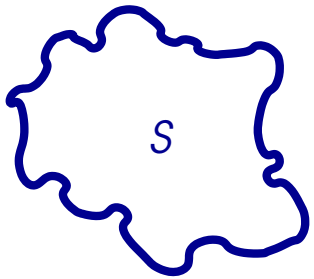
Consistency for Parametric Interval Markov Chains

Benoît Delahaye

Université de Nantes

SynCoP 2015

Context (1/2): Probabilistic Specifications



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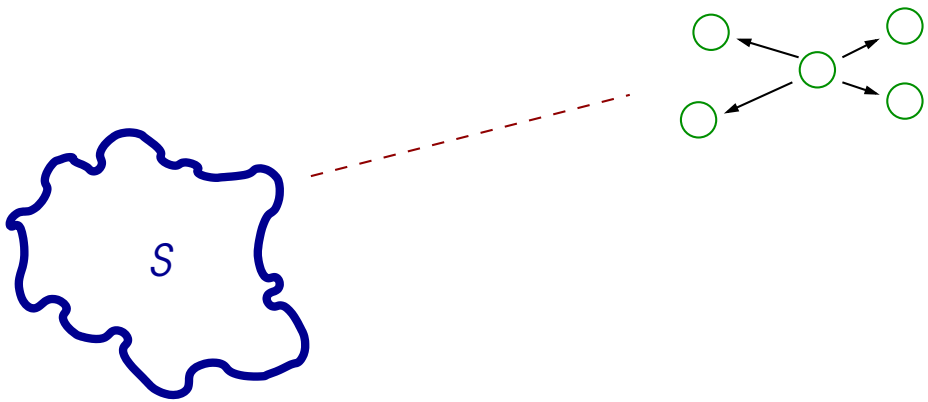

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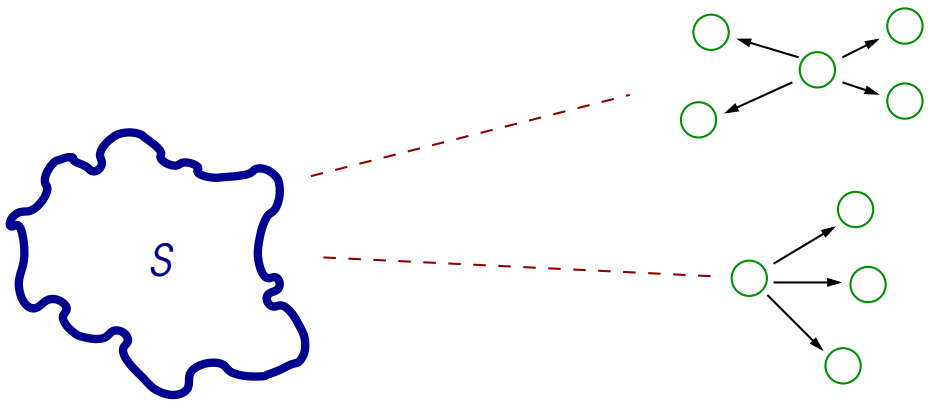

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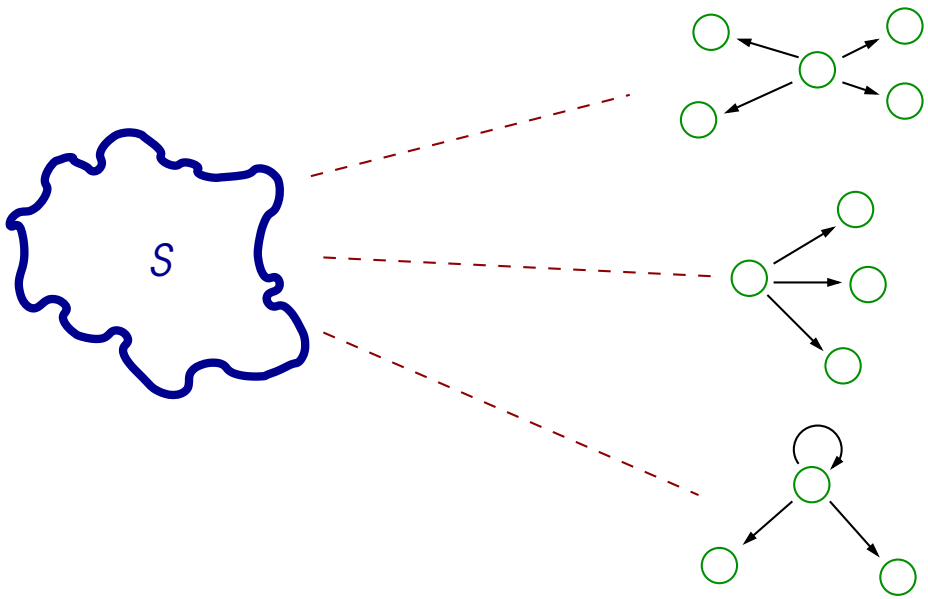

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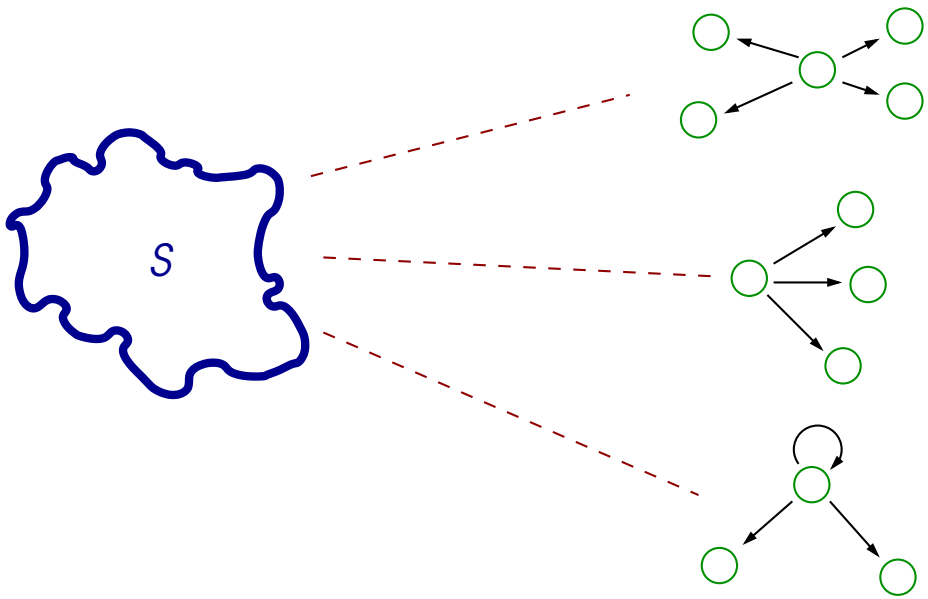

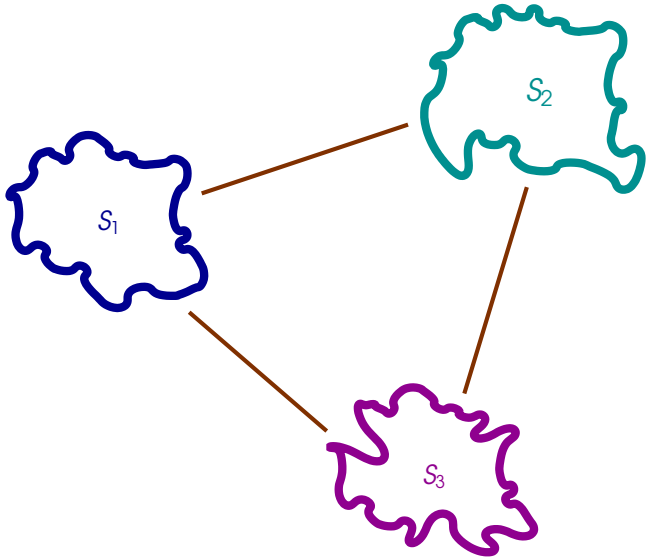

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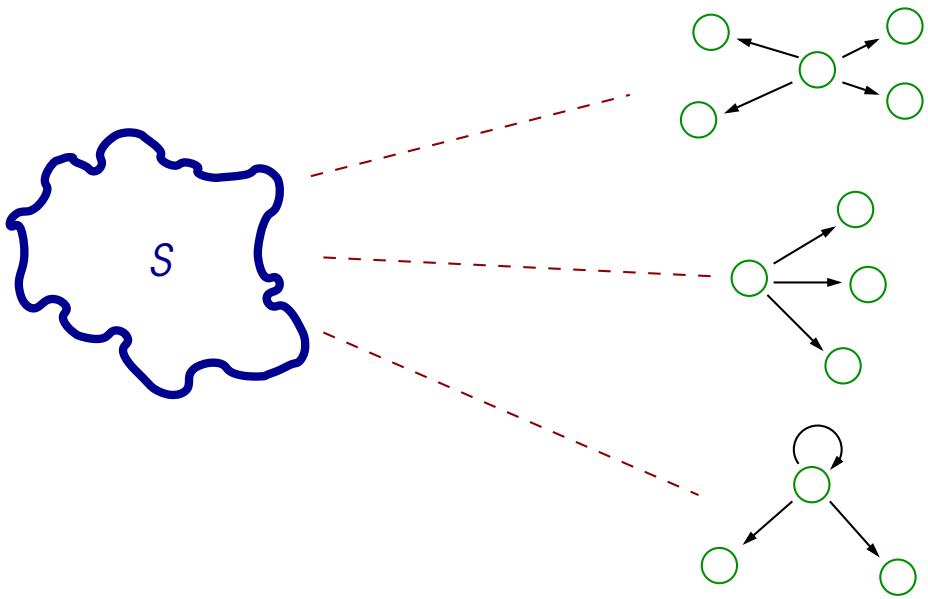

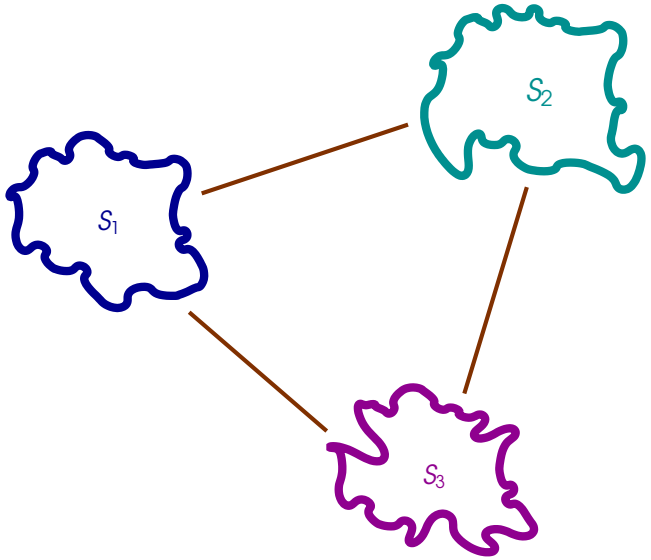

Compositional Design

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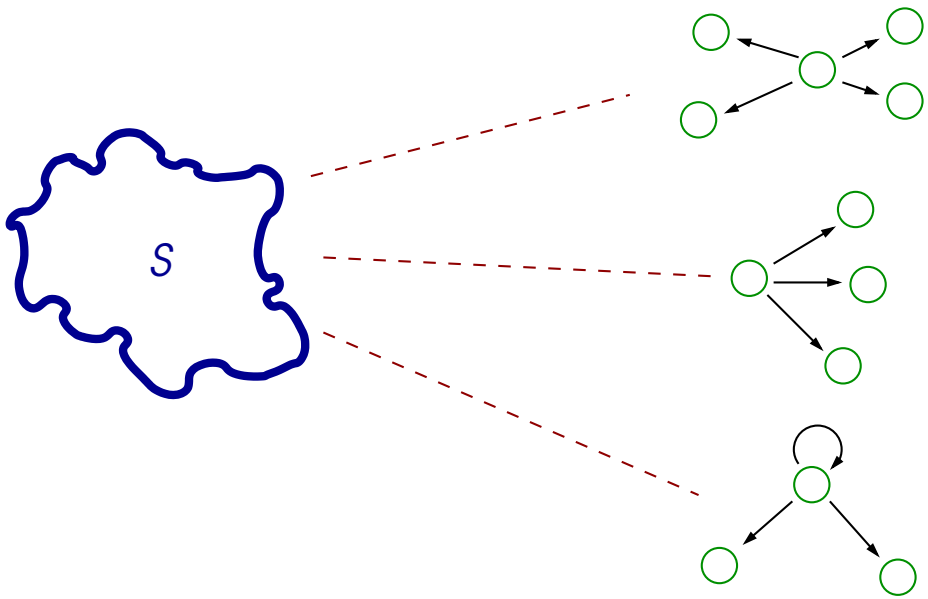

