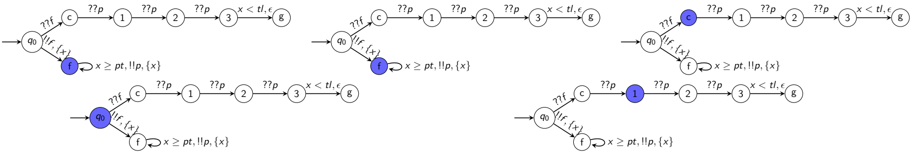

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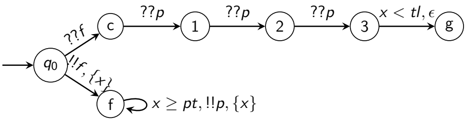

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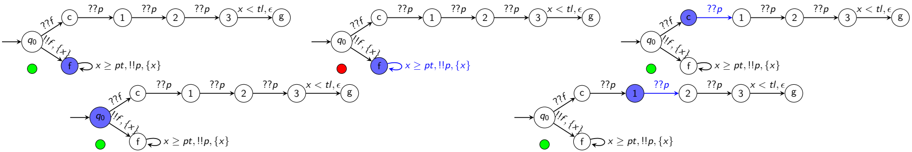

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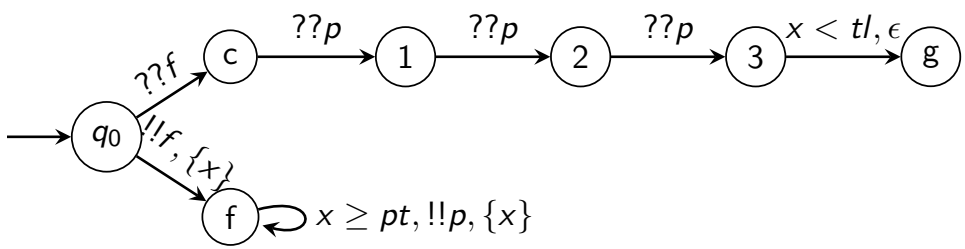

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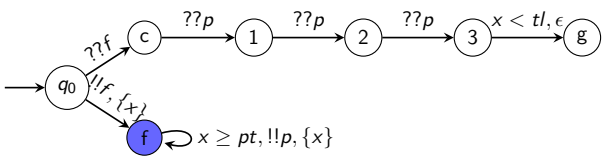

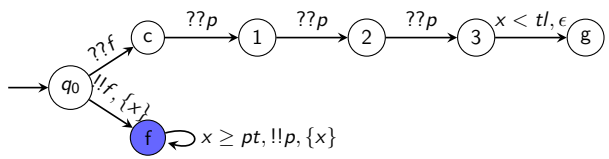

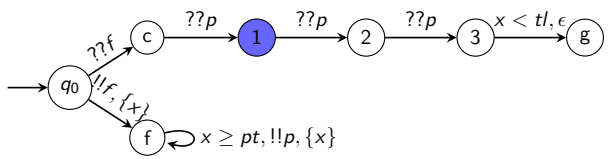

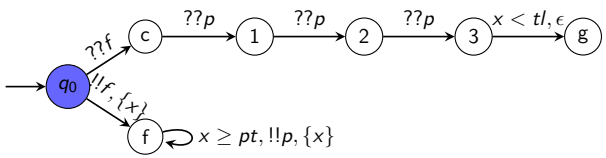

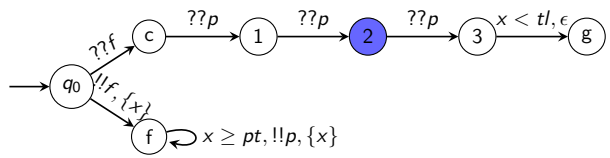

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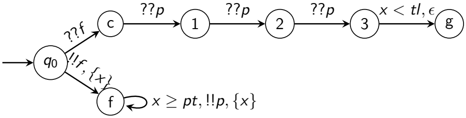
PTBP Network - Example



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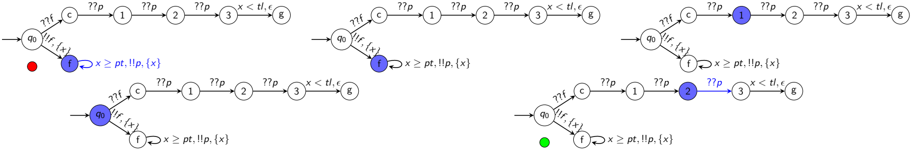

