Principal types as lambda-nets

Pietro Di Gianantonio, Marina Lenisa

Overview

- we present a (strict) connection between
  - principal types for untyped λ-terms
  - cut-free λ-nets (proof nets associated to untyped λ-term)
- show that the connections can be used to obtain
  - a TAS for principle types
  - alternative proofs and explanations of some results relating types to computation

Principle types:

A λ-term M has principal type τ if

- the set of instantiations of τ coincides with
- the set of types σ assignable to M
  $\vdash M : \sigma$ (in a suitable TAS)

Principle types synthesise by the typing algorithm of ML, Haskell

Here we consider intersection type $(\rightarrow, \wedge)$

- in most approaches, principle types defined on λ-terms in normal form.
- the synthesis of principle intersection type for arbitrary λ-terms is complex [Ronchi88] [Kfoury,Wells02]

Lambda-nets

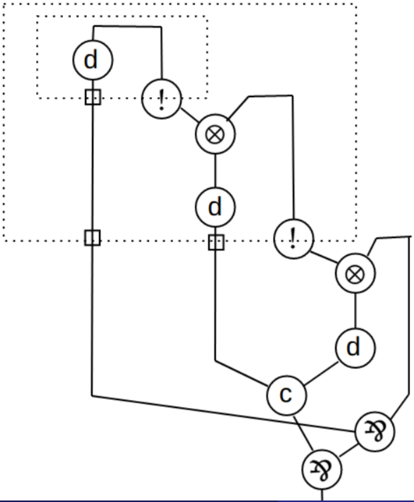
Proof-nets for λ-terms

- a λ-term M represents a proof π
- π can be represented also by proof-net P
- just some proof-nets in special form are obtained in this way
- normalisation of M (roughly) correspond to cut-elimination in P

Correspondece illustrate by an example

Take the Church number 2 $\lambda f.\lambda x.f(fx)$

the corresponding proof nets is:

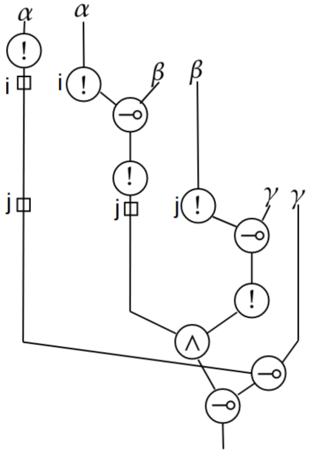


The principle type is obtained

by replacing each:

- $\otimes$ and $\parr$ by $\multimap$
- dereliction and promotion by !
- contraction by intersection
- axiom by two instances of the same variable
- box by symbols to representing box boundaries

The result is:



That is: $!(\Box_j !\beta \multimap \gamma) \wedge \Box_j\, !(\Box_i !\alpha \multimap \beta) \multimap (\Box_j \Box_i !\alpha \multimap \gamma)$

normally written as: $((\beta \to \gamma) \wedge (\alpha \to \beta)) \to (\alpha \to \gamma)$

Consequences

To design an algorithm to synthesise principle types

- principle type are primarily defined on λ terms in normal forms
- main problem: if $M : \tau$ and $N : \sigma$, what is the principle type of $M\ N$?
- a first possible solution:
  - from τ and σ, derive the lambda-nets π_1 and π_2, b
  - perform cut elimination on the composition of π_1, π_2
  - from the normalize lambda-net derive the principle type of MN
- a second solution:
  - mimic cut-elimination on the principle types

An alternative view for normalisation

If

- $\vdash M : \tau_1 \to \tau_2$
- $\vdash N : \sigma$

The principle type of *MN* is obtained by:

- defining the MGU U between τ_1 and σ
  making the two types equal by
  - instantiating variable (like in standard MGU)
  - duplicating boxes
- return the application of U to τ_2

Normalisation as an MGU algorithm

Further consequences

Relate types with computations

- $M : \tau$ (standard type) iff
- M has a principle type iff
- cut-elimination on the λ-net for M converges
- M is normalizable

Therefore:

- if M is typable, then it is normalisable
- subject reduction holds
  - reduction preserve principle types

With non-idempotent intersection $\wedge$

- to find a type for a term is as complex as normalising it
  - one build a type that is an expansion of the normal form
- types contain information about the number of reduction steps in normalisation
  - normalisation is related to cut-elimination and the number of steps cut-elimination depends on complexity of the formula
- inhabitability of types is decidable, and related to correctness of proof structures
  - a type as only a finite set of possible candidates as principle type, check for any of them if it is the representation of a term

Further consequences

Synthesise a type system for call-by-value λ-calculus

Consider:

- the encoding of the call-by-value λ-calculus in LL, through λ-nets $O \sim !(O \multimap O)$
- the corresponding principle types, through the above translation
- the set of types instantiation of principle types

Thanks for your attention