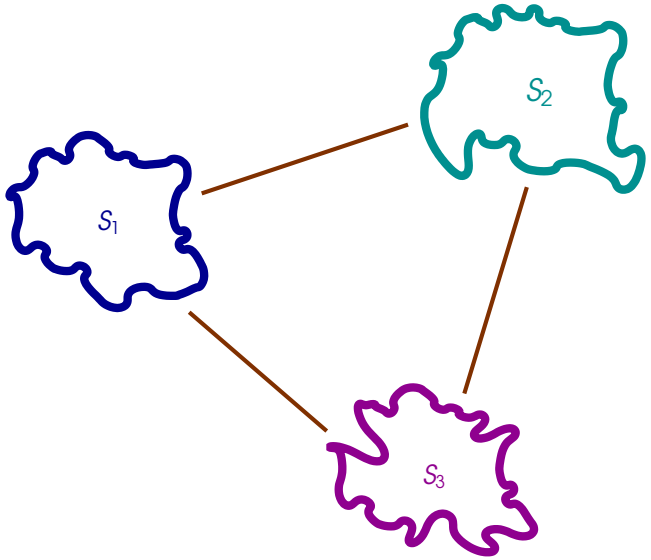

Compositional Design

Generic Results

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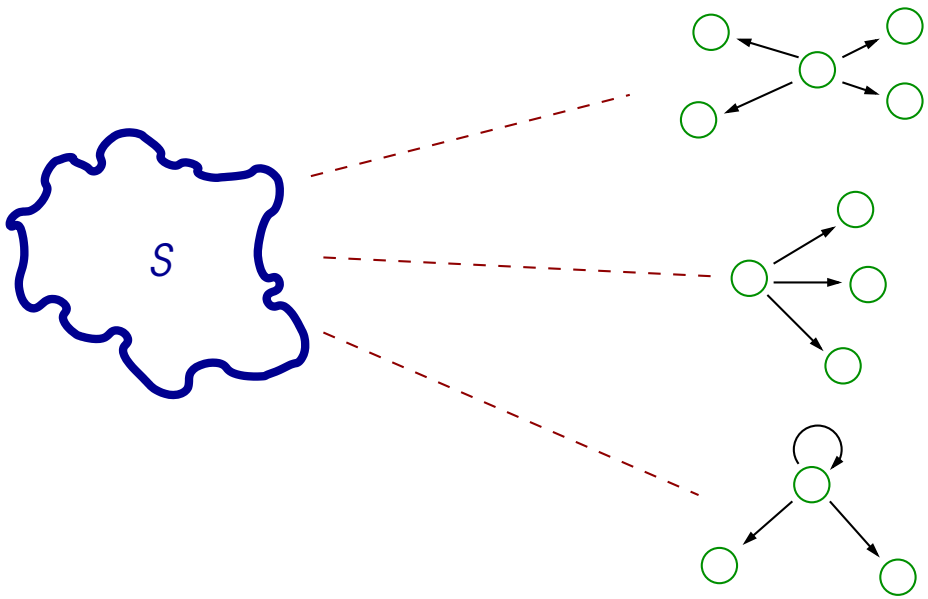
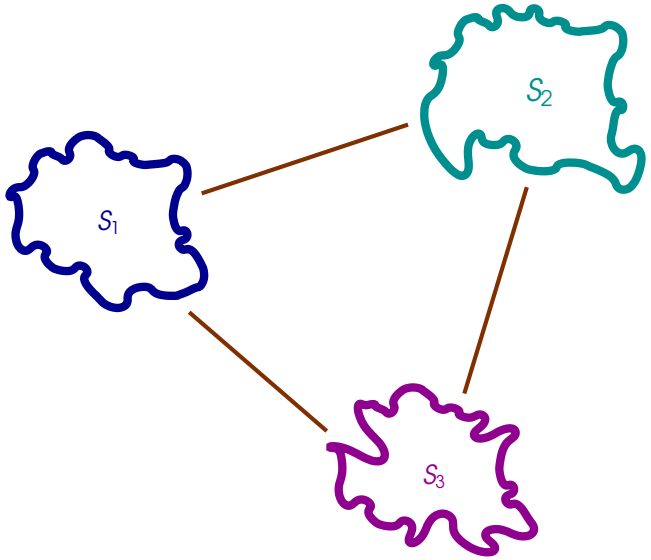


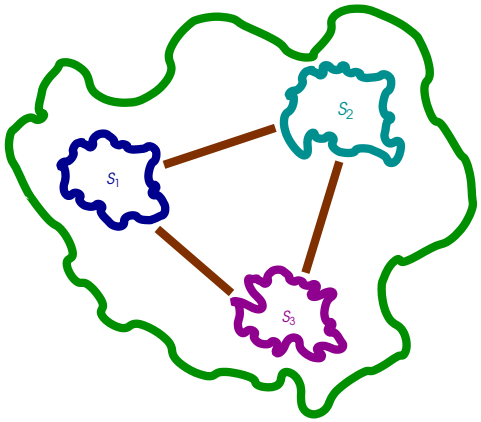


Compositional Design

Generic Results

Independent Reasoning

Context (2/2): Consistency



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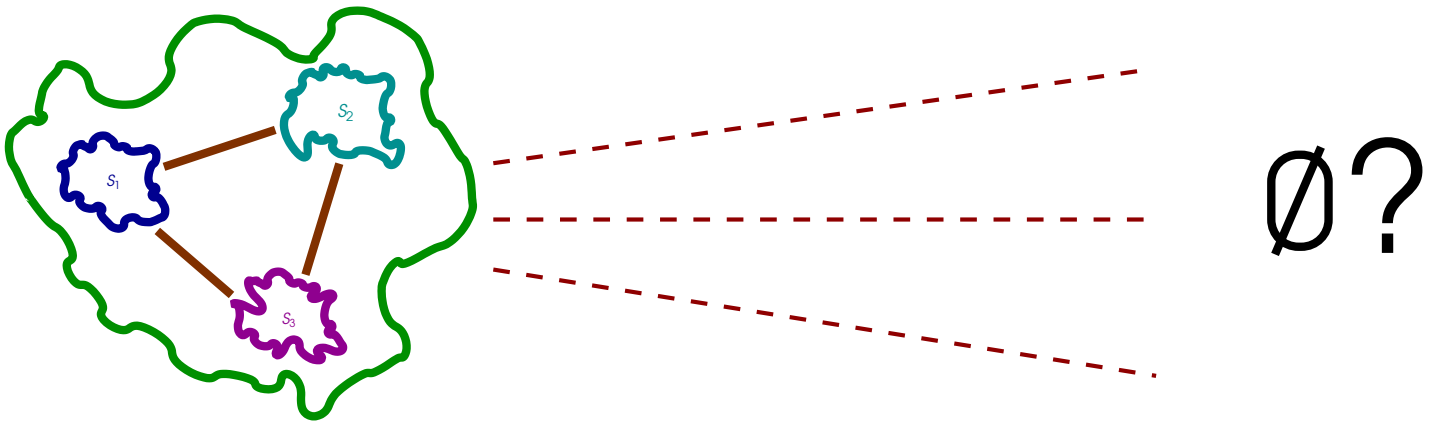

Consistency

Given a specification S, decide whether it admits at least one implementation.

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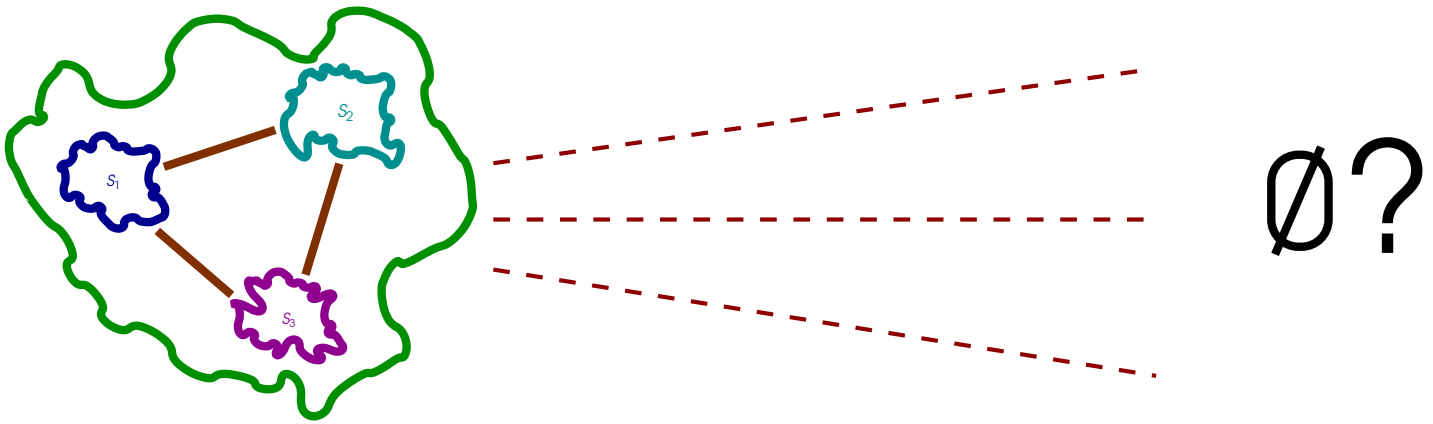

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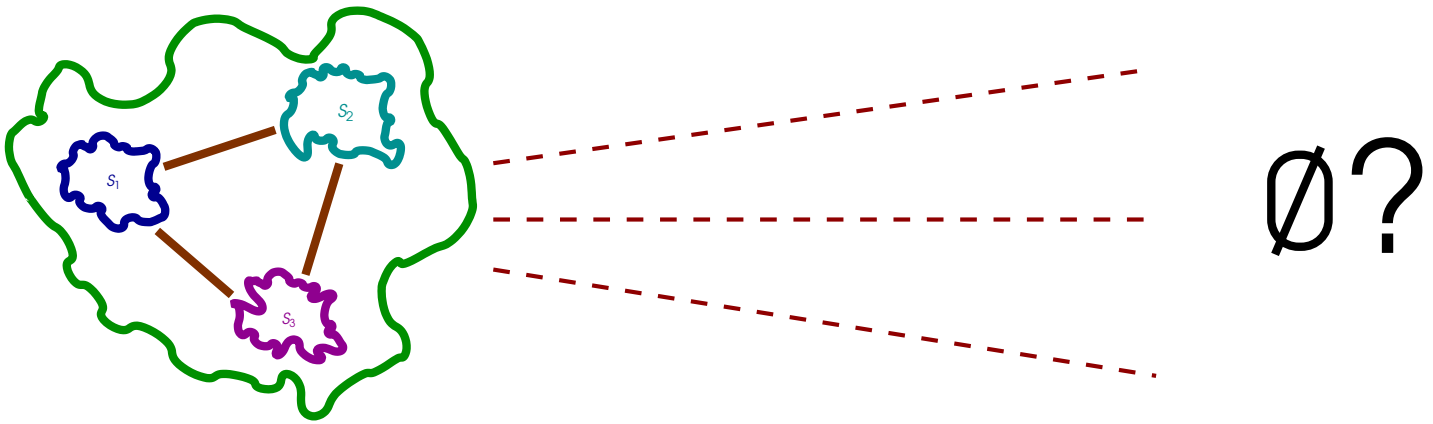

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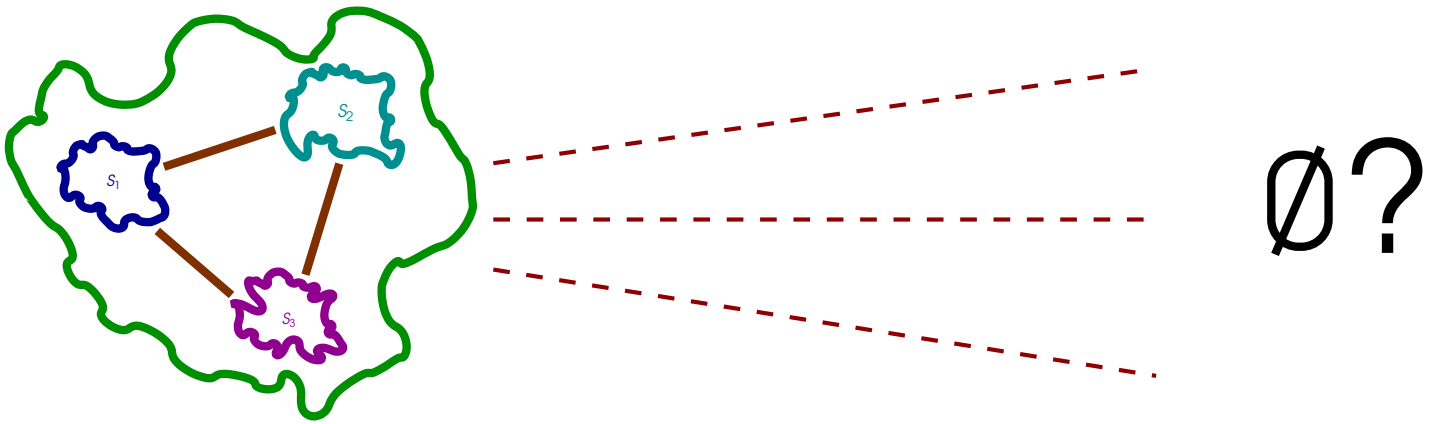

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- Common implementation
- Refinement

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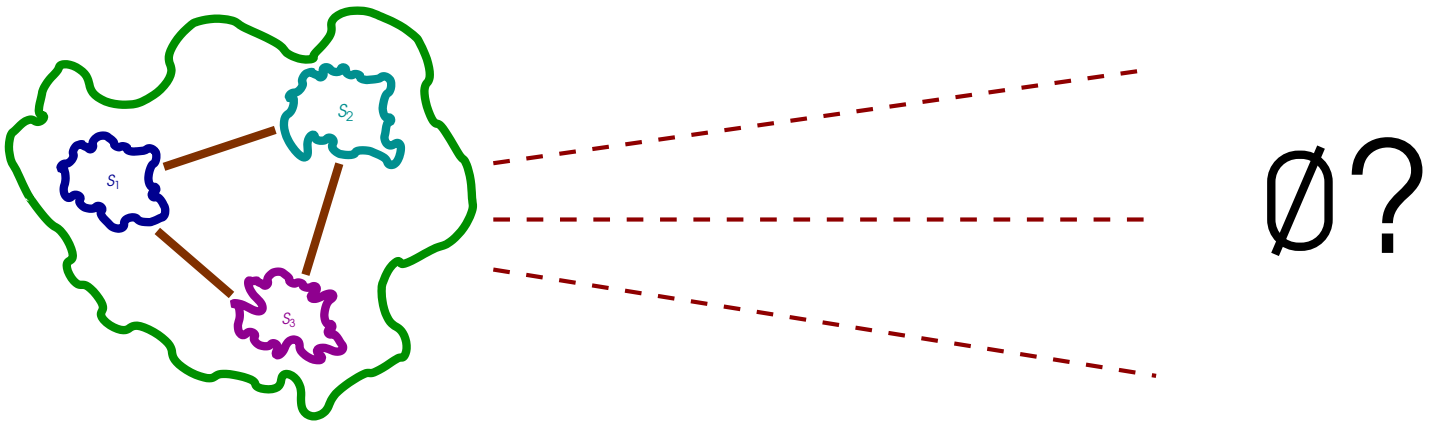


Consistency

Given a specification S, decide whether it admits at least one implementation.

