Integer Parameter Synthesis for Timed Systems

First International Workshop on Synthesis of Continuous Parameters

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Grenoble, 6 April 2014

Outline

Introduction

Parametric Timed Automata

(Un)decidability

Symbolic Semi-algorithms

Computing the Integer Valuations

Time Petri Nets

Conclusion

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Parametric Verification

- Verification:
  - Does the system satisfy the property?
  - Yes / No ;
  - Witness / Counterexample.

Parametric Verification

- Verification:
  - Does the system satisfy the property?
  - Yes / No ;
  - Witness / Counterexample.
- **Parametric** Verification:
  - What are the values of the parameter such that the system satisfy the property?
  - Flexible and/or early design;
  - Symbolic proofs;
  - Robustness.

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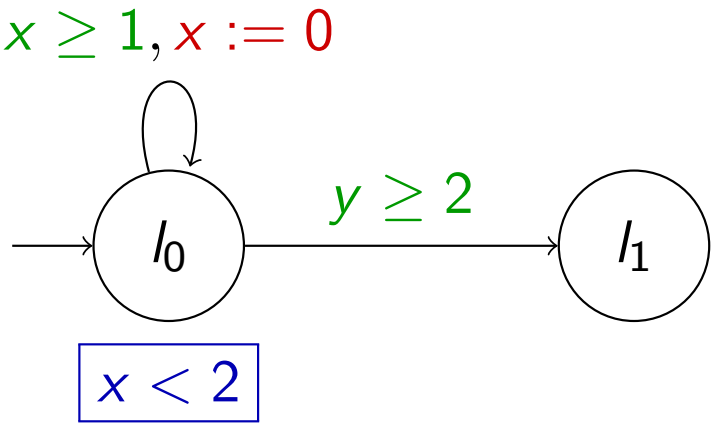
Timed Automata (TA)

- Finite automata with a finite set of clocks, guards, resets and invariants:

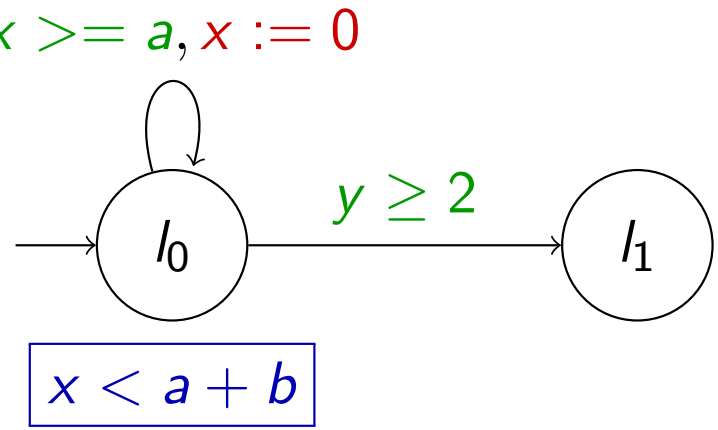


- A possible **run**:

$$(l_0,(0,0)) \xrightarrow{1} (l_0,(1,1)) \rightarrow (l_0,(0,1)) \xrightarrow{1.54} (l_0,(1.54,2.54)) \rightarrow (l_1,(1.54,2.54))$$

Parametric Timed Automata (PTA)

- Timed automata with clock constraint bounds as **linear expressions** on a finite set of parameters P:



- For any PTA $\mathcal{A}$ and any rational valuation of the parameters $v : P \to \mathbb{Q}$, "$v(\mathcal{A})$" is a **timed automaton**.

Parametric Problems

- Let Φ be some property class (reachability, unavoidability, etc.)
- Let $\mathcal{A}$ be a PTA and $\varphi \in \Phi$ be some property on $\mathcal{A}$
- Let $F_\varphi(\mathcal{A})$ be the set of valuations v s.t. $v(\mathcal{A}) \models \varphi$.

Φ-emptiness problem:

INPUTS: A PTA $\mathcal{A}$ and a property φ in Φ
PROBLEM: $F_\varphi(\mathcal{A}) = \emptyset$?

