Timed automata with parametric updates

ACSD 2018

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June 27, 2018

Introduction

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- Can have dramatical consequences in critical embedded system: autonomous car, in aeronautics...

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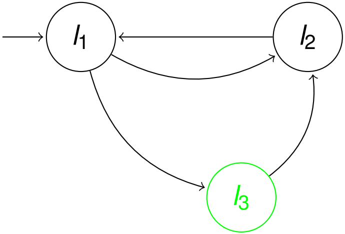
- Discovering a bug during a test of a system can be very expensive
- Can have dramatical consequences in critical embedded system: autonomous car, in aeronautics...
- Need for formal verification to ensure ahead the good behavior of a system

Model checking

- Model of a system:

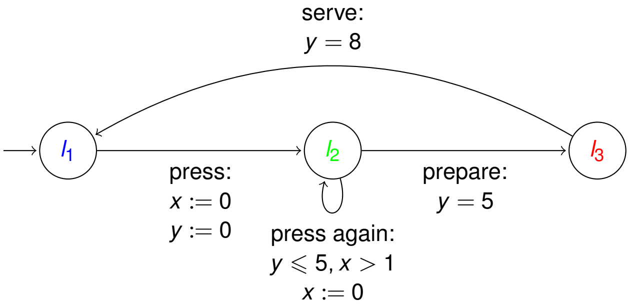


- A property of the system:  *is reachable*

- Check whether the system satisfies the property

Example of timed automaton

A **timed automaton** [AD94] which models a coffee machine



- Locations : $\{l_1, l_2, l_3\}$, clocks : $\{x, y\}$, action : {press, press again, prepare, serve}
- $Guard(\text{press again}) = \{y \leqslant 5 \wedge x \geqslant 0\}$,
  $Guard(\text{prepare}) = \{y = 5\}$, $Guard(\text{serve}) = \{y = 8\}$
- $Reset(\text{press}) = \{x, y := 0\}$, $Reset(\text{press again}) = \{x := 0\}$

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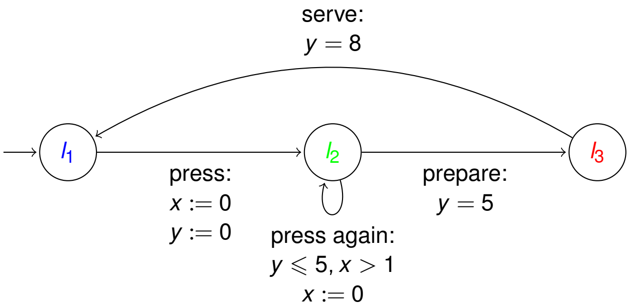


- A run : $(l_1, (0,0)) \xrightarrow[2.1]{\text{press}} (l_2, (0,0)) \xrightarrow[1.2]{\text{press again}} (l_2, (0, 1.2)) \xrightarrow[3.8]{\text{prepare}} (l_3, (3.8, 5)) \xrightarrow[3]{\text{serve}} (l_1, (6.8, 8))$
- triple (location, (value of x, value of y)) and $\xrightarrow[\delta]{\text{name}}$ discrete transition “name” after a delay δ.

Common decision problems for timed automata

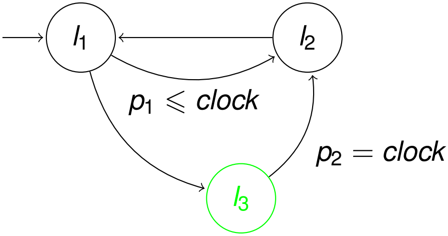
- *Reachability*: Is there a run such that the location l is reachable?
  Unavoidability: For all runs, is the location l reachable?

