Parameterised jobshop scheduling problems

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Jobshop scheduling problems

- Well known combinatorial optimisation problems
- (finite number of) **jobs**
- (finite number of) **machines**
- Each job has to accomplish some task
- which consists of operations which use some machine(s)
- A machine can only be used by one job at the same time
- The operations must obey ordering constraints

Jobshop scheduling problems

- What is the optimal schedule ?
- Easily computable by trying all schedules
- Typically NP-hard
- Here: a **parameterised** version

Parameterised jobshop scheduling

- A fixed number of machines ({a,b,c,...})
- A parameterised number of identical jobs
- Each job is given as a sequence of the machines it has to use successively
- For example: a.a.b.c.d.a.a.b.c.c.d
- Each machine can be used by one process at a given moment.
- Each step costs 1

Main problem

- Given a number of machines n, compute

  cost(n) := the number of total steps to complete
  all n jobs
- Obviously, count(n) can be computed for fixed n
- We want to compute a representation of

  {(n,count(n)) | n >= 1} in one shot

Example

a	a	b	a	b	b

a	a	b	a	b	b				
		a		a		b	a	b	b

a	a	b	a	b		b			
		a		a	b	a	b	b	

a	a	b	a	b	b						
		a		a		b	a	b	b		
					a	a	b	a		b	b

a	a	b	a	b	b									
		a		a		b	a	b	b					
					a	a	b	a		b		b		
									a	a	b	a	b	b

Example

- a.a.b.a.b.b
- Upper bound for cost(n): 6*n,
  - since each jobs takes at most 6 time units
- Lower bound for cost(n): 3*n
  - since each job must use a at least 3 times
- Therefore, 3*n <= cost(n) <= 6*n
- Here, cost(n) = 3*n+3

Some special cases

- If the job j uses the same machine all the time:

  cost(n) = |j|*n
- If the job uses |j| different machines:

  cost(n) = n + |j| - 1

In general

- Let j be a job
- Let f be the length of j
- Let m be one of the machines which is used the most
- Let g be the number of times m is used
- Clearly, g*n <= cost(n) <= f*n
- we show that cost(n) <= g*n+c for some constant c

Main result

- cost(n) is a semilinear function
  - {(n,cost(n)) | n >= 0} is a semilinear set
  - that means:

$$cost(n)=\begin{cases} d_1 \ \ if \ \ n=1 \\ \ldots \\ d_p \ \ if \ \ n=p \\ k*n+c_1 \ \ if \ \ n \bmod q=0 \\ k*n+c_2 \ \ if \ \ n \bmod q=1 \\ \ldots \\ k*n+c_q \ \ if \ \ n \bmod q=q-1 \end{cases}$$