$$
\begin{pmatrix} q_0, 0 \\ q_0, 0 \\ q_0, 0 \\ q_0, 0 \\ q_0, 0 \end{pmatrix}
\xrightarrow{0.1,1,f,\emptyset}
\begin{pmatrix} f, 0 \\ q_0, 0.1 \\ q_0, 0.1 \\ q_0, 0.1 \\ q_0, 0.1 \end{pmatrix}
\xrightarrow{4.1,2,f,\{3,5\}}
\begin{pmatrix} f, 4.1 \\ f, 0 \\ c, 4.2 \\ q_0, 4.2 \\ c, 4.2 \end{pmatrix}
\xrightarrow{1.3,1,p,\{5\}}
\begin{pmatrix} f, 0 \\ f, 1.3 \\ c, 5.5 \\ q_0, 5.5 \\ 1, 5.5 \end{pmatrix}
\xrightarrow{1.8,2,p,\{1,3,4,5\}}
\begin{pmatrix} f, 1.8 \\ f, 0 \\ 1, 7.3 \\ q_0, 7.3 \\ 2, 7.3 \end{pmatrix}
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$$

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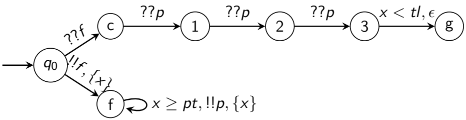

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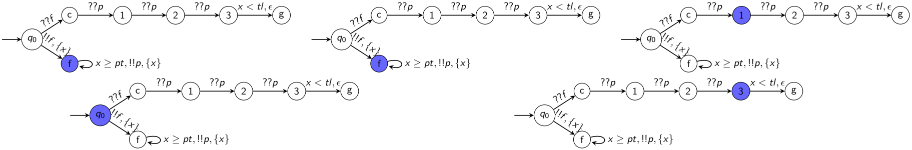
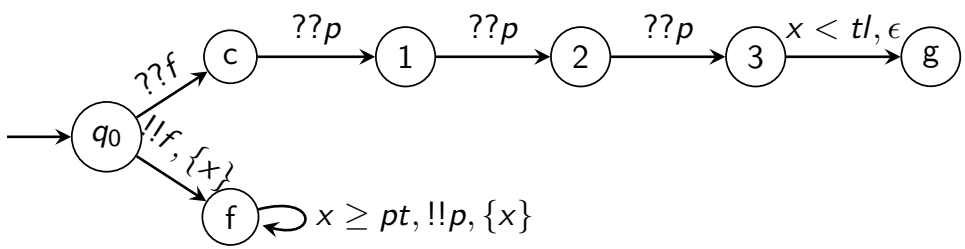
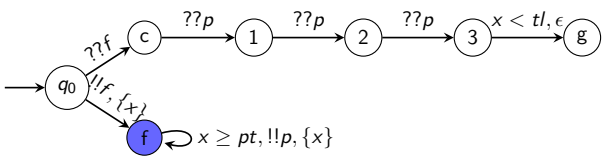


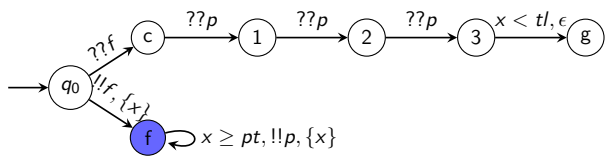
$$\begin{pmatrix} q_0,0 \\ q_0,0 \\ q_0,0 \\ q_0,0 \\ q_0,0 \end{pmatrix} \xrightarrow{0.1,1,f,\emptyset} \begin{pmatrix} f,0 \\ q_0,0.1 \\ q_0,0.1 \\ q_0,0.1 \\ q_0,0.1 \end{pmatrix} \xrightarrow{4.1,2,f,\{3,5\}} \begin{pmatrix} f,4.1 \\ f,0 \\ c,4.2 \\ q_0,4.2 \\ c,4.2 \end{pmatrix} \xrightarrow{1.3,1,p,\{5\}} \begin{pmatrix} f,0 \\ f,1.3 \\ c,5.5 \\ q_0,5.5 \\ 1,5.5 \end{pmatrix} \xrightarrow{1.8,2,p,\{1,3,4,5\}} \begin{pmatrix} f,1.8 \\ f,0 \\ 1,7.3 \\ q_0,7.3 \\ 2,7.3 \end{pmatrix} \xrightarrow{1.2,1,p,\{5\}} \begin{pmatrix} f,0 \\ f,1.2 \\ 1,8.5 \\ q_0,8.5 \\ 3,8.5 \end{pmatrix}$$