Common decision problems for timed automata

- *Reachability*: Is there a run such that the location l is reachable?
  Unavoidability: For all runs, is the location l reachable?
- Proved decidable in PSPACE [AD94]. Strategy: construct a finite automaton using an abstraction of clock valuations (clock regions)

Model checking with unknown constants

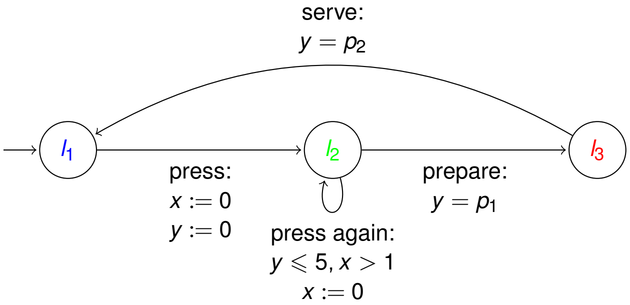
- *What if all constants are not specified ahead?*
- Model of a system with parameters:



- A property of the system: l_3 *is reachable*
- Compute the values of p_1, p_2 such that the system satisfies the property

Example of parametric timed automaton

A **parametric timed automaton** [AHV93] which models a parametric coffee machine



- A possible run if $p_1 = 2, p_2 = 3$: $\big(l_1, (0,0)\big) \xrightarrow[2]{\text{press}} \big(l_2, (0,0)\big) \xrightarrow[1]{\text{press again}} \big(l_2, (0,1)\big) \xrightarrow[1]{\text{prepare}} \big(l_3, (1,2)\big) \xrightarrow[1]{\text{serve}} \big(l_1, (2,3)\big)$
- The same run is impossible if $p_1 = 5, p_2 = 2$, or $p_1 < 1$.

Challenges for parametric timed automata

- *EF-emptiness* (decision problem): is the set of parameter valuations s.t. there exists a run reaching l in the instantiated TA empty ?

  EF-synthesis (computation problem): Compute all parameter valuations s.t. there exists a run reaching l in the instantiated TA

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Challenges for parametric timed automata

- *EF*-*emptiness* (decision problem): is the set of parameter valuations s.t. there exists a run reaching *l* in the instantiated TA empty ?
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- *EF*-emptiness problem: proved undecidable in general case [AHV93], unbounded integer-valued parameters, (un)bounded rational valued parameters and even with only one bounded parameter [Mil00]
- To recover decidability, we need to add restrictions on parameters, or restrain the PTA syntax

Where to start from ?

- Almost everything is undecidable for PTAs [And17]—especially EF-emptiness, AF-emptiness (is there a parameter valuation such that all runs reach a given location).
- Therefore, we go back to TAs.
- The reachability problem is PSPACE-complete for timed automata with updates to rational constants [BDFP04].

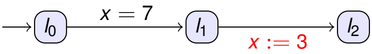


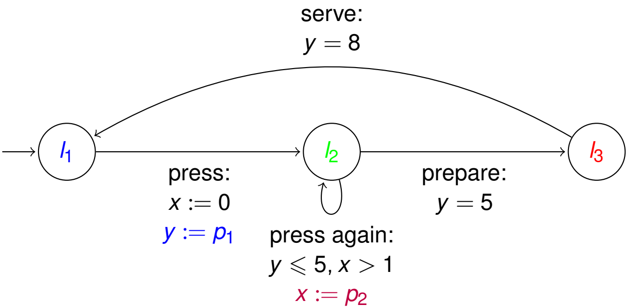
Figure: An updatable TA

Contributions

- New formalism with parametric updates of clocks: update-to-parameter TA (U2P-TA)
- Undecidability result for EF-emptiness and universality (are all parameter valuations such that there is a run reaching a given location) and AF-emptiness and universality (are all runs reaching a given location) for rational-valued parameters
- Decidability result for the same problems (in PSPACE) for integer-valued parameters, and synthesis of parameters

U2P-TA

Update-to-parameter TA (U2P-TA): TA extended with updates to rational-valued parameters.



Parametric clock updates: $y := p_1$, $x := p_2$.
Bounded parameters p_1, p_2 i.e. $p_1, p_2 \in [a, b]$ with $a, b \in \mathbb{N}$.

U2P-TA

Theorem

The EF-emptiness problem is undecidable for bounded rational-valued U2P-TAs

Proof sketch: we prove that a bounded PTA can be simulated by a bounded U2P-TA.

