The Theory of Meaningfulness in the Call-by-Value lambda-Calculus

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The untyped lambda-calculus [Chu41] is a simple and Turing-complete model of computation that represents the kernel of any functional programming language. The semantics of the untyped call-by-name lambda-calculus (CbN) is a well-developed field built around the concept of *solvable* terms, which are elegantly characterized in many different ways [Wad76, Bar77, CDC78, Bar84]. In particular, unsolvable terms provide a consistent notion of meaningless terms, and meaningful terms can be identified with the solvable ones. The semantics of the untyped call-by-value lambda-calculus (CbV, which is closer to the real implementations of programming languages [Plo75]) is instead still in its early stages, because of some inherent difficulties but also because CbV solvable terms are less studied and understood than in CbN. On the one hand, we show that a carefully crafted presentation of CbV [AP12], inspired by linear logic, allows us to recover many of the properties that solvability has in CbN, in particular qualitative and quantitative characterizations via multi types. On the other hand, we stress that, in CbV, solvability plays a different role: identifying unsolvable terms as meaningless induces an inconsistent theory [AG22]. We argue that in CbV, the correct notion of meaningful terms is captured by the concept of *potential valuability* [PR99]. In particular, terms that are not potentially valuable provide a consistent notion of meaningless terms in CbV and enjoy all the good properties (such as genericity) of unsolvable terms in CbN [AKG24].

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