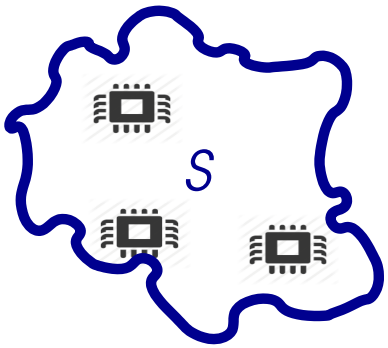
Other questions of interest

- Common implementation
- Refinement
- Composition
  Logical / Structural

Motivation: Why put parameters on probabilities?

Component failure rate

Can be chosen, with adequate cost



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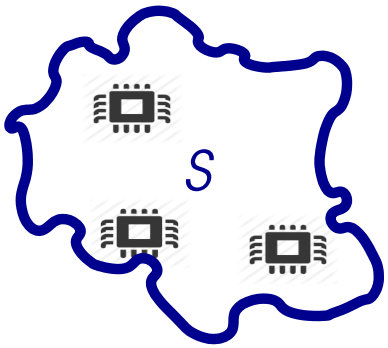

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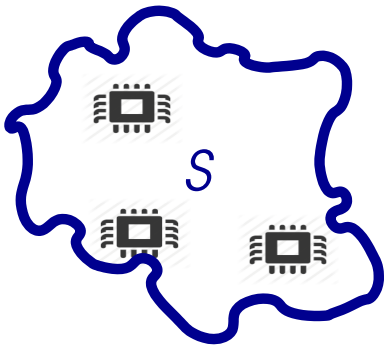

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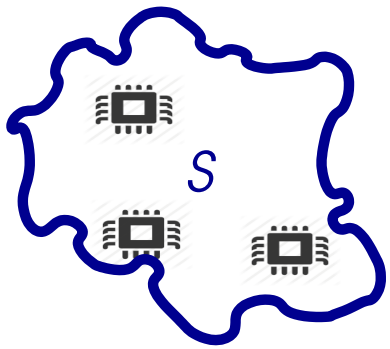

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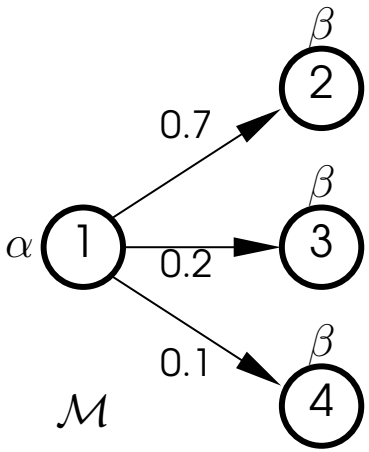
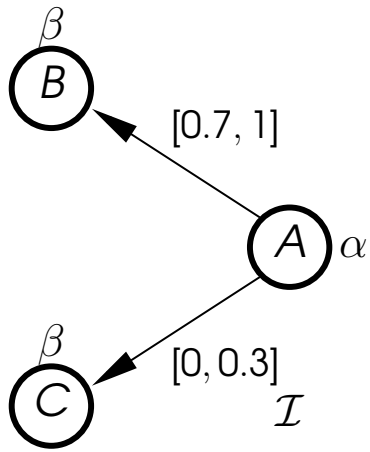
Outline

(Interval) Markov Chains





$$\mathcal{M} = (S, s_0, M, A, V)$$

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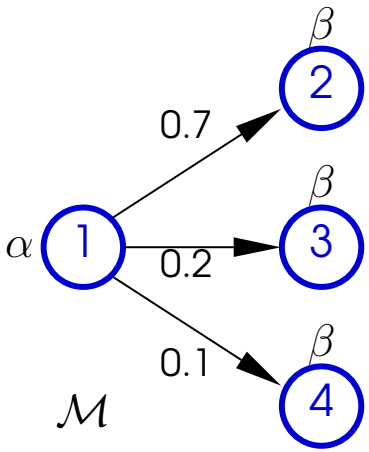
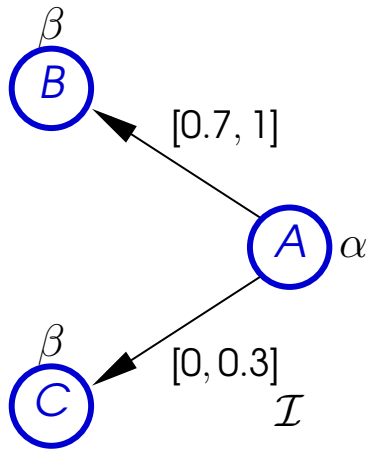
(Interval) Markov Chains





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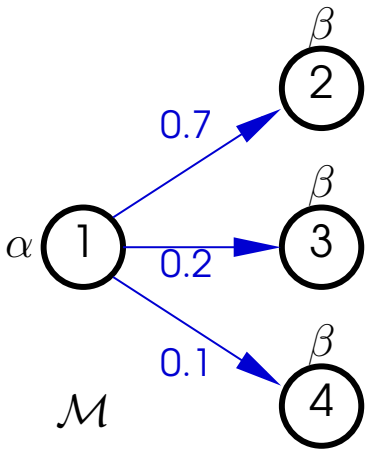

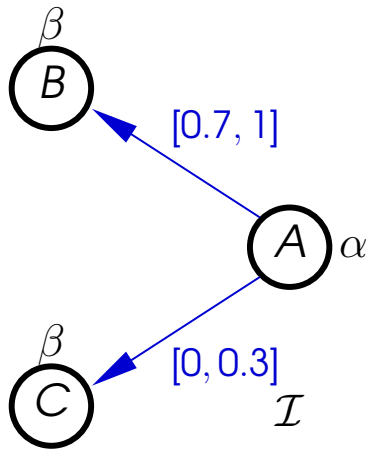

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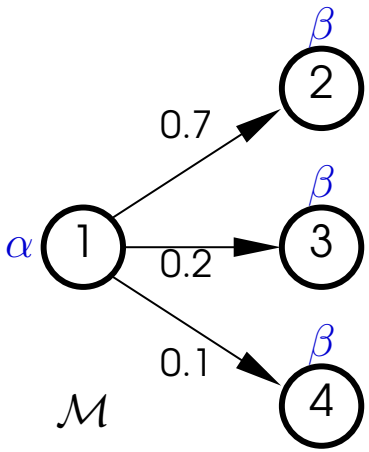

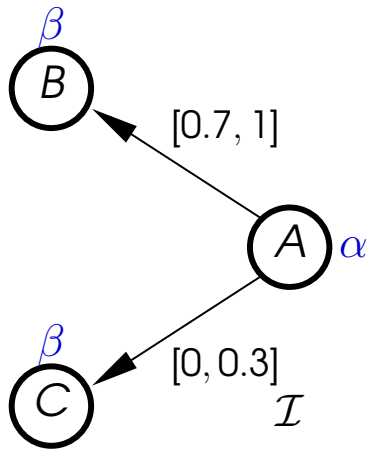

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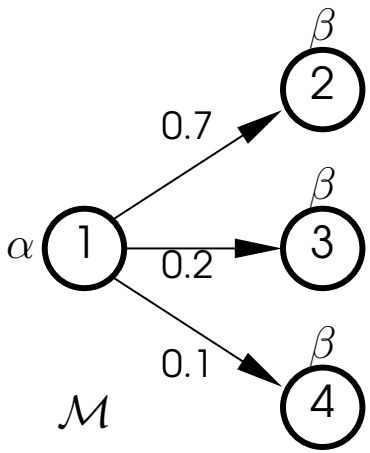
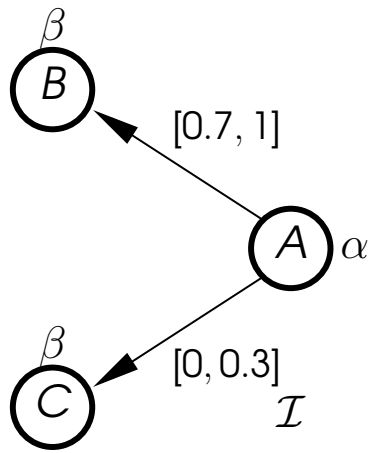
Satisfaction

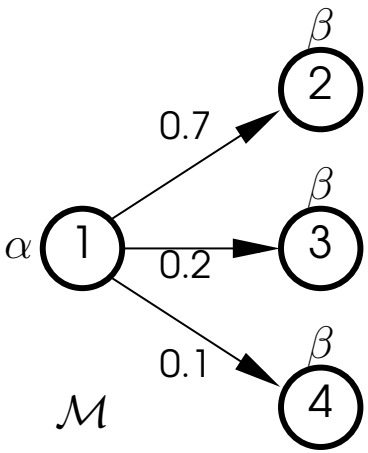


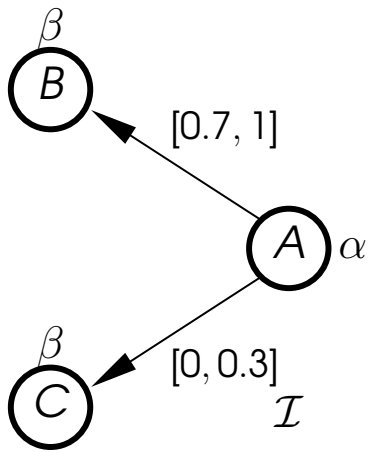


Satisfaction Relation $\mathcal{R}$

Whenever $(t, s) \in \mathcal{R}$, we have

Satisfaction





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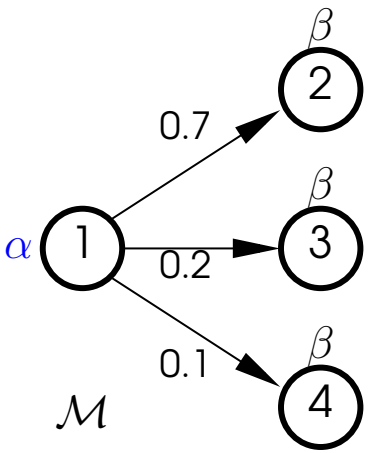

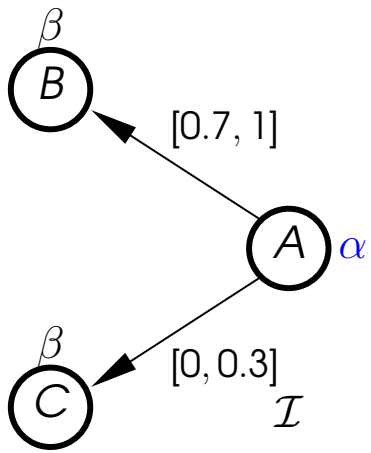

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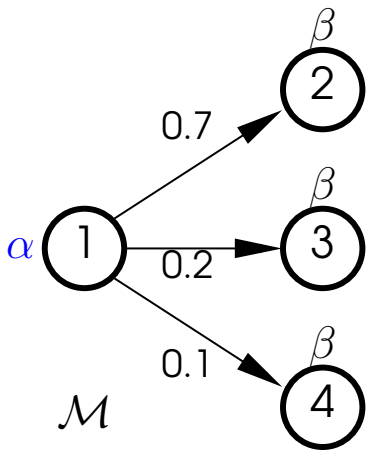