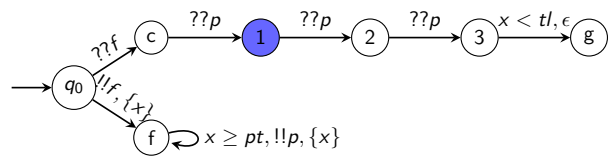
PTBP Network - Example

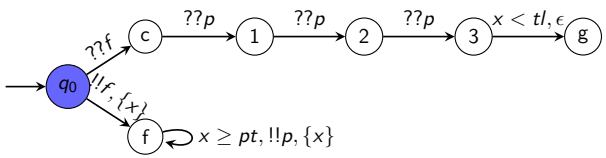


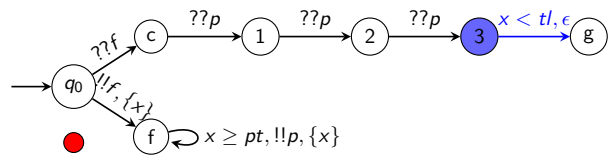
- $pt \geq 5.4$
- $pt \geq 3.1$
- $pt \geq 3$
- $tl \leq 8.5$





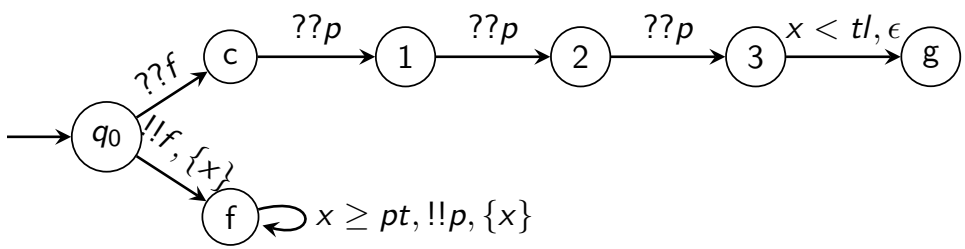






$$\begin{pmatrix} q_0,0 \\ q_0,0 \\ q_0,0 \\ q_0,0 \\ q_0,0 \end{pmatrix} \xrightarrow{0.1,1,f,\emptyset} \begin{pmatrix} f,0 \\ q_0,0.1 \\ q_0,0.1 \\ q_0,0.1 \\ q_0,0.1 \end{pmatrix} \xrightarrow{4.1,2,f,\{3,5\}} \begin{pmatrix} f,4.1 \\ f,0 \\ c,4.2 \\ q_0,4.2 \\ c,4.2 \end{pmatrix} \xrightarrow{1.3,1,p,\{5\}} \begin{pmatrix} f,0 \\ f,1.3 \\ c,5.5 \\ q_0,5.5 \\ 1,5.5 \end{pmatrix} \xrightarrow{1.8,2,p,\{1,3,4,5\}} \begin{pmatrix} f,1.8 \\ f,0 \\ 1,7.3 \\ q_0,7.3 \\ 2,7.3 \end{pmatrix} \xrightarrow{1.2,1,p,\{5\}} \begin{pmatrix} f,0 \\ f,1.2 \\ 1,8.5 \\ q_0,8.5 \\ 3,8.5 \end{pmatrix} \xrightarrow{0,5,\epsilon,\emptyset}$$

PTBP Network - Example



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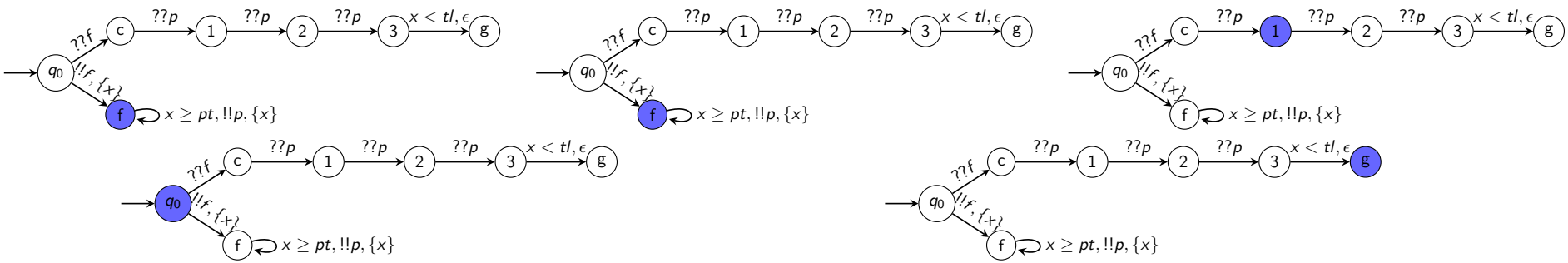

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Considered Problems

We consider *Parametric Reachability* problems, i.e. "**Does there exist a network size N satisfying a given reachability property**"?