U2P-TA

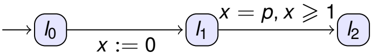


Figure: A PTA A

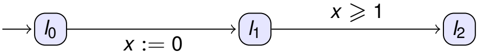


Figure: A U2P-TA obtained from A

U2P-TA

Duplicate x.

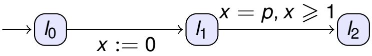


Figure: A PTA A

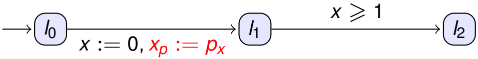
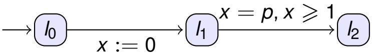


Figure: A U2P-TA obtained from A

U2P-TA

Compare x_p with C_{MAX} (maximum value between constants and parameters appearing in guards) where x is compared to p.



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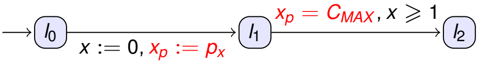

Figure: A U2P-TA obtained from A

U2P-TA

As we can simulate (w.r.t. reachability) any bounded rational-valued U2P-TA using an unbounded rational-valued U2P-TA:

Theorem

The EF-emptiness problem is undecidable for unbounded rational-valued U2P-TAs

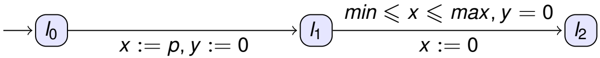

Figure: A gadget that ensures a parameter p is bounded by min and max

Integer-valued U2P-TA

U2P-TAs with integer-valued parameters over dense time.

Integer-valued U2P-TA

U2P-TAs with integer-valued parameters over dense time.

Theorem

EF-synthesis is computable for unbounded integer-valued U2P-TAs.

Integer-valued U2P-TA

Corollary

the EF-emptiness problem is PSPACE-complete for unbounded integer-valued U2P-TAs

and *unlike integer-valued PTAs* for which EF-emptiness is undecidable [AHV93,BBLS15].

Integer-valued U2P-TA

Corollary

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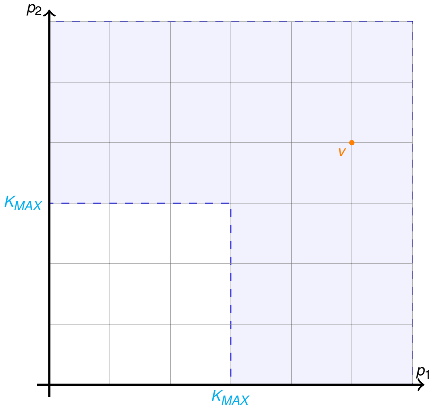
and *unlike integer-valued PTAs* for which EF-emptiness is undecidable [AHV93,BBLS15].

Proof sketch: using equivalence between parameter valuations if $> K_{MAX}$ (the maximum constant value), we enumerate parameter valuations $\leqslant K_{MAX} + 1$ as they are bounded integers.

Integer-valued U2P-TA

v and v' are equivalent.



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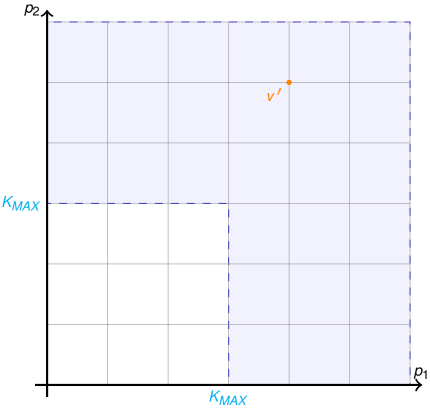
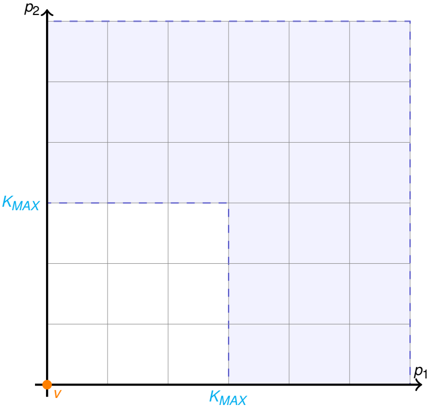


Integer-valued U2P-TA

Enumeration below $K_{MAX} + 1$.



