Some notes on the beginning of Linear Logic in Italy

ANTONIO VEREDICE

Sapienza Università di Roma
antonio.veredice@uniroma1.it

ABSTRACT

The idea of this talk comes from our research work on the rebirth of mathematical logic in Italy in the second half of the twentieth century. [5]. One of the main topic of this work is the relationship between logic and computer science in the italian research community between the seventies and the eighties. We realized that at the crossroad of the researches of theoretical computer science and proof theory that took place in Italy during the eighties, are some of the seeds of the birth of Linear Logic.

During the seventies, Italian logic was characterized by the presence of two research communities that act in parallel, without substantial exchanges.

On the one hand, the community that was born from Ludovico Geymonat's CNR group in 1962: Evandro Agazzi, Ettore Casari, Marialuisa Dalla Chiara, Corrado Mangione. This group is responsible for the rebirth of logic in Italy after the period of emptiness that followed Peano's time. Thanks to these people, in a few years, various research groups rise in Italian universities, such as the Siena School led by Roberto Magari.

On the other side there is the school created by Corrado Böhm which made important contributions to the study of λ-calculus. The importance of these contributions was recognized by international logicians (Hindley, Seldin, Plotkin, Barendregt) but not by the Italian logicians of the Geymonat group.

The lack of communication between the two communities is documented in our research work [5] through testimonies of the protagonists and analysis of some works of the time.

In that period Michele Abrusci, a former Casari's student, took interest in Proof Theory and was impressed by Jean-Yves Girard's work in that field. In particular the Π_2^1-logic (also called β-logic) and dilators. A dilator is a functor from ON (the category of *ordinals*) to ON such that preserves direct limits and pull-backs. While the *usual* Σ_1^0 logic considers *finite* proofs and the ω-logic (Π_1^1) considers ω-proofs, β-logic (Π_2^1) considers β-proofs in order to characterize the concept of *truth in every model* (where "ordinals" and "well foundness" are standard) from a syntactic point of view. Abrusci becomes aware of Π_2^1-logic and decides to go to France in 1981 to deepen the study of Π_2^1 logic and dilators. It was the starting point for a collaboration between Girard and the italian community.

Starting from the work on dilators Girard conceived important applications for the study of structures like Scott domains with their role as untyped λ-calculus models.

Girard found in the italian community a good environment for the diffusion and development of his ideas, especially in Torino - where the computer science research group started by Böhm had been carried out by Mario Coppo, Mariangiola Dezani, Simonetta Ronchi della Rocca and others - and in Pisa where, during the meetings of De Giorgi's seminar, a new approach to foundational issues was taking hold. It happened thanks to the contribution of Giuseppe

Longo, Furio Honsell and Marco Forti, interested in topics that concern both logic and computer science.

During the 1980s congresses were organized by the Italian logic community which brought the world of logic closer to that of theoretical computer science. We refer to the Logic Colloquium held in 1982 in Florence devoted to Model Theory, Category Theory and λ-calculus and to the SILFS congress held in 1983 in San Gimignano. Here the different tendencies of Italian logic in the broadest sense where all represented and wide space was given to the themes of proof theory and denotational semantics. This is evident in the talks held by Girard, Longo, Böhm, Dezani, Abrusci and Martin Löf.

A similar situation concerns the logic meetings organized by the Siena School. The VIII meeting takes place in 1985 on the theme *Proof Theory and Intuitionism* and includes a short course held by Girard, while the X meeting of the 1986 is about *Logic in Computer Science*.

The Siena school meetings were an important training opportunity for young scholars entering the world of research at that time.

In 1985 Girard published *The system F of variable types fifteen years later* capturing the attention of young italian researchers interested in the relationship between logic and computer science.

The work on denotational semantics and coherent spaces led Girard in those years to conceive a new approach to proof theory. According to the evidence collected in [5], some of the ideas that led to the genesis of Linear Logic arose during Girard's italian frequent visits.

The year of publication of the paper Linear Logic (1987) coincides with the foundation year of AILA, the Italian association for logic and its applications. It is an act of institutionalization of the italian logical community in which theoretical computer science is formally recognised as a part of it.

The aim of this talk is to propose to the participants of the TLLA 2023 a shared reflection on the contribution of the italian logic community to the birth of linear logic. In this context it might be interesting to know what the participants think about the study and the rediscovery of some old themes that gave rise to the current research lines.

References

[1] Abrusci, V. M. *Sistemi di notazione ordinale e dilatatori.* Atti degli incontri di Logica Matematica. Scuola di Specializzazione in Logica Matematica, Dipartimento di Matematica. Università di Siena. 1982

[2] Girard, J.-Y. *The system F of variable types fifteen years later.* Theoretical Computer Science 45 (1986) 159-192 North-Holland.

[3] Girard, J.-Y. Π_2^1-*logic. Part 1: dilators.* Annals of Mathematical Logic 21 (1981) 75-219. North Holland Publishing Company.

[4] Girard, J.-Y. *Proofs and types.* cambridge University Press. 1989.

[5] Veredice, A. *La rinascita della Logica in Italia nella seconda metà del '900: aspetti storici e didattici.* PhD Thesis, Sapienza University of Rome, to be defended in 2023.