# Liveness in L/U-Parametric Timed Automata

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# Introduction

- Parametric timed automata (PTA) allow for flexible, abstract, and robust modelling;
- The answer to **parametric** model-checking is appealing;
- Many undecidability results exist for safety / reachability properties;
- And a few decidable subclasses:
  - L/U PTA [HRSV02];
  - IP-PTA [ALR16];
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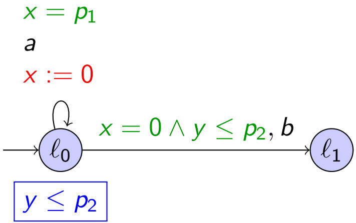
  - bounded integer PTA [JLR15].
- What about **liveness**?

# Parametric Timed Automata [AHV93]



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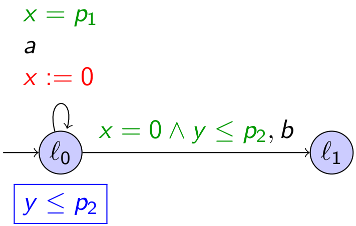
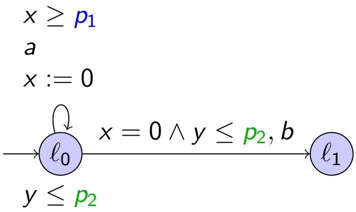



For $p_1 = 1.2$ and $p_2 = 4$:

$$\begin{matrix} \ell_0 \\ x = 0 \\ y = 0 \end{matrix} \xrightarrow{1.2} \begin{matrix} \ell_0 \\ x = 1.2 \\ y = 1.2 \end{matrix} \xrightarrow{a} \begin{matrix} \ell_0 \\ x = 0 \\ y = 1.2 \end{matrix} \xrightarrow{b} \begin{matrix} \ell_1 \\ x = 0 \\ y = 1.2 \end{matrix} \xrightarrow{2.4} \begin{matrix} \ell_1 \\ x = 2.4 \\ y = 3.6 \end{matrix}$$

# L/U Parametric Timed Automata [HRSV02]




- Parameters are used either as lower bounds or as upper bounds, never both.
- **Monotonicity**: increasing upper bounds or decreasing lower bounds gives **more** behaviours.

# Liveness in (Parametric) Timed Automata

- Our **liveness** properties concern **maximal** paths:
  - Existence of an **infinite** maximal path (discrete **cycle**, denoted EC);
  - Existence of a **finite** maximal path (**deadlock**, denoted ED);
  - Existence of a maximal path preserving some property (CTL **EG** property).

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- **Parametric** properties:
  - $\phi$-emptiness: is the set of parameter valuations s.t. $\phi$ holds empty?
  - $\phi$-universality: is the set of parameter valuations s.t. $\phi$ holds universal?

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# Results from the Literature

| Class | PTA | L/U PTA |
|---|---|---|
| EC-emptiness | open | **PSPACE-c.**[1] |
| ED-emptiness | open | open |
| EG-emptiness | open | open |

[1] Integer parameters [BL09].

# EC-emptiness is PSPACE-c for L/U PTAs

- There exists a **rational** parameter valuation s.t. there is a cycle iff there exists an **integer** valuation.
- Use the **monotonicity** property of L/U PTAs: **round** up for upper bounds, down for lower bounds to get a good **integer** valuation.

# EC-emptiness is undecidable for PTAs

- Reduce from the **counter boundedness** problem of **2-counter machines**
  - Finite-state machine + 2 non-negative integer counters;
  - **increment** some counter and go to some state;
  - **if** some counter is **zero** then **decrement** it and go to some state; otherwise go to some other state;

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- States of the machines are encoded by locations $q_i$;
- Counters are encoded by clocks $y, z$ and one parameter $p$: when clock $x$ is null,

$$y = 1 - c_1 p$$
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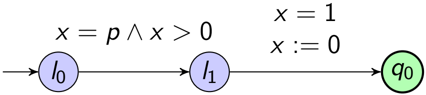
- Initialisation:



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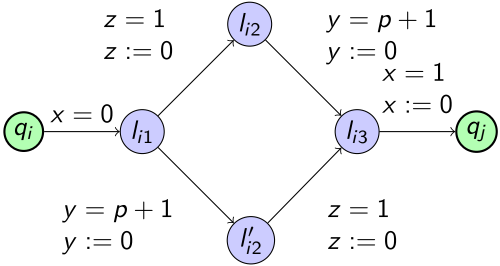

