Polylogarithms at non-positive (i.e. negative) multi-indices

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Abstract

We extend the definition and construct several bases for polylogarithms Li_T, where T are recognizable, by a finite state (multiplicity) automaton and of alphabet $X = \{x_0, x_1\}$[1]. The kernel of the "polylogarithmic map" $\mathrm{Li}_\bullet$ is also characterized and provides a rewriting process which terminates to a normal form. We mostly concentrate on the algebraic aspects of this extension.

[1] Gérard H. E. Duchamp, Hoang Ngoc Minh, Ngo Quoc Hoan, *The algebra of Kleene stars of the plane and polylogarithms*, hal-01267134.

[1] The space of rational series considered here is $(\mathbb{C}\langle X\rangle \sqcup\!\sqcup \mathbb{C}^{\mathrm{rat}}\langle\langle x_0\rangle\rangle \sqcup\!\sqcup \mathbb{C}^{\mathrm{rat}}\langle\langle x_1\rangle\rangle, \sqcup\!\sqcup, 1_{X^*})$.