Efficient synthesis algorithms for Parametric Timed Games with merging
Extended Abstract

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Many real-time systems, such as embedded controllers or network devices, must be verified under uncertainty — whether due to environmental factors or unknown hardware delays. Parametric Timed Automata (PTA) [2] are a formalism that extends timed automata (TA) [1] with timing parameters, enabling analysis and reasoning about timed systems with unknown or uncertain constants. PTA can be used for reachability analysis, synthesizing the set of parameters for which a target location is reachable in the automaton. Another aspect of real-time systems that is useful to capture is differentiating whether an action is within control of the system or something uncontrollable caused by the environment.

This leads us to Timed Games (TG) [11]. TG extends TA by partitioning the actions into controllable (player) and uncontrollable (environment) actions. The goal is to determine whether a strategy exists for the player that guarantees reaching a target state, regardless of the environment's decisions. This is called a *winning strategy*. An on-the-fly algorithm was introduced in [6], to decide whether a timed game is won by the player. This algorithm forms the basis of Uppaal Tiga [5].

Finally, Parametric Timed Games (PTG) combine the ideas of PTA and TG and were introduced along with a semi-algorithm, that synthesizes the set of parameters for which a winning strategy exists [9,10]. This algorithm was extended and implemented for the first time in [7], with the theoretical guarantee that it enumerates all constraints on clock parameters, for which the game is won. The implementation is an extension to the Imitator model checker [3], supporting a wide selection of algorithms for PTA. While [7] synthesizes parameters, it does not output a strategy describing how to win the game. This was the focus in [8], where the implementation and theory is extended to synthesize a concrete strategy as well as a controller PTA that enforces it.

Unfortunately, these PTG reachability algorithms, like their PTA counterparts, suffer from *state space explosion*. The classical approach constructs a Parametric Zone Graph (PZG), a symbolic graph where states are pairs of discrete locations and convex constraints over parameters and clocks. In the PTA setting, *merging* of convex zones has been shown to be an effective method to tame the state space for reachability objectives [4]. However, directly applying the same method to PTG is nontrivial for two main reasons:

1. Various bookkeeping for e.g. backward propagation of winning states is informed by and relies on the structure of the PZG
2. The exploration/update order is not specified for current PTG algorithms, where it must be layer-based to apply the merging method.

We adapt the efficient merging from [4] to PTG in a way that preserves correctness of the synthesized parameter sets and strategy synthesis. The adaptation involves modifications of the exploration order in the base algorithm, as well as the merge procedure. The merging is integrated into an existing on-the-fly PTG algorithm that already supports inclusion checking and other optimizations [7,8].

To assess the practical benefits of merging in this setting, we introduce a novel PTG case study inspired by an adversarial IoT scenario involving uncertain communication delays. A central scheduler receives communication requests from multiple devices and must continually grant them while ensuring that only a certain number can communicate at a time to avoid congestion. An adversary can delay requests to manipulate their timing and force synchronization. The model is parameterized by the communication duration, the maximum delay between request grants, and the maximum adversarial delay. The case study also motivates the addition of a new *Reach-Avoid* objective, which we show simplifies modelling.

Finally, we demonstrate that applying merging in PTG yields an additional benefit: a reduction in the size of synthesized controllers. Experimental results show improved runtime and scalability. The resulting controllers are smaller, making them easier to interpret and implement.

References

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