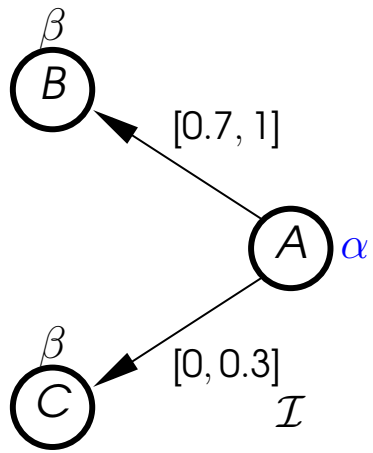

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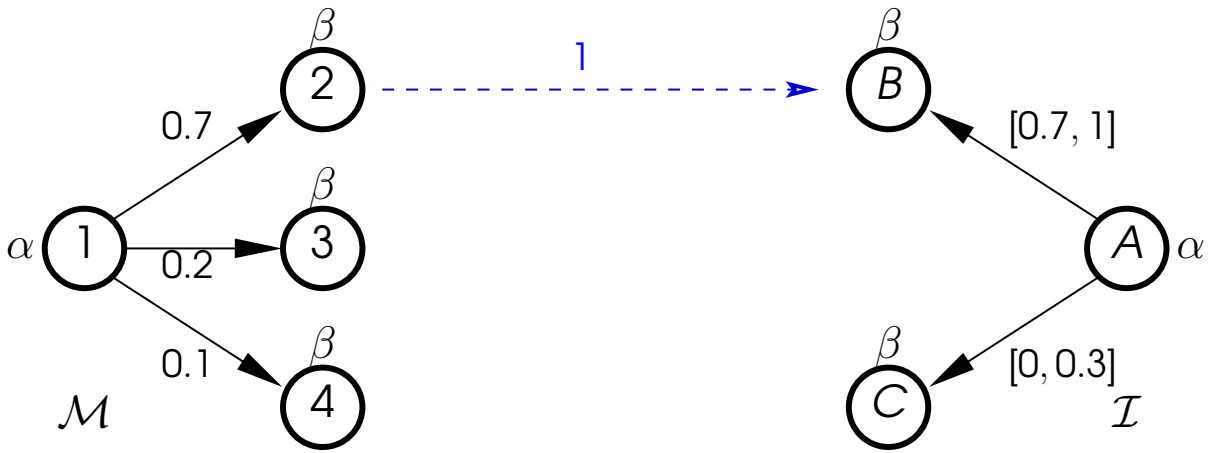

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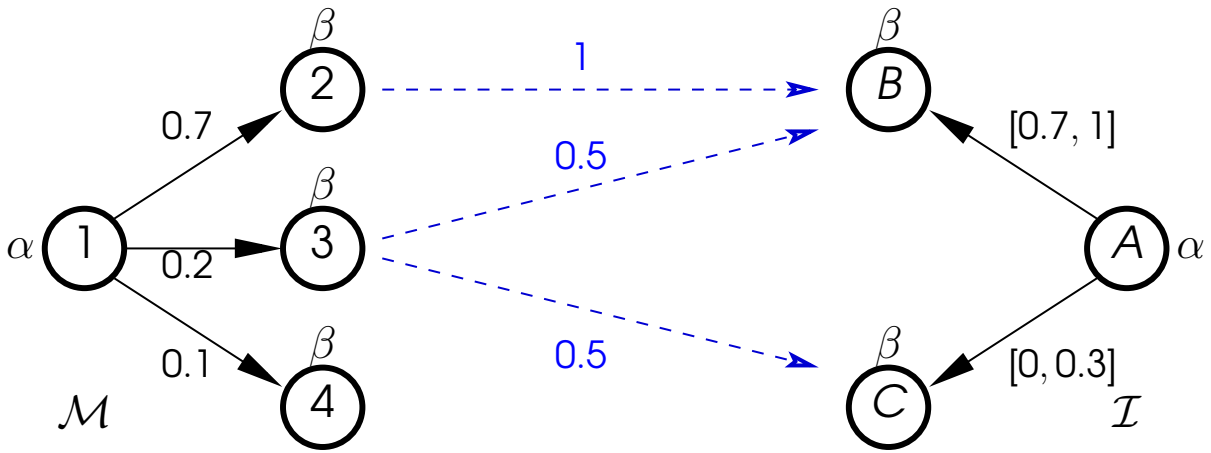

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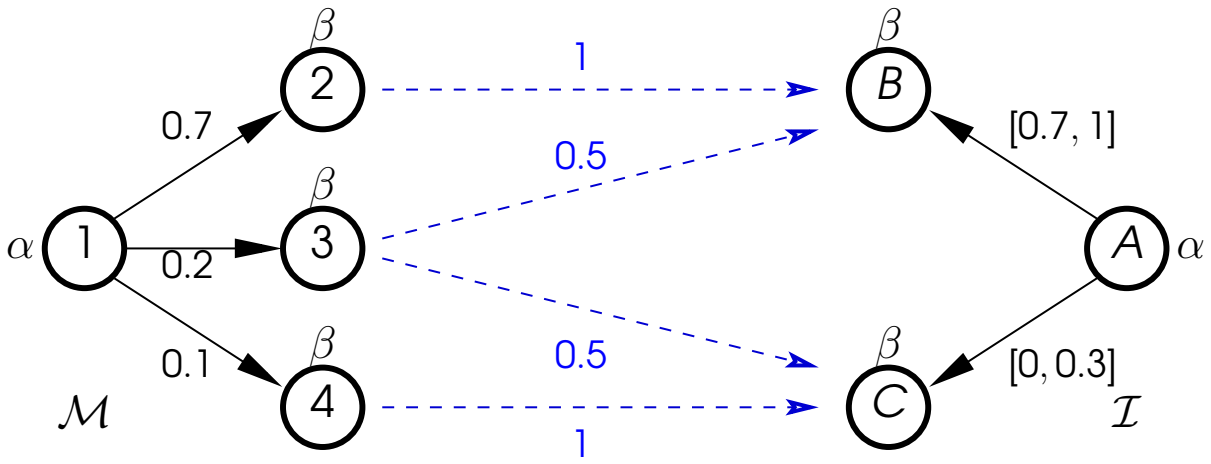

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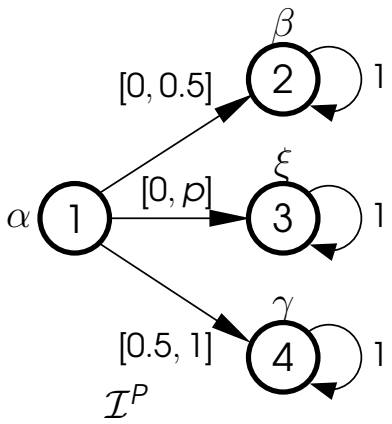
Parametric Interval Markov Chains

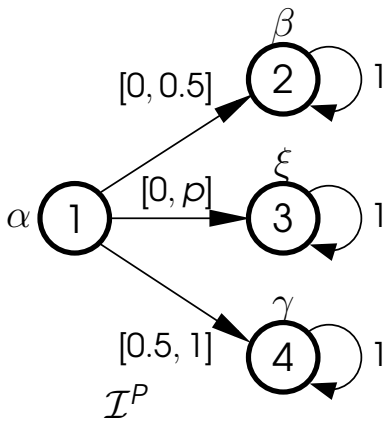


pIMCs

$$\mathcal{I}^P$$

Intervals can contain parameters from set P

Parametric Interval Markov Chains



pIMCs

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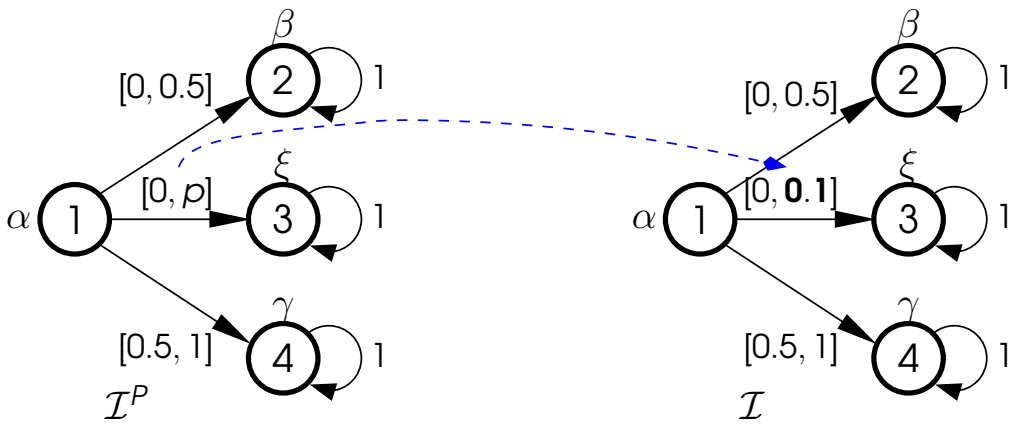
Intervals can contain parameters from set P

Instantiation

$$\mathcal{I} \vdash \mathcal{I}^P$$

Assign values to parameters

Parametric Interval Markov Chains



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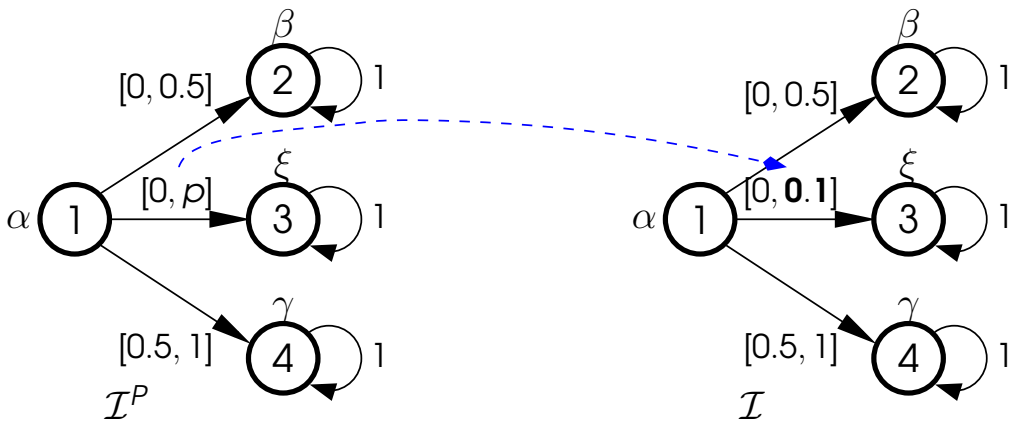

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Instantiation

$\mathcal{I} \vdash \mathcal{I}^P$

Assign values to parameters

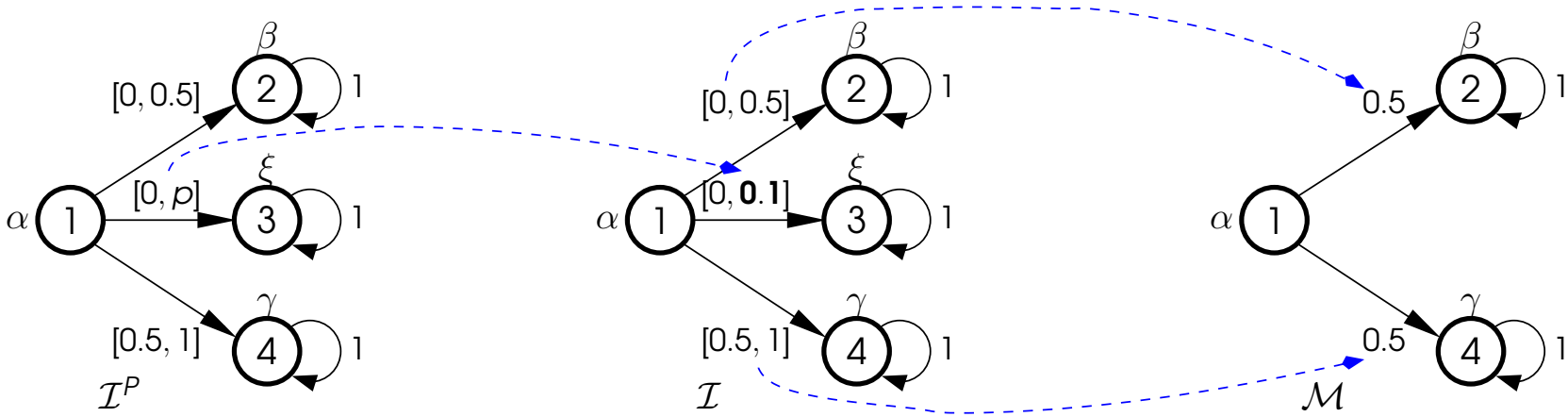
Satisfaction

$\mathcal{M} \models \mathcal{I}^P$

iff $\exists \mathcal{I}$ s.t.