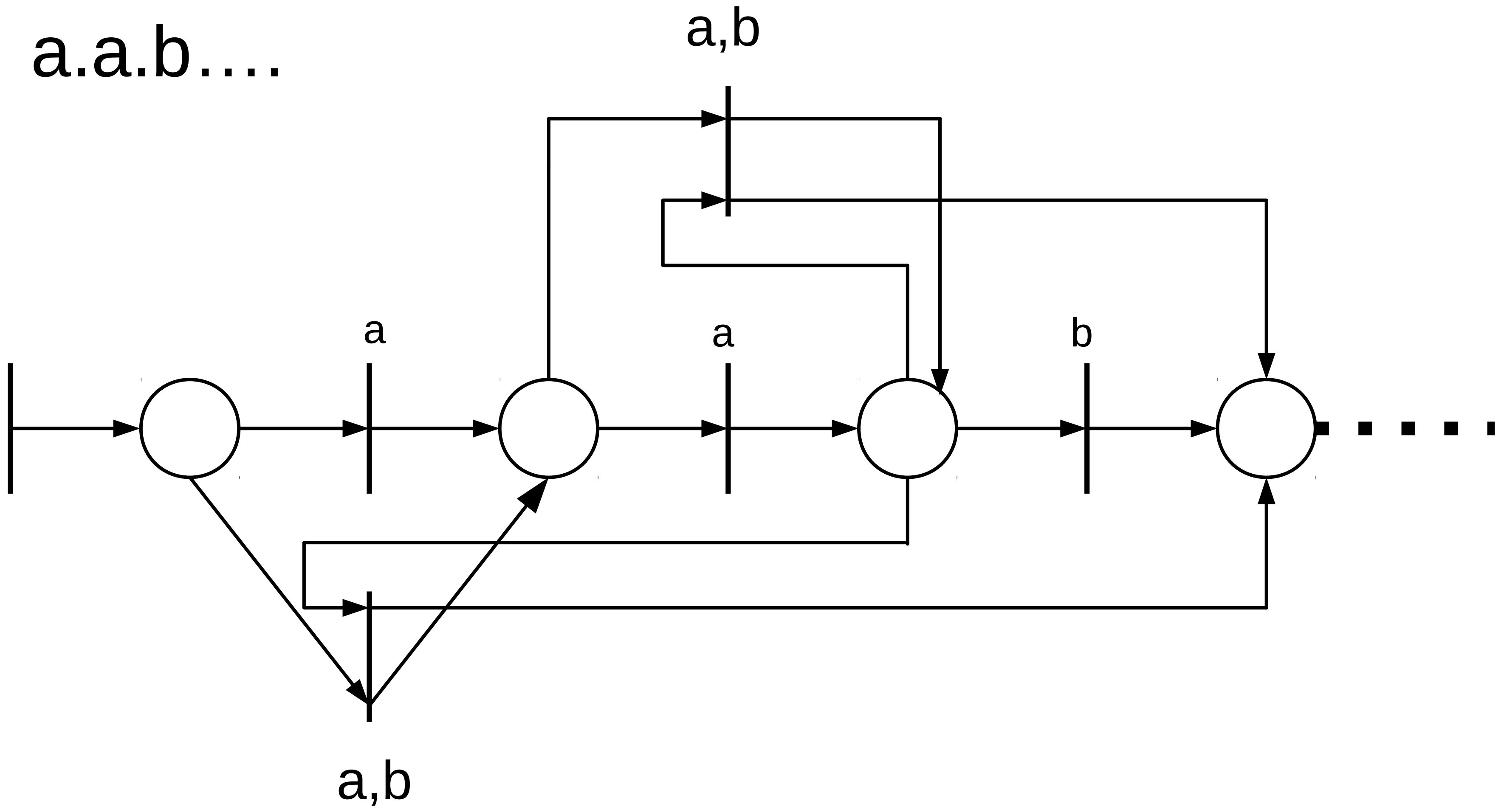
- Solution: transformation to a Petri Net problem

Transformation to a PN problem

- Counting abstraction
  - Each position in the job corresponds to a control state
  - Consider number of jobs in each state
- Construct an equivalent PN N
  - Each position in the job corresponds to a place in N
  - Transitions of N are moving tokens ahead
  - Each transition is labeled by the corresponding set of machines
- Initially, n tokens or a generating transition
- Each transition is counted for the cost

Example

- a.a.b….