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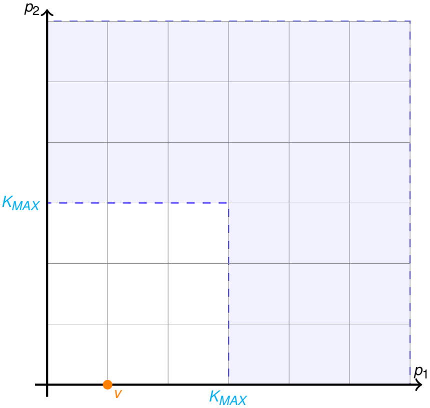

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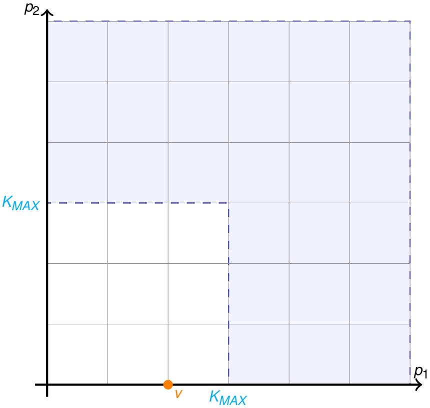

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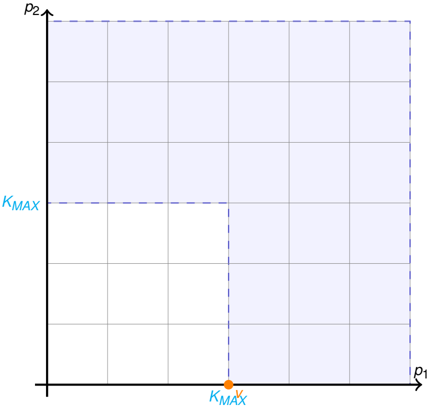

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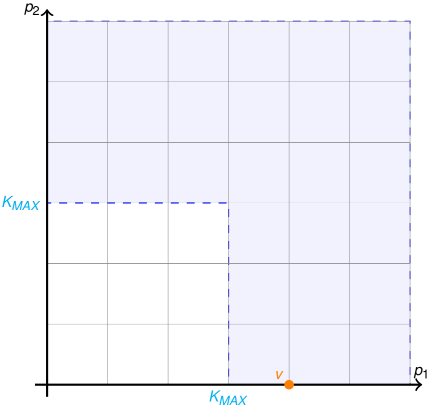

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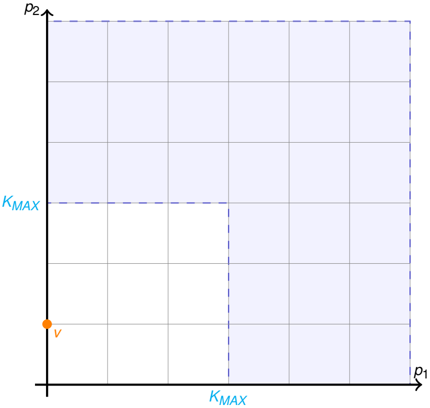

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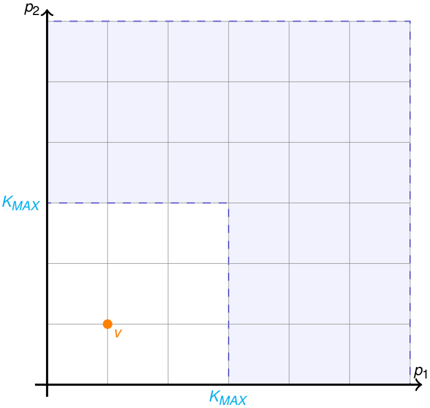

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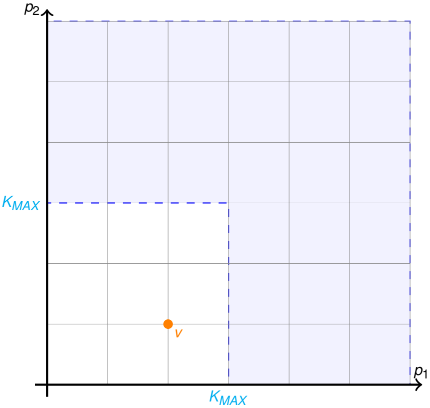