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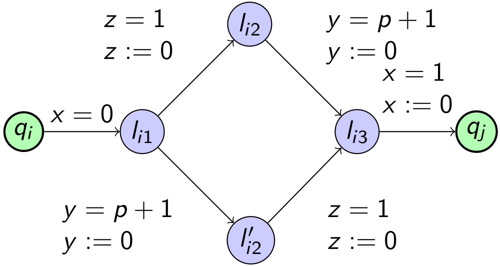


$$\begin{array}{l} q_i \\ x = 0 \\ y = 1 - c_1 p \\ z = 1 - c_2 p \end{array} \xrightarrow{0} \begin{array}{l} l_{i1} \\ x = 0 \\ y = 1 - c_1 p \\ z = 1 - c_2 p \end{array} \xrightarrow{c_2 p} \begin{array}{l} l_{i2} \\ x = c_2 p \\ y = 1 - (c_1 - c_2) p \\ z = 0 \end{array} \xrightarrow{(c_1 - c_2 + 1) p}$$

$$\begin{array}{l} l_{i3} \\ x = (c_1 + 1) p \\ y = 0 \\ z = (c_1 - c_2 + 1) p \end{array} \xrightarrow{1 - (c_1 + 1) p} \begin{array}{l} q_j \\ x = 0 \\ y = 1 - \mathbf{(c_1 + 1)} p \\ z = 1 - c_2 p \end{array}$$

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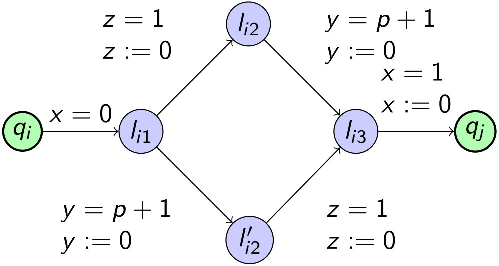


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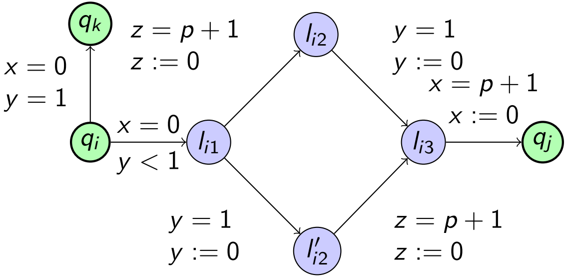
- implies $p \leq \frac{1}{c_1 + 1}$ otherwise it **blocks** at $l_{i3}$.

# EC-emptiness is undecidable for PTAs

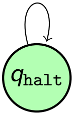
- Zero-test and decrement:




- $c_1 = 0$ iff $y = 1$.
- Decrement is similar to increment.

# EC-emptiness is undecidable for PTAs

- Halting:



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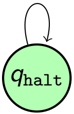


- There is a (discrete) cycle in the PTA iff the counter are bounded:
  - if the machine halts, $q_{\mathtt{halt}}$ is reachable $\rightarrow$ cycle;
  - if the machine does not halt but the counters are bounded, there is a parameter valuation **small enough** to have a cycle among the instruction widgets;
  - if the counters are unbounded, for any valuation, the PTA will eventually **block** in the increment widget.

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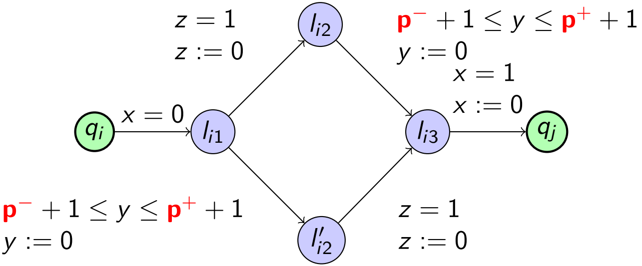
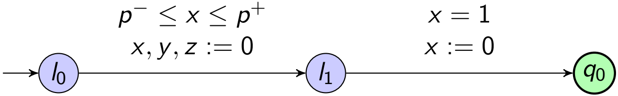
- Reduce from the **halting** problem of 2-counter machines;
- Change previous construction to "split" parameters and get an L/U PTA:



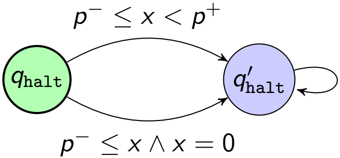

- We use the deadlock property to **enforce** $p^- = p^+$.