4 potential declinations:

- $\exists$-EF-emptiness: $\exists N \in \mathbb{N}, \exists p \in \mathit{bounds}, \exists \rho \in \mathsf{Execs}, \rho \models \Diamond q_f$
- $\exists$-EF-universality: $\exists N \in \mathbb{N}, \forall p \in \mathit{bounds}, \exists \rho \in \mathsf{Execs}, \rho \models \Diamond q_f$
- $\exists$-AF-emptiness: $\exists N \in \mathbb{N}, \exists p \in \mathit{bounds}, \forall \rho \in \mathsf{Execs}, \rho \models \Diamond q_f$
- $\exists$-AF-universality: $\exists N \in \mathbb{N}, \forall p \in \mathit{bounds}, \forall \rho \in \mathsf{Execs}, \rho \models \Diamond q_f$

Outline

AF-problems in Reconfigurable semantics

AF-Problems in Reconfigurable Semantics

∃-AF-emptiness and ∃-AF-universality are decidable for 1 clock PTBP but undecidable for (L/U)-PTBP with 3 clocks or more.

Intuition: In this context, AF problems are equivalent to the same problems in networks of size 1 (because receiver sets can be empty).

EF-problems in Reconfigurable semantics

We knew from timed networks (trivial encoding):

Without timing parameters

- $\exists$-EF is decidable for PTBP without parameters and with one clock per process and undecidable with two clocks per process.
- $\exists$-EF-emptiness and $\exists$-EF-universality problems are undecidable for PTBP with two clocks.

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We prove that

EF-Problems in Reconfigurable Semantics

The $\exists$-EF-emptiness and $\exists$-EF-universality problems are undecidable for PTBP with at least one clock.

Proof: Reduction of the halting and boundedness problems for two-counter machine.

Undecidability in Clique semantics

From previous work, we know that

AF-Problems in the Clique semantics

The $\exists$-AF problem is undecidable for PTBP with no clock in the clique semantics.

Intuition: Reduction of the halting problem of a two-counter machine. The values of the counters are encoded by the number of processes in given states.

EF-Problems in Clique Semantics

The $\exists$-EF-emptiness and $\exists$-EF-universality problems are undecidable for PTBP with at least one clock.

Same proof as for reconfigurable semantics

Outline

1-clock L/U PTBP

Since the problems are undecidable in timed Networks for 2 clocks with no parameters, we focus on L/U-PTBP with a single clock.

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Why are they interesting?

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Why are they interesting?

Monotonicity

In L/U-PTBP, increasing the value of U-parameters and decreasing the value of L-parameters does not remove behaviors (preserves reachability).

Decidable EF-problems for L/U-PTBP with 1 clock

Recall that the EF-Problem is decidable for PTBP with a single clock and no timing parameters. As a consequence,

Decidability of (some) EF-Problems

- The $\exists$-EF-universality problem is decidable for closed bounded L/U-PTBP with one clock in both semantics.
- The $\exists$-EF-emptiness problem is decidable for (open or closed) bounded L/U-PTBP with one clock in both semantics.

Intuition: Replace the parameters by their maximal (U-parameters) and/or minimal (L-parameters) values. Use monotonicity to solve the problems.

Undecidable EF-problems for L/U-PTBP with 1 clock

When the parameters are open-bounded, or unbounded, $\exists$-EF-universality becomes undecidable.

Undecidability for open-bounded/unbounded L/U-PTBP

- The $\exists$-EF-universality problem is undecidable for open bounded L/U-PTBP with one clock in the clique semantics.
- $\exists$-EF-universality in the clique semantics is undecidable already with a single clock for U-PTBP with open bounds on the left, and L-PTBP with infinity as right bound.

Intuition: Reduction from the halting problem of a two-counter machine.

Outline

Conclusion and Future Work

	1-c	2-c	3-c	1-L/U cb	1-L/U ob	2-L/U	3-L/U
∃-EF-empt.	*Th3*			**L5**		*L1*	
∃-EF-univ.	*Th3*			**L4**	*open*	*L1*	
∃-AF	**Th1**	*open*	*Th1*	**Th1**		*open*	*Th1*

(a) Reconfigurable semantics

	PTBP	L/U cb	L/U ob	L or U cb	L or U ob
∃-EF-empt.	*L2*	**L5**		**L5**	
∃-EF-univ.	*L2*	**L4**	*Th5*	**L4**	*L6*
∃-AF	*Th4*				

(b) Clique semantics for 1 clock

Table: Summary of our contributions

Conclusion and Future Work

	1-c	2-c	3-c	1-L/U cb	1-L/U ob	2-L/U	3-L/U
∃-EF-empt.	*Th3*			**L5**		*L1*	
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∃-AF	**Th1**	*open*	*Th1*	**Th1**		*open*	*Th1*

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Table: Summary of our contributions

- Open problems (2-clocks, reconfigurable is well-known)
- Exact Synthesis for decidable subclasses
- Other communication topologies/semantics
- Universality for network sizes

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∃-AF	*Th4*				

(b) Clique semantics for 1 clock

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- Open problems (2-clocks, reconfigurable is well-known)
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Thank you for your Attention