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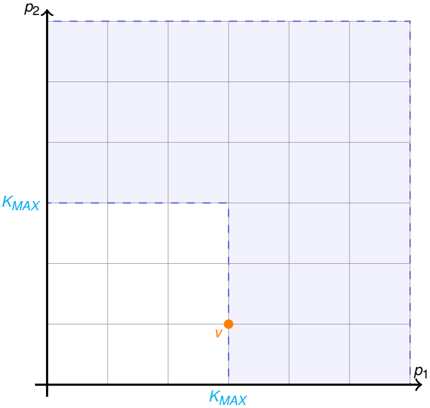

Conclusion

- Two new subclasses of PTAs: rational-valued U2P-TAs for which the *EF*-emptiness problem is *undecidable*, and integer-valued U2P-TAs for which it is *decidable*.
- In fact we have the same results for *EF*-universality, *AF*-emptiness/universality.
- We also can perform *parameter synthesis*.

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- We also can perform *parameter synthesis*.

Future work:

- Find syntactic restrictions in order to find a decidability result for rational parameter valuations
- Adapt our formalism to hybrid systems, in which clocks can evolve at different rates

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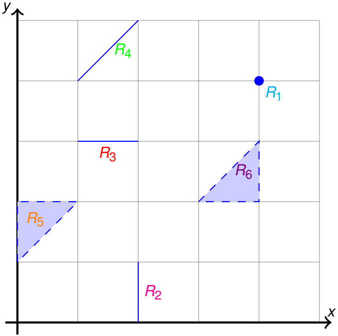
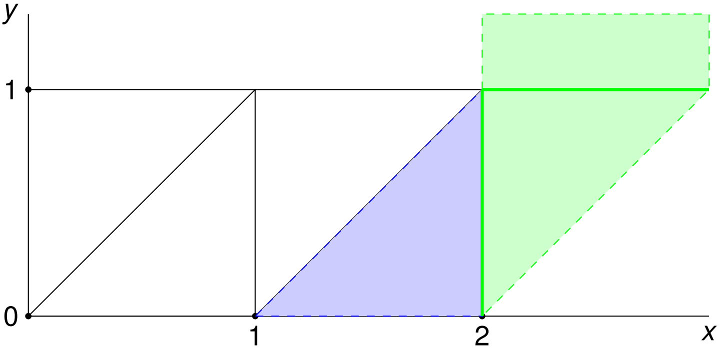
Clock regions



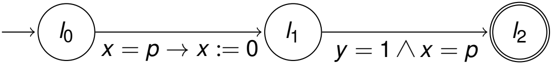
- The corner point: $R_1 = \{(4,4)\}$
- The vertical line: $R_2 = \{(x,y) \mid x = 2, 0 < y < 1\}$
- The horizontal line: $R_3 = \{(x,y) \mid y = 3, 1 < x < 2\}$
- The diagonal: $R_4 = \{(x,y) \mid x = y - 3, 4 < y < 5\}$
- The upward triangle: $R_5 = \{(x,y) \mid 0 < x < y - 1, 1 < y < 2\}$
- The downward triangle: $R_6 = \{(x,y) \mid y + 1 < x < 4, 2 < y < 3\}$

Clock regions

Two clocks x, y, max constants $c_x = 2$, $c_y = 1$.
Time successors of the blue region $\{0 < y < 1, 0 < y < x - 1\}$ different of itself: four regions in green: $\{0 < y < 1, x = 2\}$, $\{0 < y < 1, x > 2\}$, $\{y = 1, x > 2\}$ and $\{y > 1, x > 2\}$

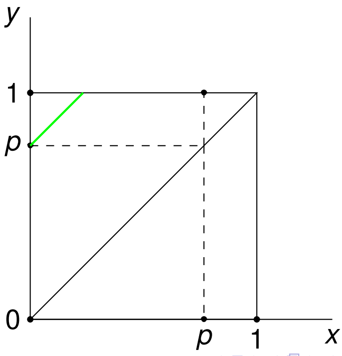


Using regions for parametric timed automata ?