$\mathcal{M} \models \mathcal{I} \vdash \mathcal{I}^P$

Parametric Interval Markov Chains



pIMCs

$\mathcal{I}^P$

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$\mathcal{I} \vdash \mathcal{I}^P$

Assign values to parameters

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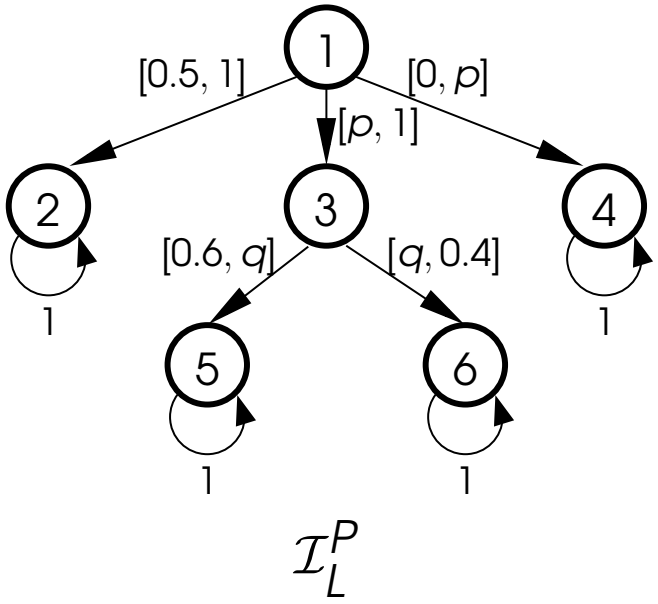
$\mathcal{M} \models \mathcal{I}^P$

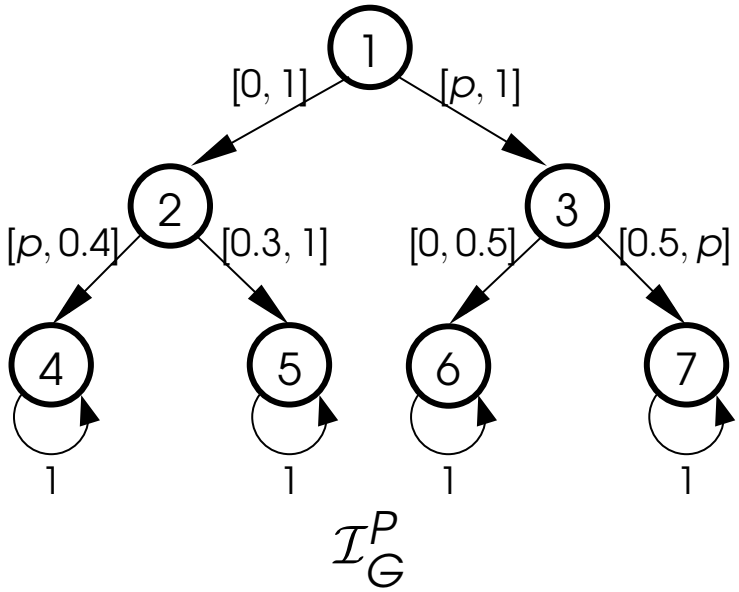
iff $\exists \mathcal{I}$ s.t.

$\mathcal{M} \models \mathcal{I} \vdash \mathcal{I}^P$

Local vs Global Parameters



$\mathcal{I}_L^P$



$\mathcal{I}_G^P$

Local Parameters

p is **local** if it appears in outgoing transitions of **at most 1 state**

Global Parameters

p is **global** if it appears in outgoing transitions of **at least 2 state**

Outline

Consistency of IMCs

IMC $\mathcal{I}$ is consistent iff there exists at least one MC $\mathcal{M}$ such that

$$\mathcal{M} \models \mathcal{I}$$

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- Decidable ([JLAP'12])
- (Complex) Algorithm based on *Consistency Relations*

Local state-consistency

A state is **locally consistent** iff there exists a distribution ρ matching its outgoing intervals.

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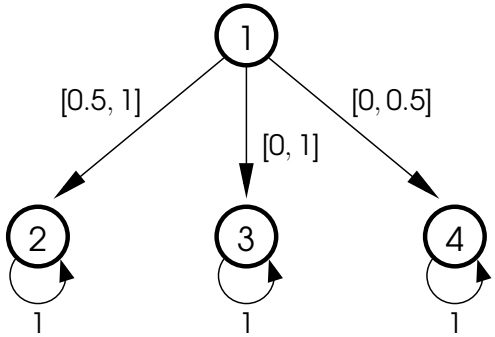

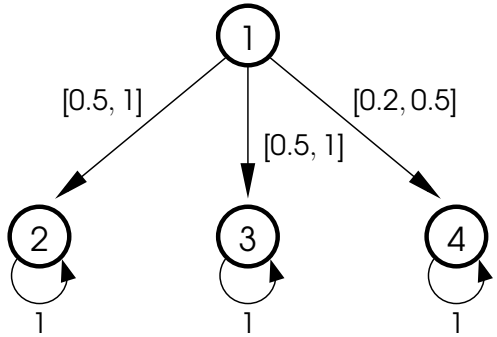

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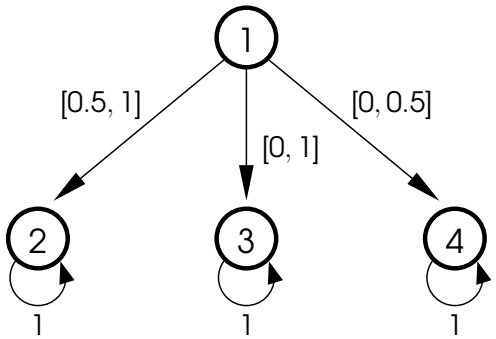

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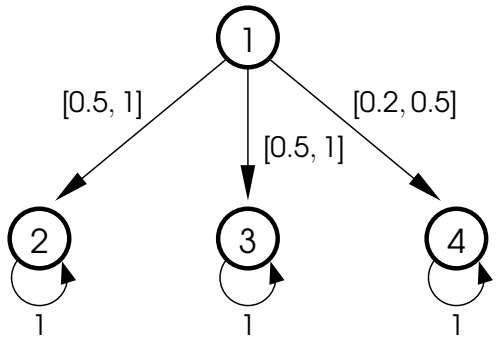

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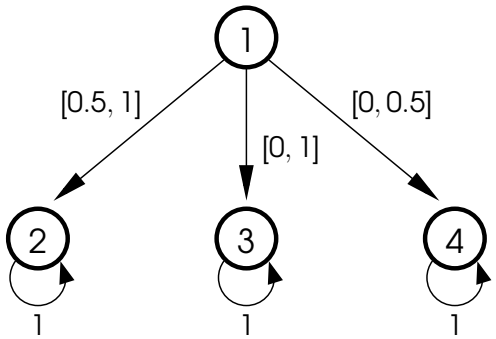

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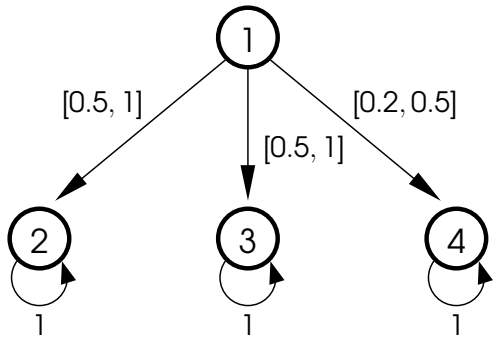

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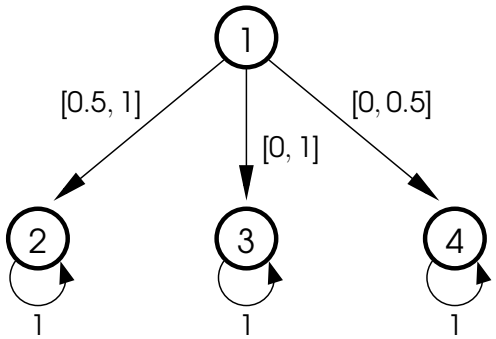

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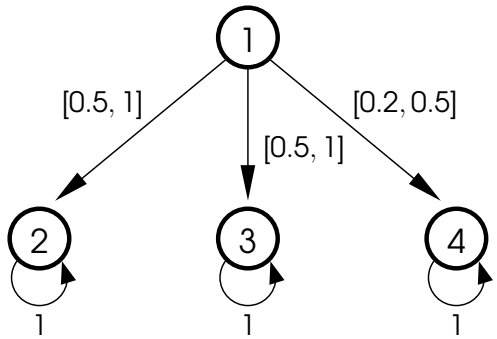

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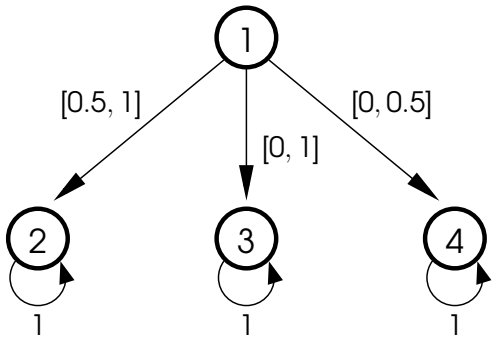

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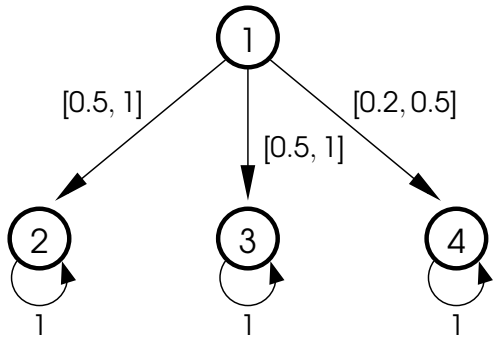

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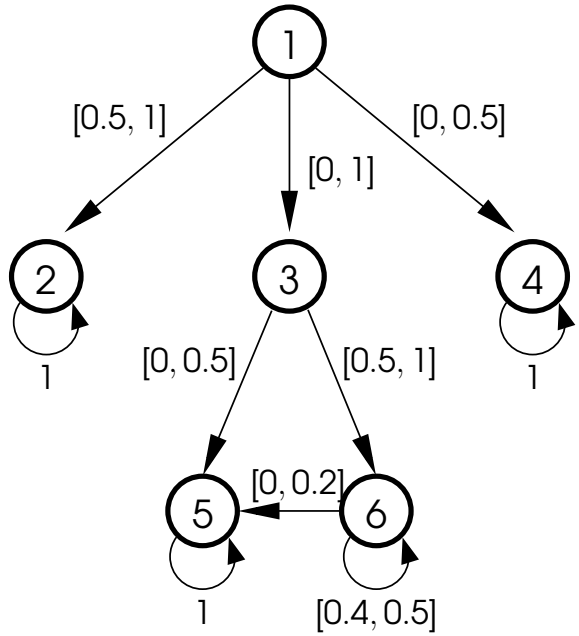
(if all intervals are **well-formed**)

Pruning operator for IMCs

Remove all inconsistent behaviors



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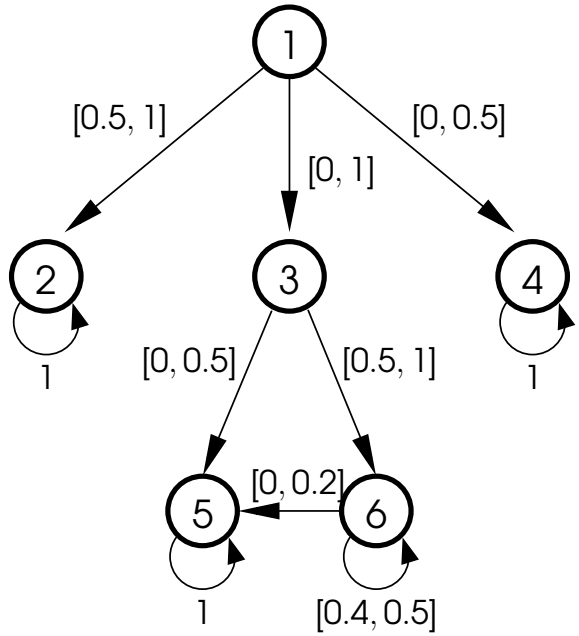

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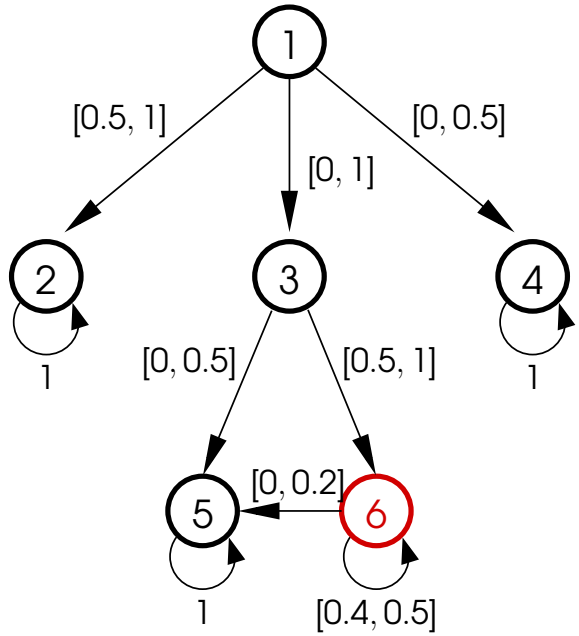

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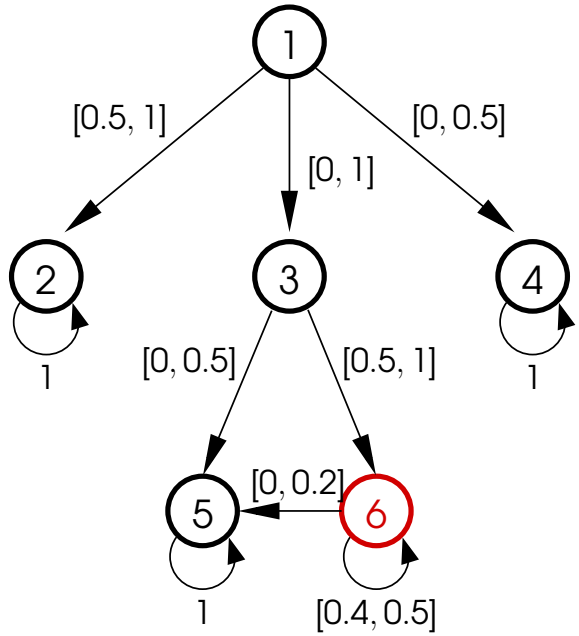

