Internship topic

Quantum Alternating Operator Ansatz with dedicated mixers for Constrained Binary Quadratic Problems

Context

Quantum computing is increasingly being explored by the optimization community to address combinatorial optimization problems. Specifically, the main problems tackled currently are Quadratic Unconstrained Binary Optimization (QUBO) problems using quantum heuristics (Quantum Approximate Optimization Algorithm (QAOA) [1], more general variational quantum methods [2, 3], Quantum Annealing [4] etc.). Constrained problems, representing a significant part of everyday challenges, are under-studied. The classical Lagrangian relaxation, or similar methods, coupled with quantum heuristics for unconstrained problems are mainly invoked for such constrained problems [5], in parallel of reformulations of the initial problem [6]. However, other approaches dealing with constraints at the core of the algorithm exist, such as the Quantum Alternating Operator Ansatz algorithm [7] where constraints are characterized by mixers (Mixer-QAOA). It is these particular methods that interest us here.

Mixers for constraints with QAOA

The algorithm at stake for this internship is Mixer-QAOA [7]. This is a hybrid heuristic which consists in alternating between a quantum and a classical part. The quantum part produces a state $|\theta\rangle$ by a quantum circuit parametrized by $\theta \in \mathbb{R}^d$, $d \in \mathbb{N}$. The classical part optimizes over θ driven by the aim that $|\theta\rangle$ has a high probability of being measured as the optimal solution of the initial problem. The new idea of Mixer-QAOA, compared to the seminal QAOA dealing with unconstrained problems only, is to express the constraints as mixers in the quantum circuit such that the latter manipulates only feasible states. Such mixers have been found for specific constraints (e.g. constant Hamming weight [8, 9]) but building mixers for other general constraints remains an open question.

In this internship, we aim at solving the Quadratic Knapsack Problem with cardinalty constraint (kQKP) [10] with Mixer-QAOA, meaning designing mixers for two types of constraints (capacity and cardinality). In the long term, it will lead to the study of more generic problems, where many variations are possible (in terms of objective function, types of contraints, domains of the variables). Notice that we will also be able to investigate other methods to solve the kQKP by integrating Mixer-QAOA in classical decomposition methods [11].

From a practical point of view, Mixer-QAOA (in the gate-based model) on small instances of kQKP will be implemented and simulated with Qiskit. If it is relevant, some experiments will also be conducted on quantum hardware accessible on the cloud.

A continuation in a PhD after this internship may be considered.

Practical information

Employer:	Laboratory LIPN within the framework of an academic internship agreement
Internship location:	LIPN (Laboratoire d'Informatique de Paris Nord), UMR CNRS 7030 Université Sorbonne Paris Nord 99 avenue Jean-Baptiste Clément 93430 Villetaneuse
Duration:	4-6 months at spring 2026
Remuneration:	Internship compensation
Required level:	Second year of Research Master or third year of Engineering School
Profil:	Combinatorial optimization, Quantum computing, Computer science, Applied mathematics, Python

Advisors

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References

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