In l_1: $(x, y) = (0, p)$
But after letting some time elapse, depending on the value of $0 < p < 1$ we reach different regions:

- region $y = 1, 0 < x < p$ if $1 > p > \frac{1}{2}$



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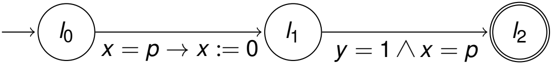

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But after letting some time elapse, depending on the value of $0 < p < 1$ we access different regions:

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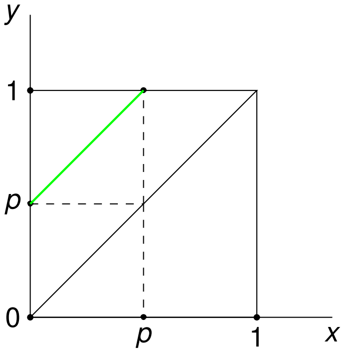

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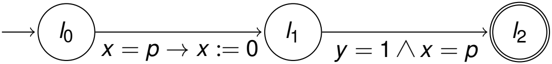
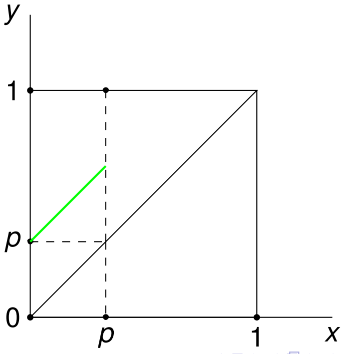
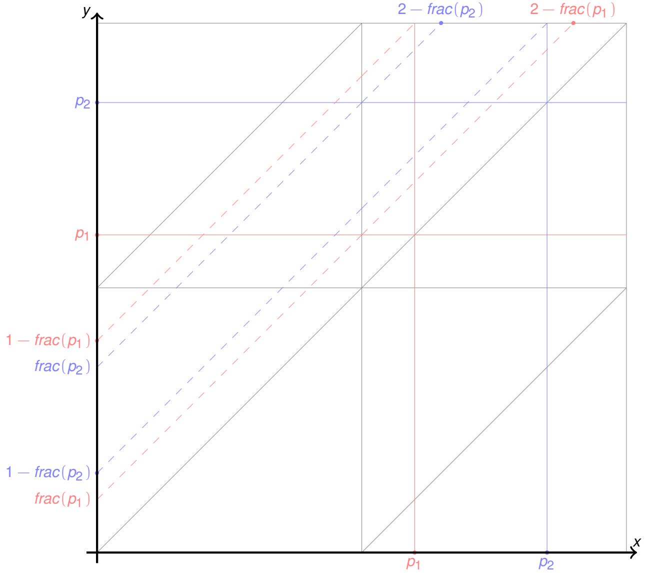


In l_1: $(x, y) = (0, p)$
But after letting some time elapse, depending on the value of $0 < p < 1$ we access different regions:

- region $p < y < 1, x = p$ if $p < \frac{1}{2}$



U2P-TA



2 — frac(p2)
2 — frac(p1)
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1 — frac(p2)
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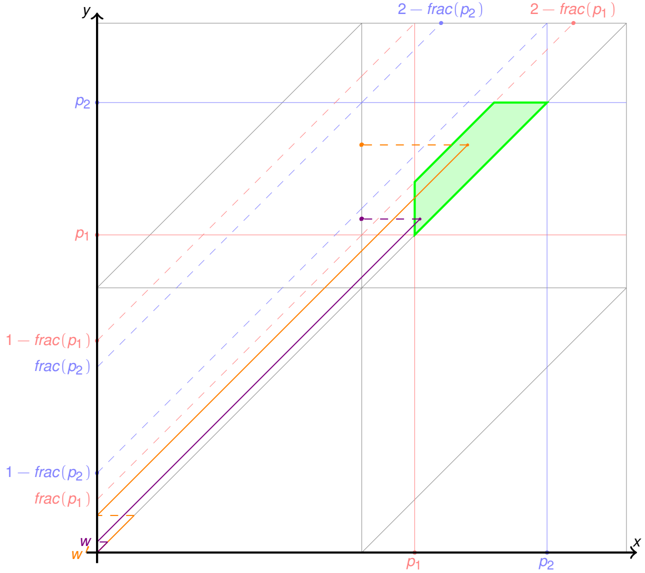
U2P-TA