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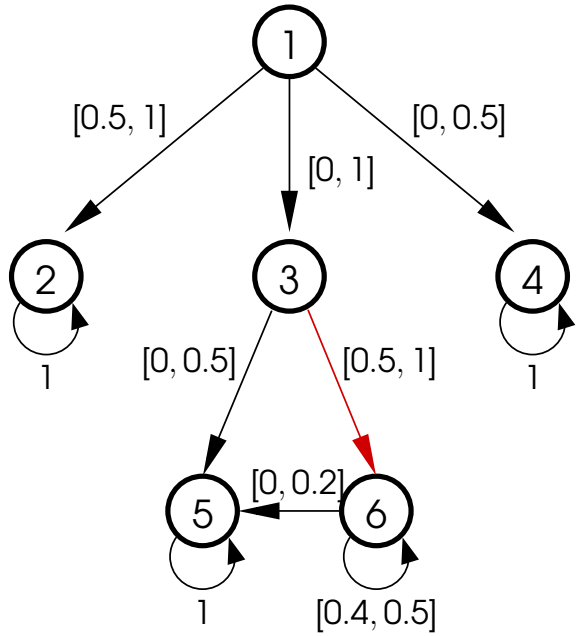

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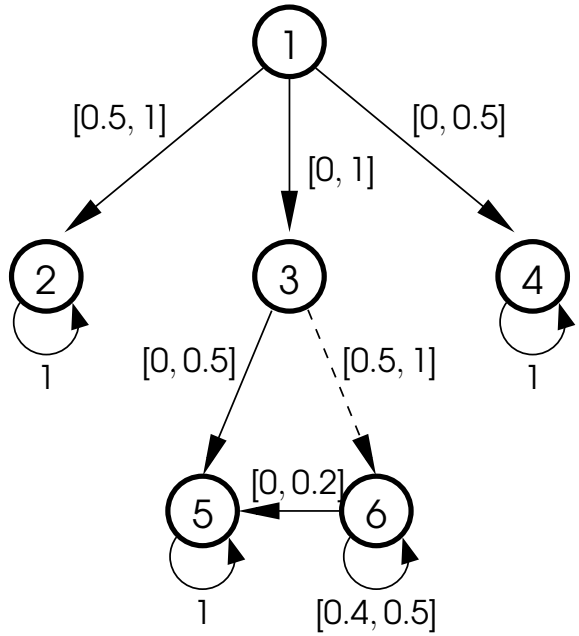

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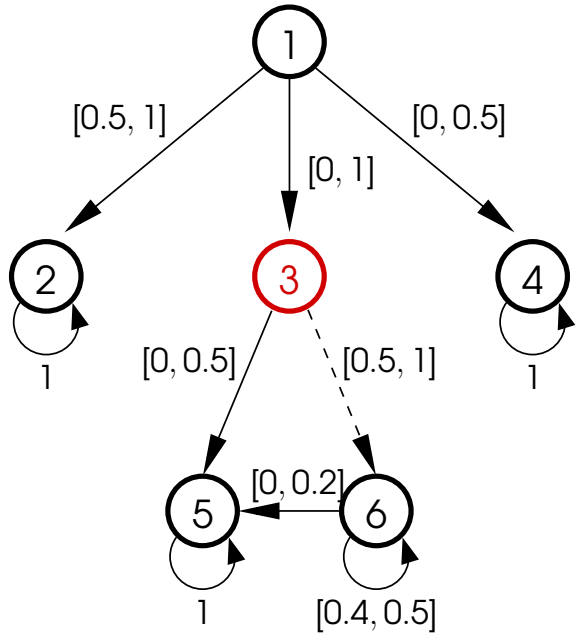

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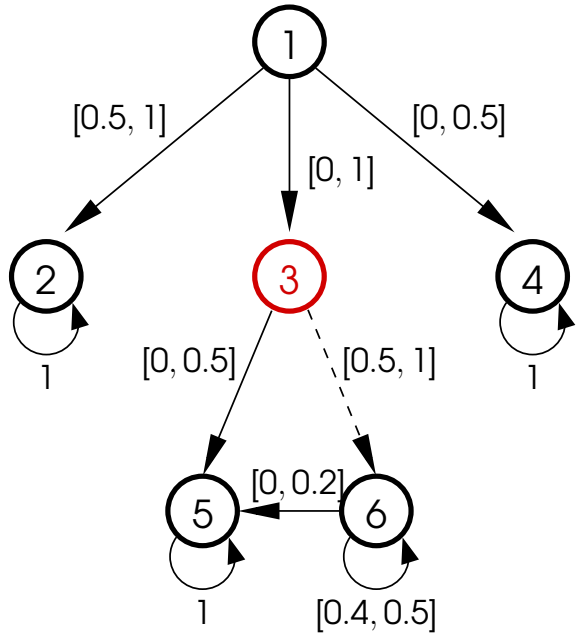

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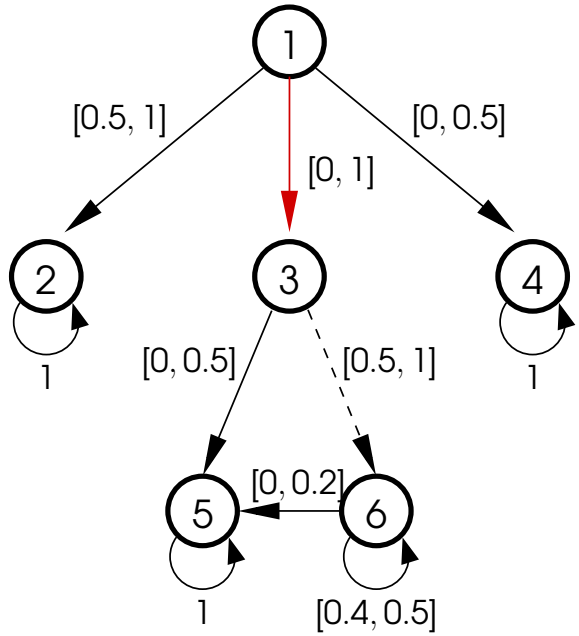

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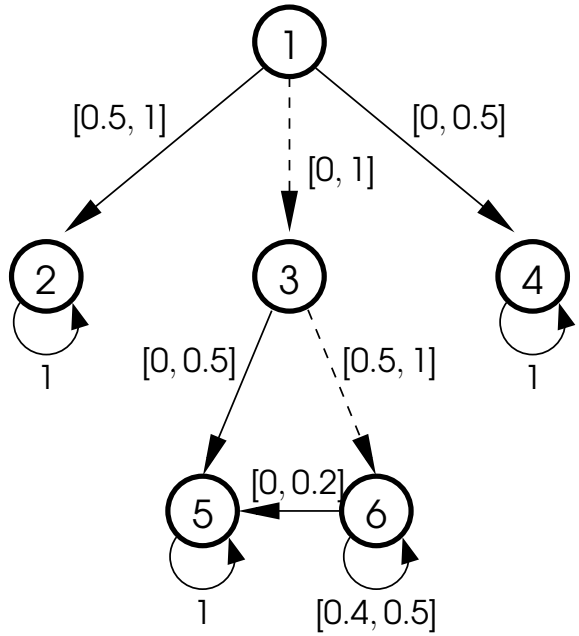

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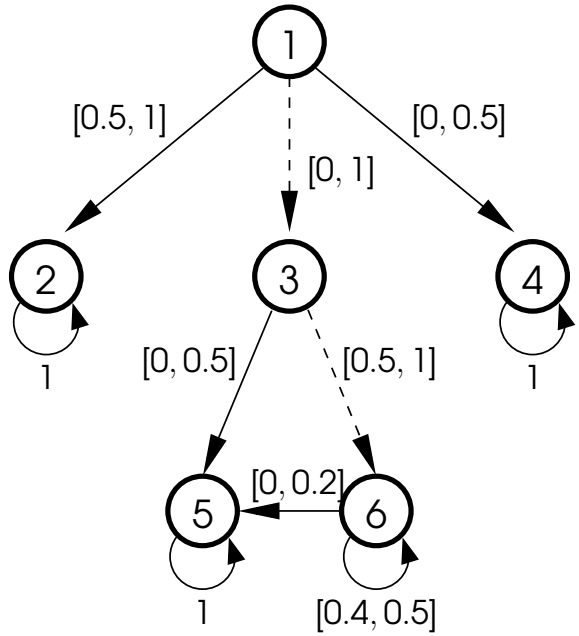

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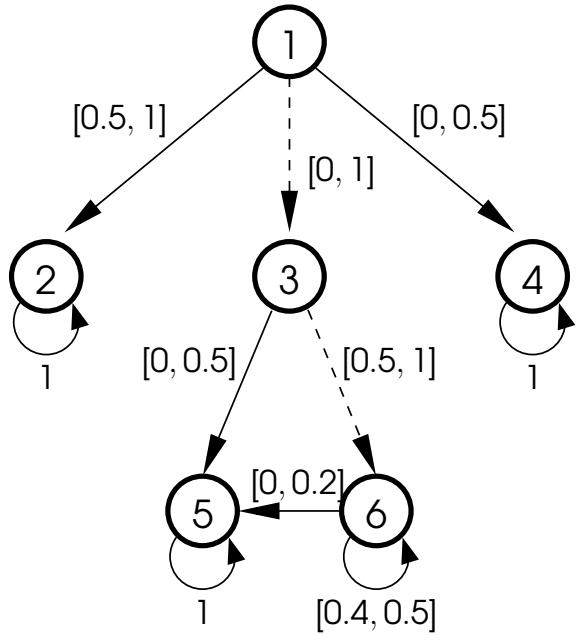

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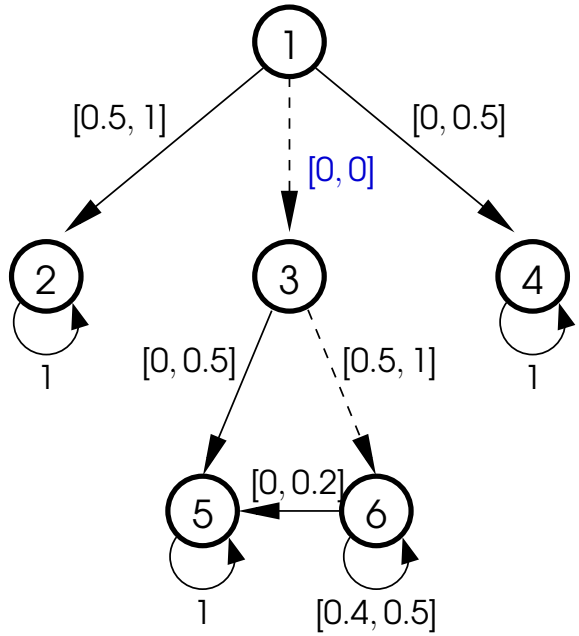

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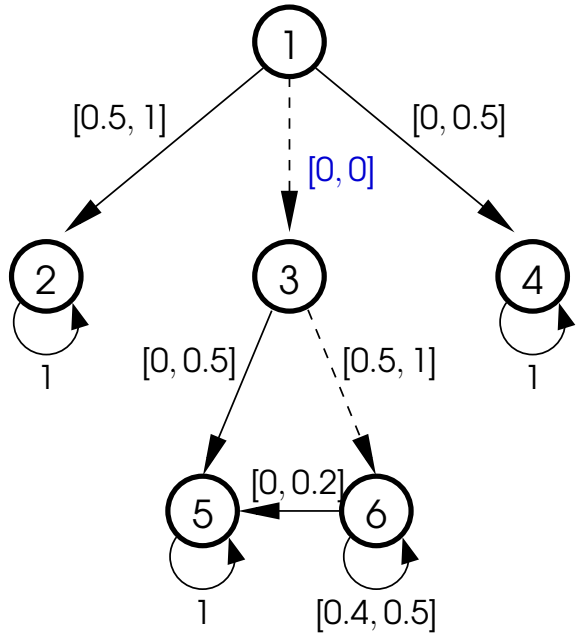

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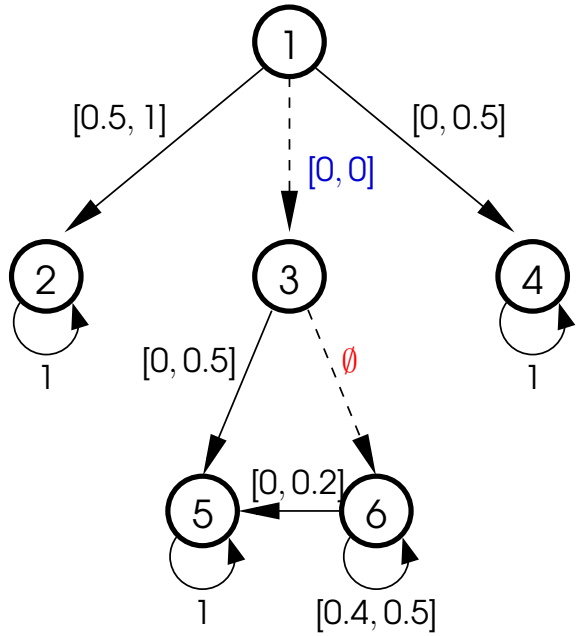

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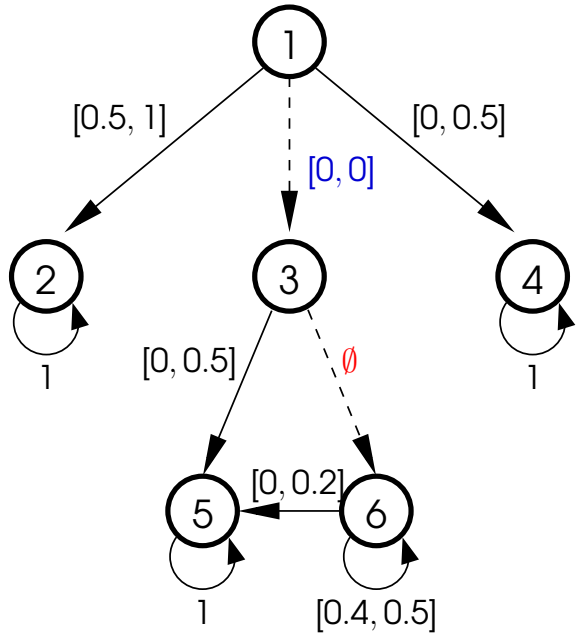

