French-British Workshop on Analytic Combinatorics

Venue: Lecture Room 102, Department of Statistics, 1 South Parks Road.

Wednesday, September 5th

9:30-11:00 Elementary Concepts in Analytic Combinatorics I
(J. Lumbroso, Paris 6)
11:00-11:20 Coffee break
11:20-12:50 Elementary Concepts in Analytic Combinatorics
(B. Morcrette, Paris 6)
13:00-14:00 Lunch at Linacre College
14:00-18:00 Problem session, work in groups (SPR 1)
18:30 Dinner (King's Arms)
20:30 Caleidh dancing (New College)

Thursday, September 6th

9:30-11:00 Distribution and Parameter Analysis using Singularity Extraction
(J. Lumbroso, Paris 6)
11:00-11:20 Coffee break
11:20-12:50 Distribution and Parameter Analysis using Singularity Extraction
(B. Morcrette, Paris 6)
13:00-14:00 Lunch at Linacre College
14:00-15:00 Results on Concentration (C. McDiarmid, Oxford)
15:00-16:00 Problem session, work in groups (SPR 1)
16:30-18:00 Punting
19:00 Dinner (Turf Tavern)

Friday, September 7th

9:30-10:00 Introduction to Random Generation
(J. Lumbroso, Paris 6)
10:00-11:00 Automatic Random Generation of Combinatorial Objects
(A. Jacquot, Paris 13)
11:00-11:20 Coffee break
11:20-12:50 Automatic Random Generation of Combinatorial Objects
(A. Jacquot, Paris 13)
13:00-14:00 Lunch at Linacre College
14:00-15:30 Problem sessions, work in groups (SPR 1)
15:30-16:00 Final Discussion

1 Enumeration Problems

1.1 Permutations

Proposed by Cécile Mailler and Neil O'Connell: Increasing subsequences in permutations

An increasing subsequence of a permutation σ is a finite set I of indices such that for all i, j in I,

$$i < j \Rightarrow \sigma(i) < \sigma(j).$$

For example, if $\sigma = (3, 1, 2, 4)$ we can find 9 increasing subsequences : 3, 1, 2, 4, 34, 12, 14, 24, 124.

Pick a permutation σ of length n uniformly at random. Let X_n denote the number of increasing subsequences in σ and let L_n be the length of its longest subsequence. It is known that

$$\lim_{n\to\infty} \frac{L_n}{\sqrt{n}} = 2$$

holds in expectation and as well as in probability.

Problem 1. *Determine the expected number of increasing subsequences in a random permutation. It is widely conjectured that*

$$\lim_{n\to\infty} \mathbb{E}\left[\frac{\ln X_n}{\sqrt{n}}\right] = 2$$

eq:

should hold.

References

Lifschitz [1] V. Lifschitz and B. Pittel, The number of increasing subsequences of the random permutation, *Journal of Combinatorial Theory, Series A*, bf 31-(1981) 1-20.

Zeilberger [2] M. Petkovsek, H. Wilf, D. Zeilberger, A=B, (1995).

Barrucand [3] P. Barrucand, Quelques intgrales relatives aux fonctions de Bessel et au sommes de carrs des coefficients multinomiaux, *Comptes Rendus de l'Acadmie des Sciences de Paris*, 260 (1965), pp. 5439-5441.

Watson [4] G.N. Watson, A Treatise on the Theory of Bessel Functions, *Cambridge Mathematical Librairy*, (1941).

1.2 Set partitions

Proposed by Julien David: Enumerating/Generating Particular Set Partitions

This problem is linked to Automata Theory. I came up with the question last week and I haven't work on it yet, so it might be really easy. If it is, there are other questions that are immediately related.

Problem 2. *Let n and k be two integers and let k be fixed. What is the number of partitions of $\{1, \dots, k \cdot n\}$ into n non-empty subsets such that for any set P in the partition and any elements $x, y \in P$, we have*

$$x \equiv y \bmod k?$$

Observe that in general, it is known that the number of set partitions of size kn into n non-empty subsets are counted by Stirling Number of the second kind. It would be interesting to study the proportion of such partitions amongst all set partitions of size kn into n non-empty subsets.

1.3 Families

Proposed by Julien David: Enumerating/Generating (Co-)Moore Families

Problem 3. *A family $\mathcal{F}_n \subseteq \mathcal{P}(\{1, \dots, n\})$ is said to be a* Moore Family *if for all subsets $X, Y \in \mathcal{F}_n$, we have $X \cap Y \in \mathcal{F}_n$ (basically it is closed under intersection). A family closed under union is called a The co-Moore Family.*

- *Given a fixed integer n, how many (co-)Moore Families are there, up to isomorphism?*
- *Can we obtain an efficient random generator for Moore families?*

The first question is really hard in my opinion. The latest paper on the subject are [1, 2], which answers the first question for $n = 1, \dots, 7$. Improving the lower and upper bounds even a little bit would be great. For the random generation part, using Markov Chains seems like a good idea.

References

ref1 [1] P. Colomb, A. Irlande, O. Raynaud and Y. Renaud About The Recursive Decomposition of The Lattice of Moore co-families, ICFCA (2011).

ref2 [2] P. Colomb, A. Irlande and O. Raynaud Counting of Moore Families for n=7., ICFCA : Agadir(2010), Marocco.

2 Graph Theory

Proposed by Sera Cakiroglu:

Problem 4. *Does the complete bipartite graph with an added edge have the maximum number of spanning trees among all connected graphs with the same number of edges?*

One would want to check the Matrix Tree Theorem (or Kirchhoff's theorem) to study this conjecture.

References

Sedlacek [1] J. Sedlacek, On the minimal graph with a given number of spanning trees, *Canad. Math. Bull.*, 13 (1970) 515-517.

Proposed by Kerstin Weller: Excluding a path as a minor

A graph H is called a *minor* of a graph G if H is isomorphic to a subgraph that can be obtained by contracting and/or deleting edges of G (loops and parallel edges are deleted). A class of graphs$\mathcal{A}$ is *minor-closed* if for any $G \in \mathcal{A}$ every minor of G is also in $\mathcal{A}$. Furthermore $\mathcal{A}$ is called *proper* if it does not contain all graphs. A graph F is an *excluded minor* of a minor-closed class $\mathcal{A}$ if F is not in $\mathcal{A}$ but every minor of F is.
The famous Robertson and Seymour Theorem then states that every proper minor-closed class of graph can be characterized by a finite set of excluded minors [2]. Let $Ex(P_k)$ denote the set of graphs excluding the path of lenght $k \in \mathbb{N}$ as a minor. In Bernardi et al. [1] it was shown that the corresponding exponential generating function is entire, hence its radius of convergence is infinite. On the other hand the exponential generating function is not known explicitly nor is it known if it is algebraic.

Problem 5. *Can we deduce the exponential generating function of $Ex(P_K)$ for fixed $k \in \mathbb{N}$? Can we say something about the asymptotic properties of those classes of graphs, i.e. about the number of graphs on n vertices, the limiting law of the number of components, etc.?*

References

Bernardi [1] O. Bernardi and M. Noy and D. Welsh, Growth constants of minor-closed classes of graphs, *Journal of Combinatorial Theory, Series B*, **100** (2010) 468–484.

Diestel [2] R. Diestel *Graph Theory.* Graduate Texts in Mathematics, Springer, third edition, 2005.