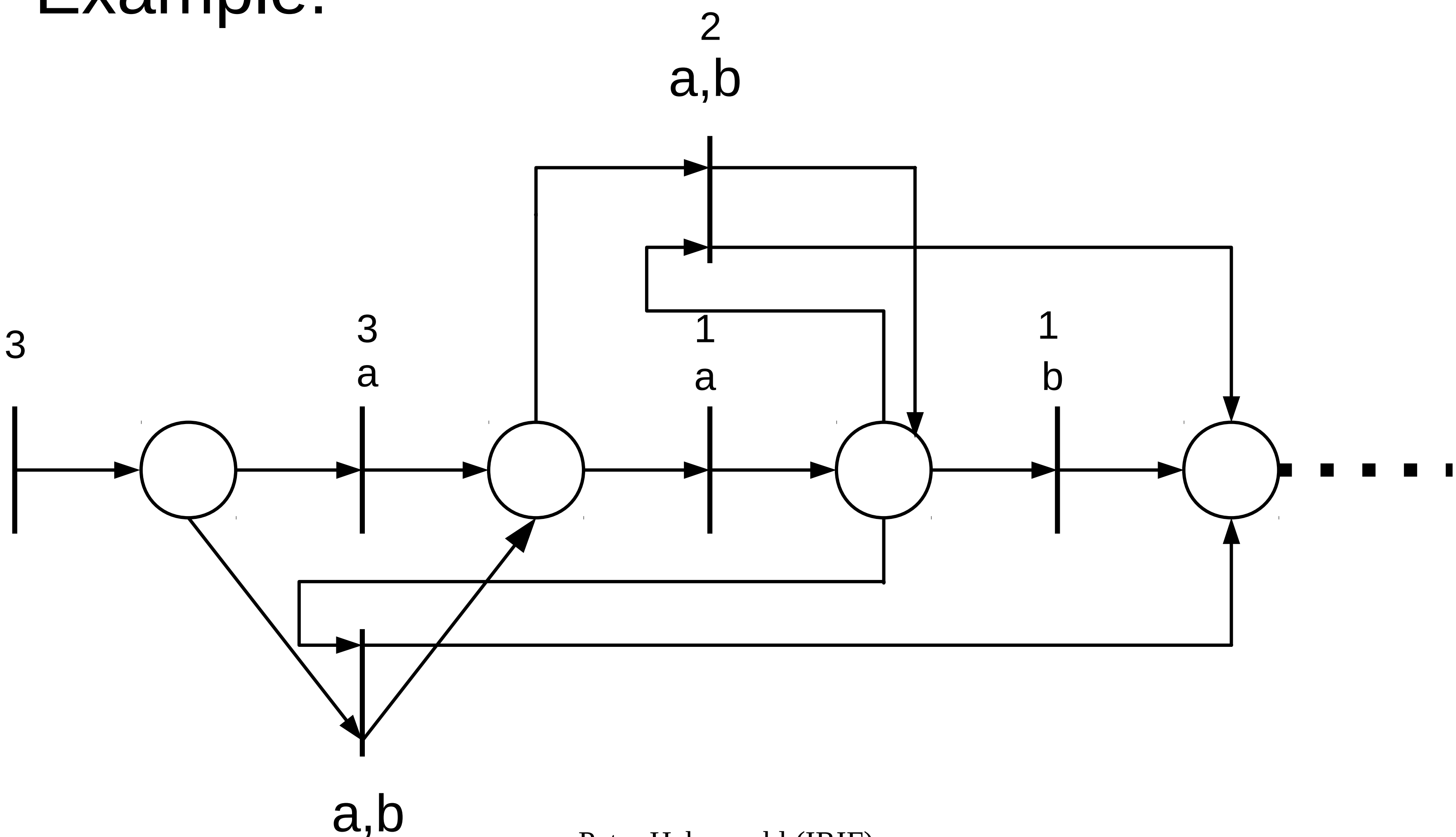
Transition invariants

- Let M be the incidence matrix of N
- A transition invariant is a vector t (multplicities of transitions)

  such that $Mt = 0$
- Executing a sequence of transitions corresponding to t keeps the token counts constant

Transition invariants

Example:

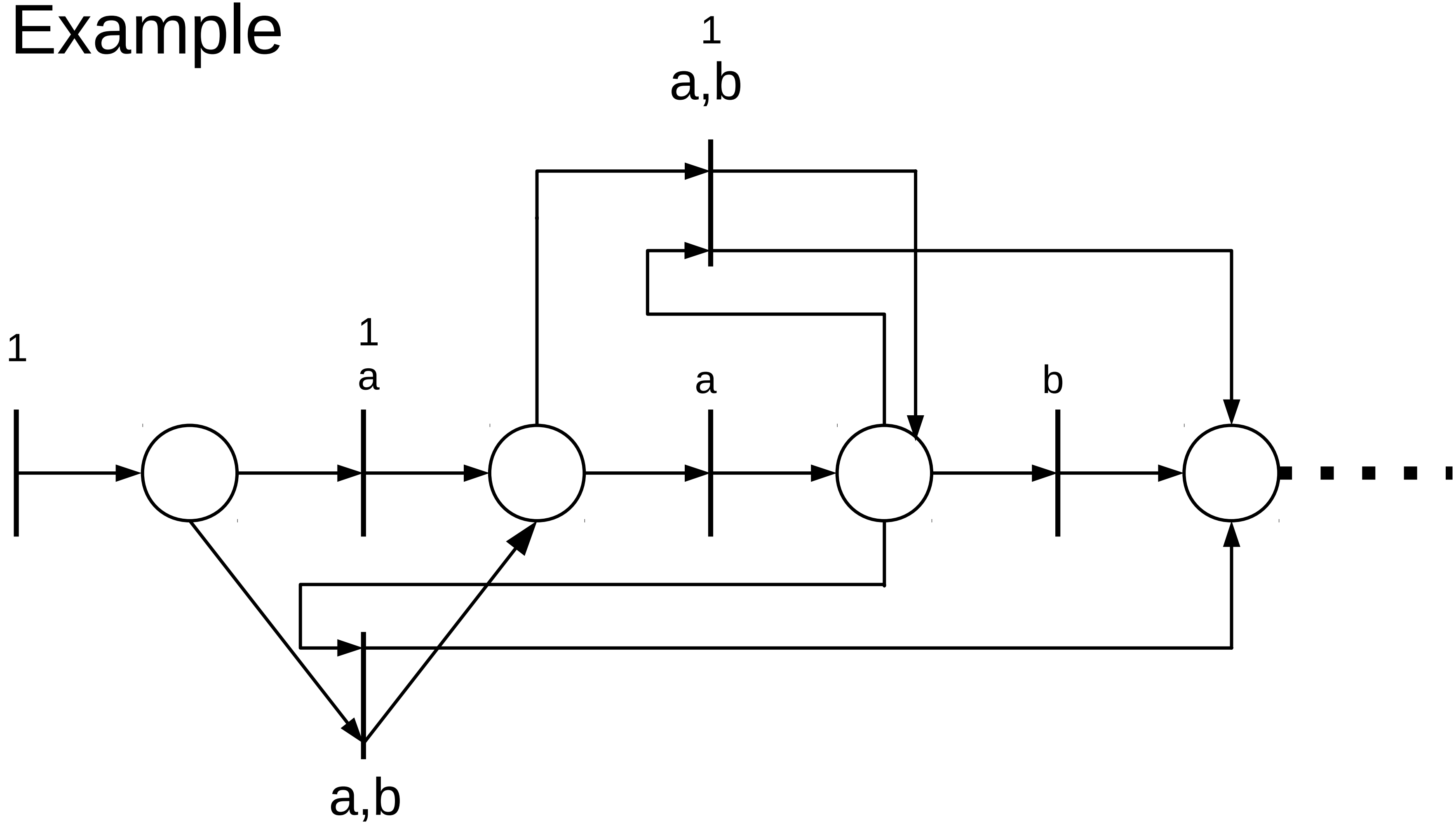


Transition invariants

- Here all transition invariants t are realisable
- which means, there exists a reachable marking, s.t. from there a sequence of transitions with count t can be executed

Transition invariants

- Example



Transition invariants

- One can compute all transition invariants
  - finite number of minimal transition invariants
- Compute **optimal** transition invariants
  - the machine m is always in use
- For any number n we can construct a run where almost all the time transitions from an optimal transition invariant are used

Example

a	a	b	a	b	b

- Optimal transition invariant:

a	a	b	a	b	b

- Realisation:

a	a	b	a				b	b								
				a	a	b		a		b	b					
						a	a		b		a		b	b		
									a	a		b		a		b
												a	a		b	

Computing cost(n)

- We obtain k*n <= cost(n) <= k*n + c
- It remains to compute for each c’ with 0 <= c’ <= c:

  {n | cost(n) = k*n+c’}
- Modify PN N:
  - Generate k*n+c’ tokens in a “counting” place and n tokens in the initial place
  - Remove one token of the “counting” place for each transition
  - Define a PN language with one-letter: reach empty marking
  - Since one-letter PN languages are regular (Hauschildt/Jantzen 94), we have that {n | cost(n) = k*n+c’} is semilinear.

Boundedness conjecture

- For each execution of the PN N, there is an execution with same or better cost, where the number of tokens are bounded
- Would imply easily the result

Extensions

- Steps which cost different from 1:
  - Cost k: k steps of cost 1
  - Rescheduling
- A job can choose from several sequences:
  - For example: aababb or bbabab or aabb or bbaa
  - Here we still have just one parameter n
  - The same reasoning can be applied

Extensions

- Several parameters:
- n_1 jobs of type 1, n_2 jobs of type 2, etc.
- Compute cost(n_1,n_2,...n_i)
- We still have that the optimal cost can be computed up to a constant c
- but the same reasoning as with cost(n) can not be applied