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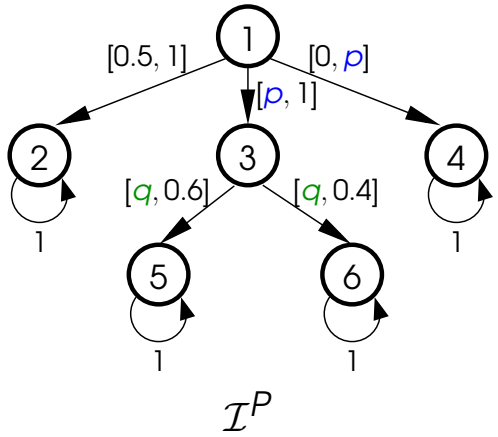
Consistency of pIMCs



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pIMC $\mathcal{I}^P$ is consistent iff there exists at least one IMC $\mathcal{I}$ and one MC $\mathcal{M}$ such that

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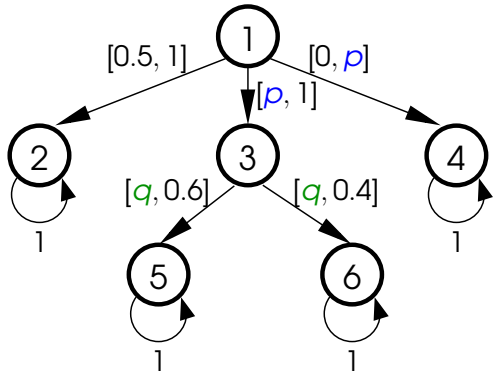

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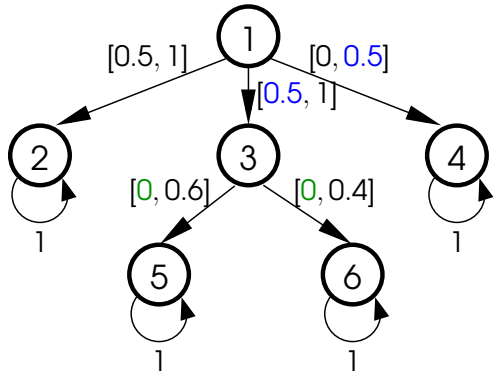

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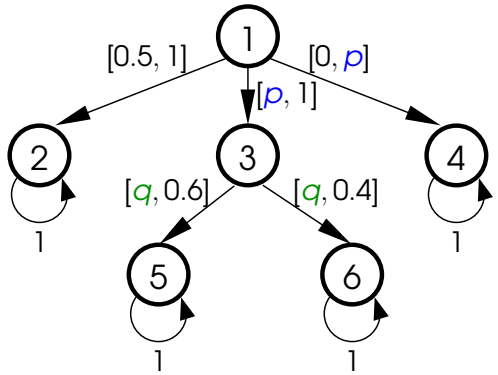

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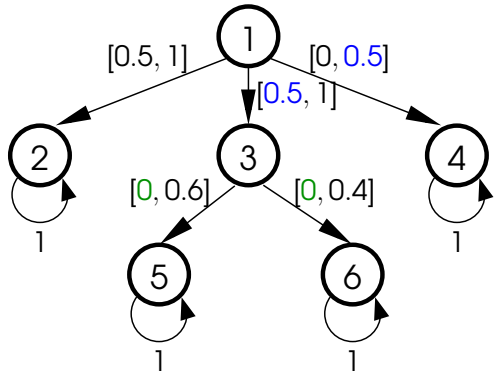

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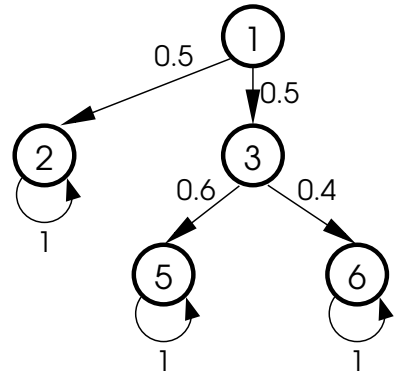

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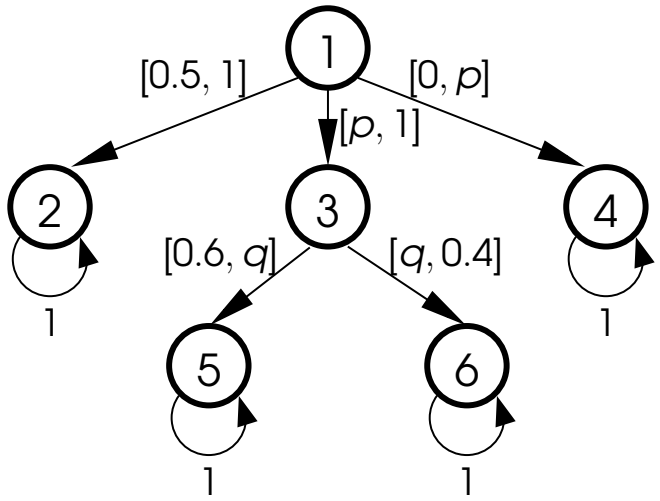

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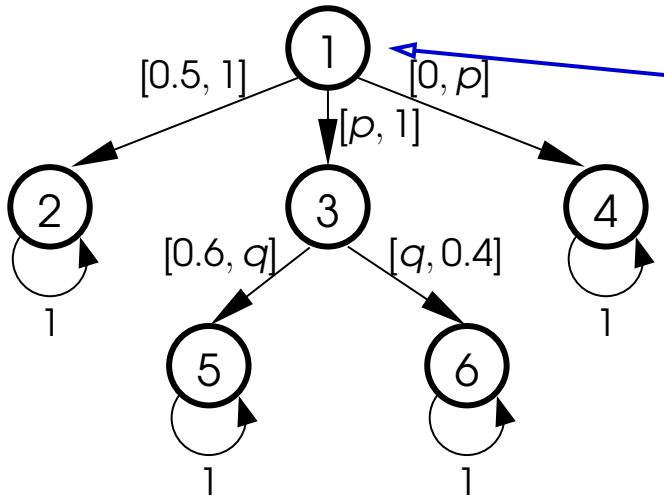

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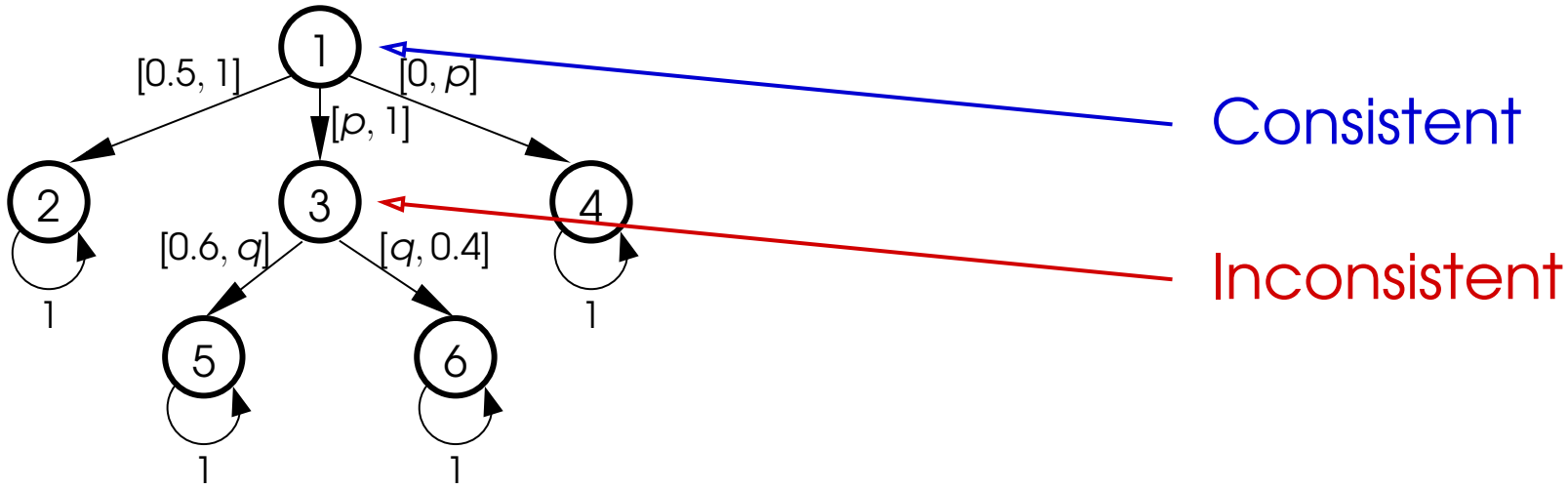

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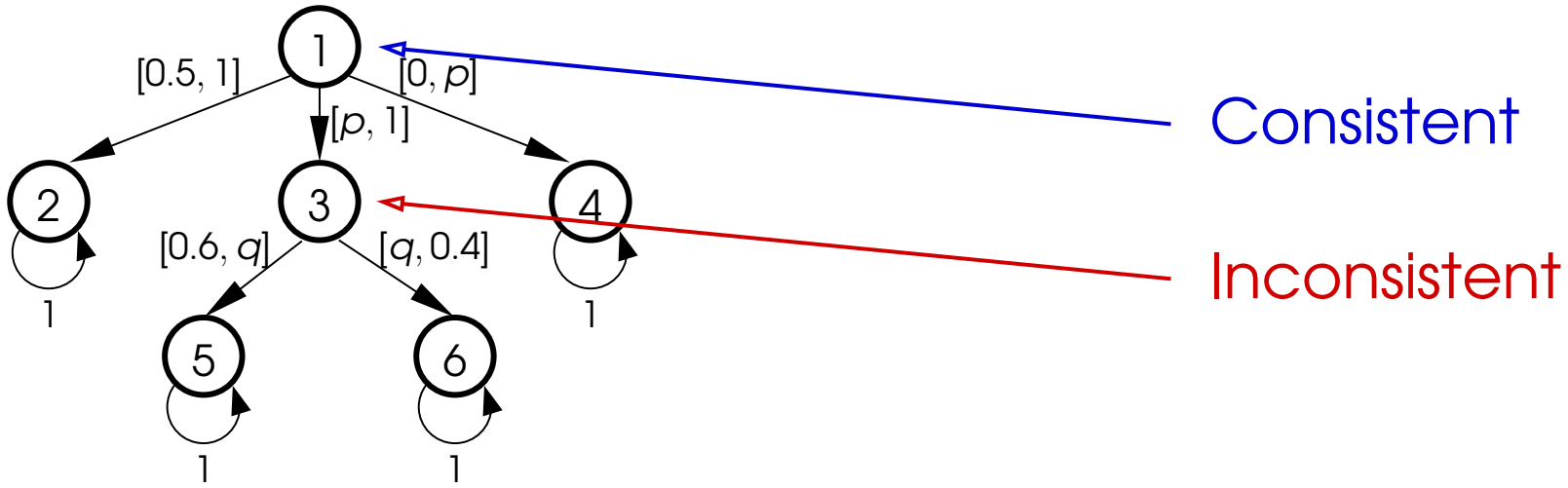

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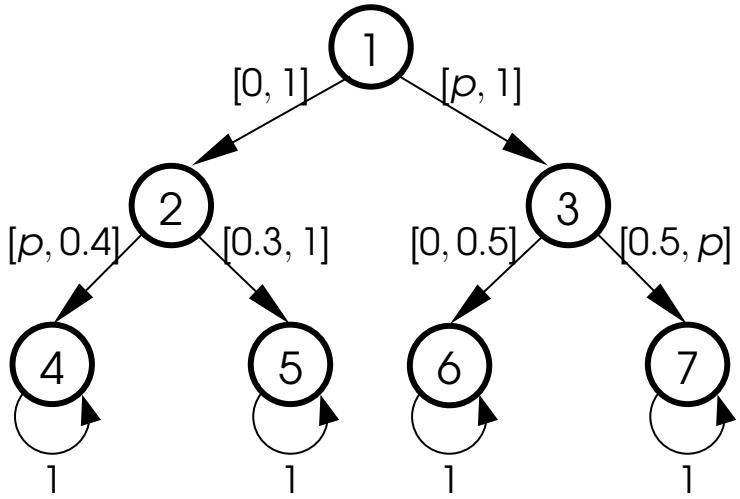

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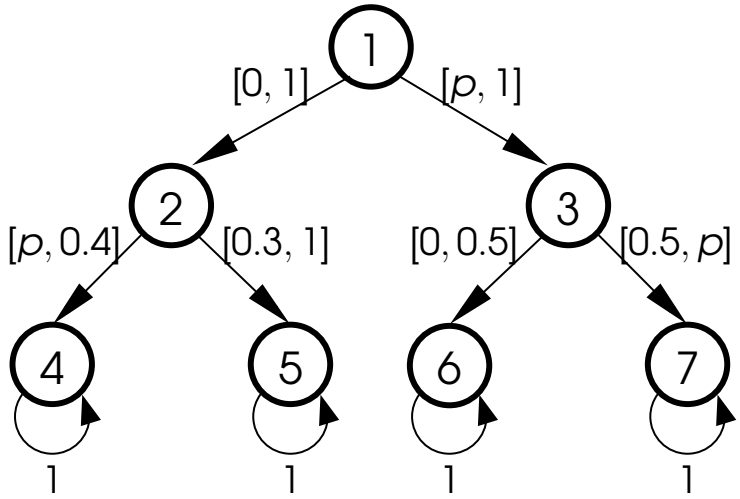