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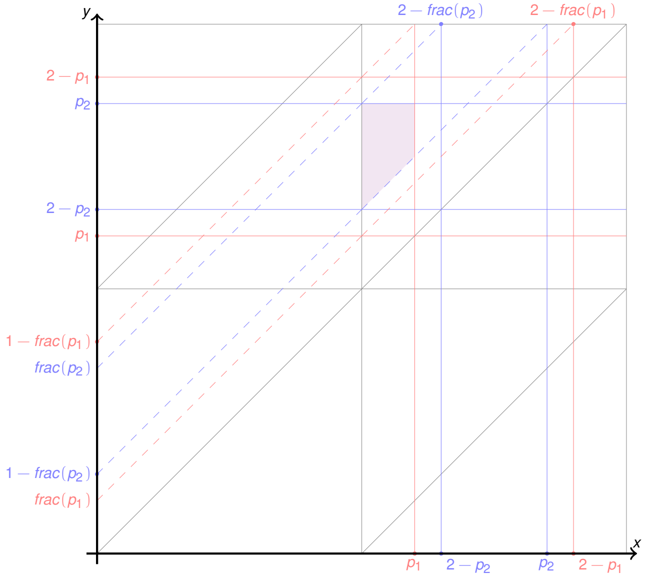

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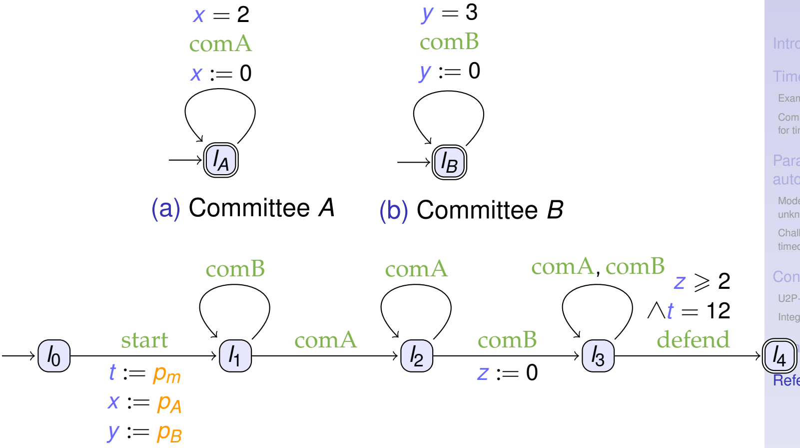
Example



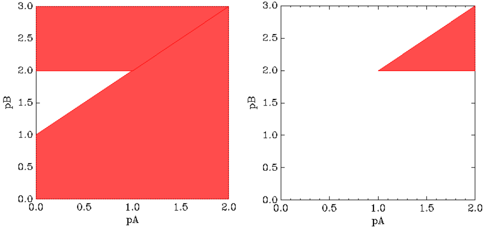
(a) Committee A

(b) Committee B

(c) A PhD student's defense workflow

Figure: A motivating example of integer-valued U2P-TA

Example



Graphical visualization in two dimensions of the parameter synthesis of with $p_m = 6$ (left) and $p_m = 9$ (right) provided by IMITATOR. Constraints are:

$$p_A \leqslant 2 \wedge p_B \leqslant p_A + 1$$
$$\vee$$
$$p_B \geqslant 2 \wedge p_B \leqslant 3 \wedge p_B \geqslant p_A + 1$$

with $p_m = 6$

$$p_B \geqslant 2 \wedge p_A \leqslant 2 \wedge p_A + 1 \geqslant p_B$$

with $p_m = 9$