# ED-emptiness is undecidable for L/U PTAs

- Initialisation, enforce $p^- \leq p^+$:




- Halting, there is a **deadlock** in $q_{\mathtt{halt}}$ iff $p^+ \leq p^-$ (and $p^- > 0$):




- Add a transition with guard true from all locations but $q_{\mathtt{halt}}$;
- the machine **halts** iff there exists a valuation such that $p^- = p^+$ and there is a **deadlock** in the PTA.

# EG-emptiness is undecidable for L/U PTAs

- by reduction from the **halting** problem of 2-counter machines;
- similar to the ED-construction with a different encoding adapted from [BBLS15];
- the main idea is to eliminate cycles by:
  - making sure all widgets execute in 1 t.u.;
  - add a global invariant limiting the **total execution time** so that it does not exceed some parameter $p_2$;
  - then the PTA can only execute **at most** $p_2$ instructions and $p_2$ has to be **big enough** for executing a halting sequence.

# Results up to now

| **Class** | **PTA** | **L/U PTA** |
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| EC-emptiness | *Undec.* | **PSPACE-c.** |
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- We can find some decidability by considering parameters are **bounded** (each takes its values in some bounded interval);
- Changes nothing for **PTAs**;
- We consider both (topologically) **closed** and **open** parameter domains.

# EG-emptiness is decidable for closed bounded L/U PTA

1. Test if there is an infinite path preserving $\phi$ in the TA obtained by setting:
   - **lower** bounds to their **minimum** value,
   - and upper bounds to their maximal values.

   i.e. verify CTL property "EG ($\phi \wedge$ EX true)" on the **region graph** of the TA.
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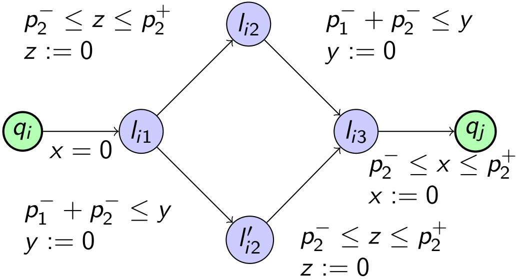
2. if yes we are done
3. otherwise all paths preserving $\phi$ are finite: explore them symbolically, using the **symbolic polyhedral abstraction** of **linear hybrid automata**;
4. test all symbolic states on those paths for **deadlocks**:
   - consider all states that can reach some guard (classic **past** operator)
   - check if those states **cover** the whole symbolic state (polyhedral union and inclusion).

# EG-emptiness is undecidable for open bounded L/U PTA

- Reduce from the **halting** problem of **2-counter machines**
- Make sure all widgets execute in $[p_2^-, p_2^+]$ t.u. (instead of 1);




- use the **open** parameter domain to enforce $p_2^- > 0$;
- add a **global invariant** so that the whole PTA can only execute for 1 t.u. to **eliminate cycles**;
- the machine **halts** iff there exists a parameter valuation s.t. $p_1^- = p_1^+$ and $p_2^- = p_2^+$ and there is a **deadlock** in the PTA.

# Final Results

| Class | PTA | L/U PTA | closed b. L/U | open b. L/U |
|---|---|---|---|---|
| EC-empt. | *Undec.* | **PSPACE-c.** | **PSPACE-c.** | open |
| ED-empt. | *Undec.* | *Undec.* | *Undec.* | *Undec.* |
| EG-empt. | *Undec.* | *Undec.* | **Dec.** | *Undec.* |

- The other results follow directly from the previous constructions;
- We conjecture that EC-emptiness for open bounded L/U PTAs is **decidable** with techniques similar to [San11].

# Conclusion and Perspectives

- Summary:
  - We have exhibited a very thin border of decidability for **liveness** properties;
  - It depends on the **boundedness** of the parameters and the topological **closure** of their initial domain.
- Future work:
  - Prove that EC-emptiness for open bounded LU PTAs is **decidable**;
  - Complete the results for the **universality** problems;
  - Find the **complexity** of EG-emptiness for closed bounded L/U PTA.

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