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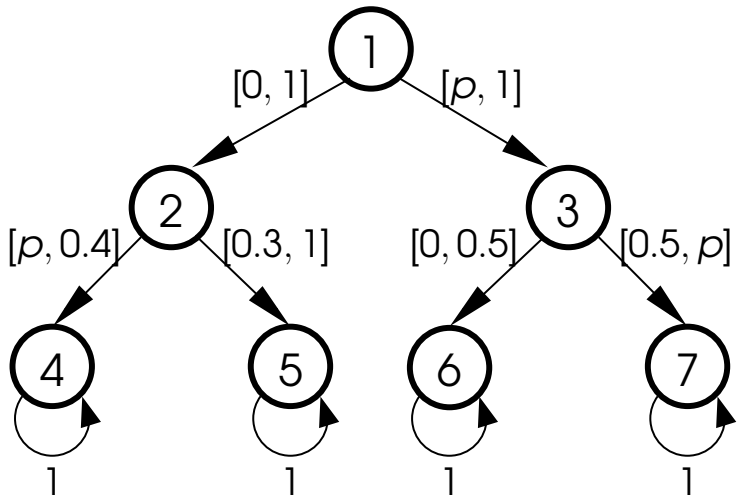

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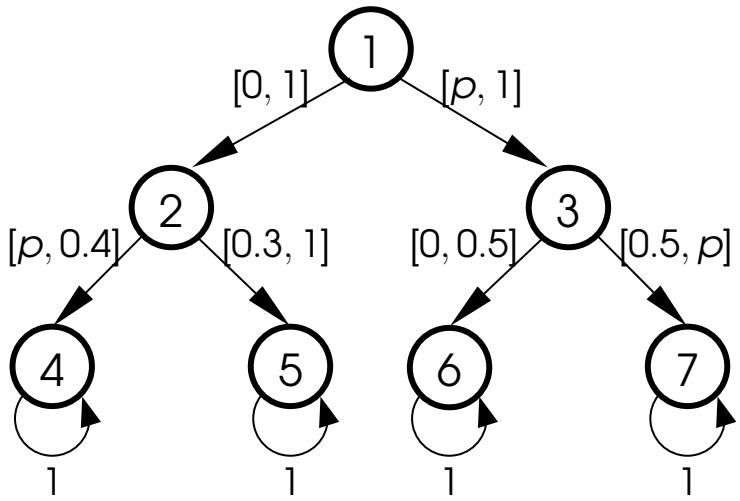

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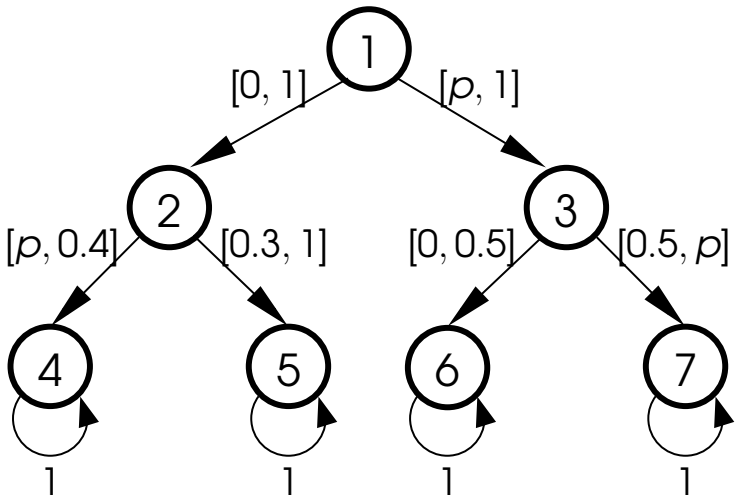

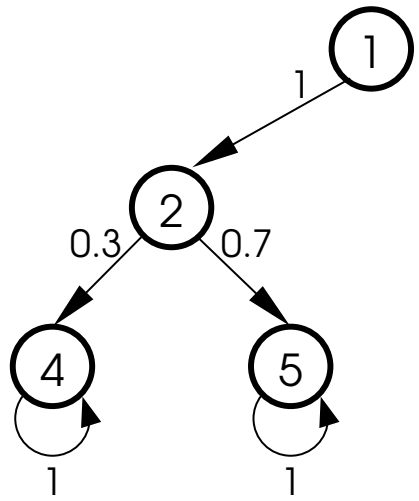

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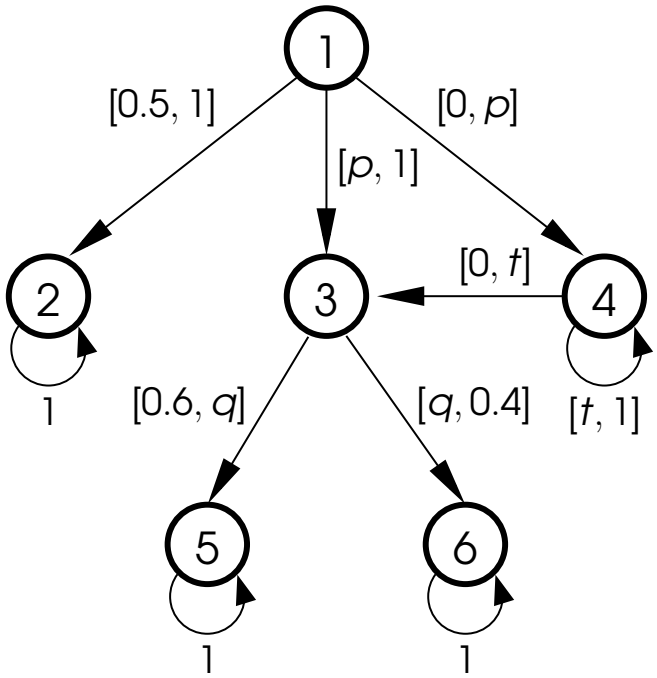
- Repeat until no more inconsistent states are reachable

How to make state s_i unreachable?

For all other states $s \neq s_i$, three cases:

- If $\varphi(s, s_i) = [0, .]$, then $\varphi'(s, s_i) = [0, 0]$
- If $\varphi(s, s_i) = [p, .]$, then $\varphi'(s, s_i) = [0, 0]$ **and $p = 0$ in all other intervals**
- If $\varphi(s, s_i) = [x, 0]$ with $x > 0$, then $\varphi'(s, s_i) = \emptyset$

Example



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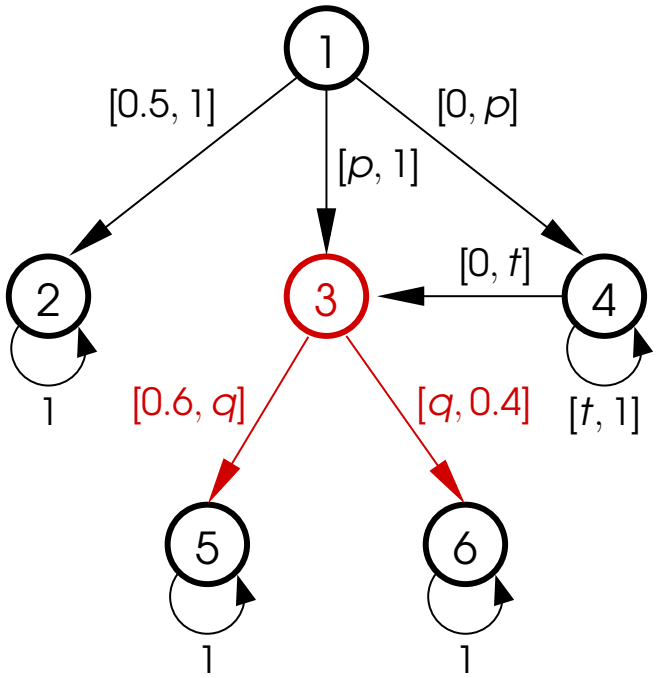

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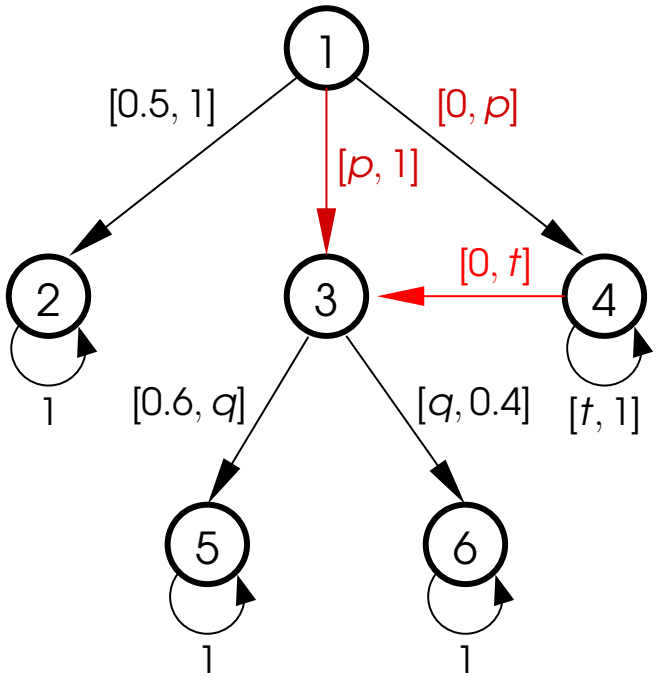

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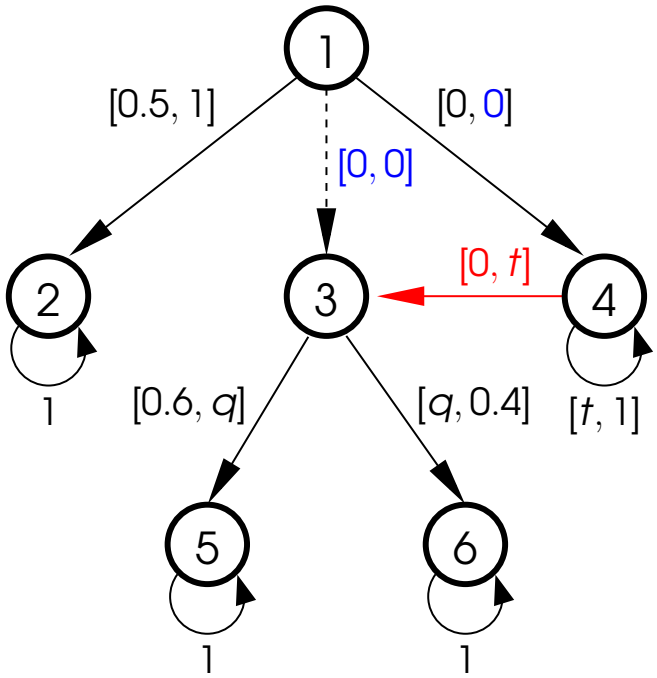

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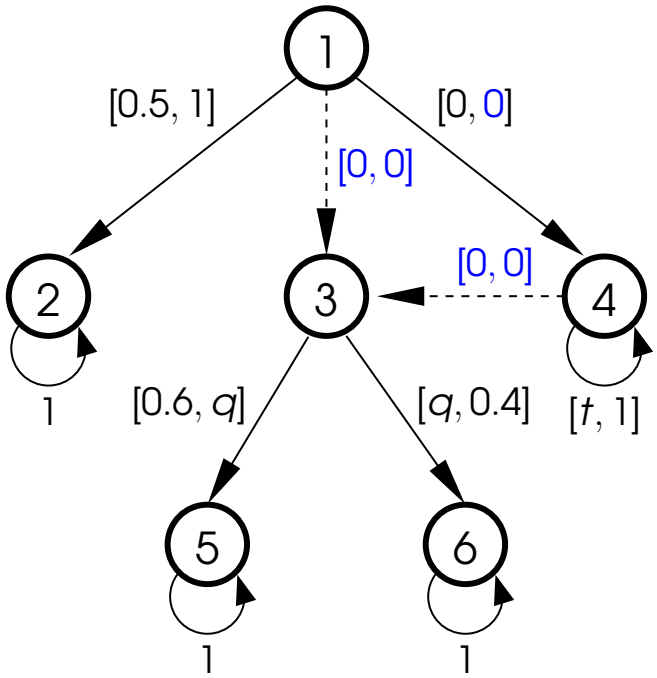

Consistency of global pIMCs - Hints

Brute force algorithm

Inter-consistency: Parameters are local to *groups of states* $\Rightarrow$ combine their systems of inequations.

Consistency of global plMCs - Hints

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- Identify inconsistent *groups of states*

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  - If consistency is proven $\Rightarrow$ ok
  - If not, backtrack and try another state

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- Make one state of the group unreachable and start over
  - If consistency is proven $\Rightarrow$ ok
  - If not, backtrack and try another state
- If all possible combinations have failed, the pIMC is inconsistent

Other problems of interest

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- Synthesize *all* parameter values ensuring consistency

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The end...

Thank you for your attention!

Questions?