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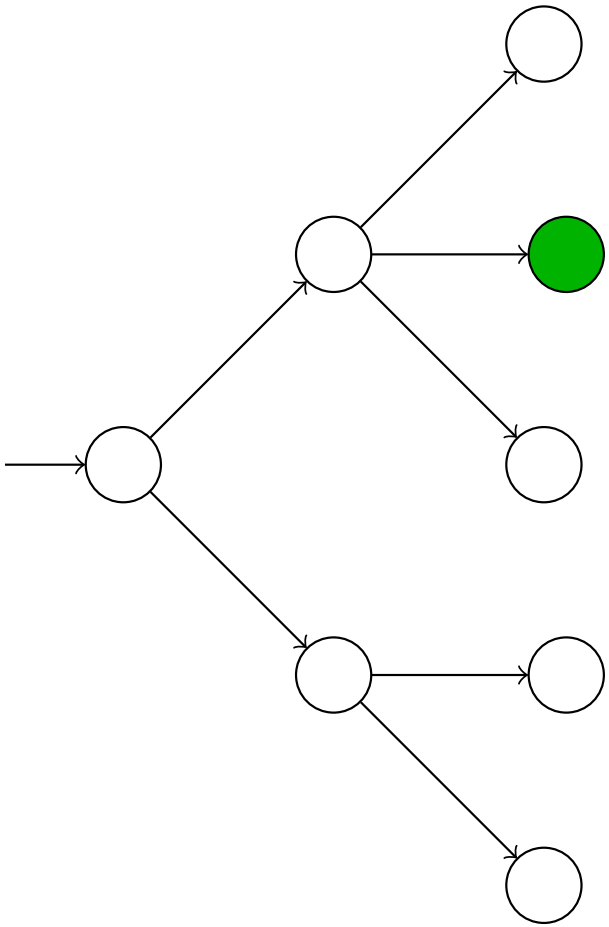
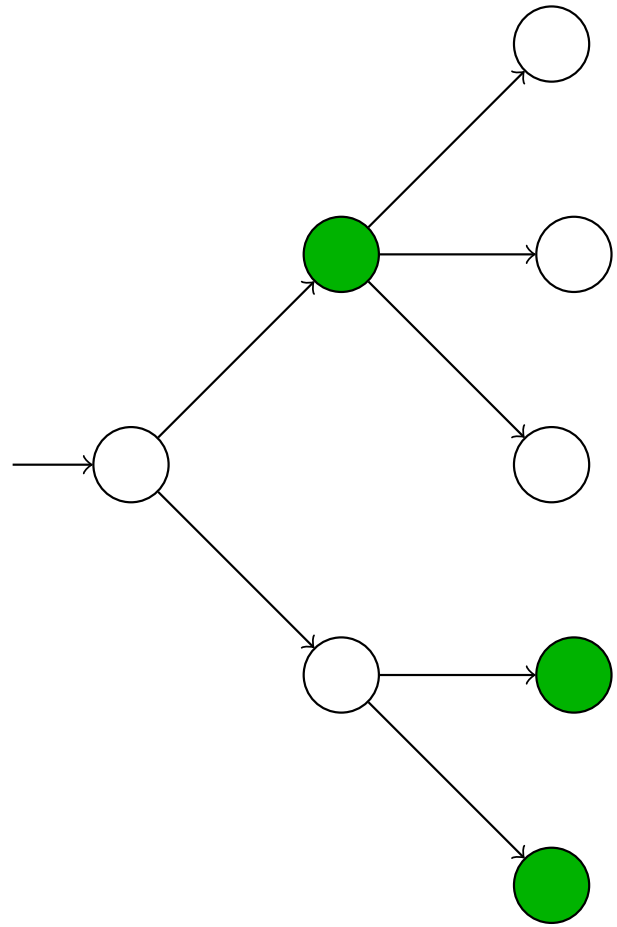
Φ-synthesis problem:

INPUTS: A PTA $\mathcal{A}$ and a property φ in Φ
PROBLEM: Compute $F_\varphi(\mathcal{A})$

Reachability and Unavoidability



Reachability (EF)



Unavoidability (AF)

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Parametric Verification is Hard!

The EF-emptiness problem for PTA is:

- in dense-time:
  - undecidable with one bounded parameter, one parametric clock, and three regular clocks [Mil00];
  - decidable with one clock [Mil00];
  - "robustly" undecidable [Doy07] (with only open constraints).
- in discrete-time [AHV93]:
  - undecidable with three parametric clocks;
  - decidable with one parametric clock.

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Theorem ([AHV93])

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- Reduction of the 2-counter machine halting problem;

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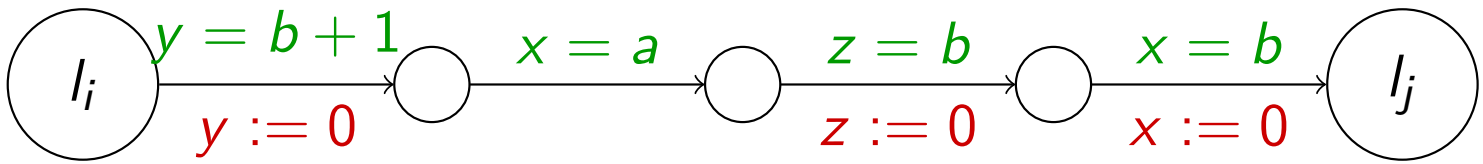
- Increment of C_1



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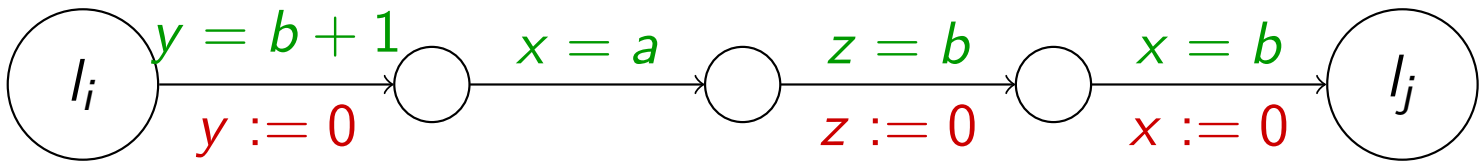

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- Zero-test of C_1: a guard $y = b$ between l_i and l_j

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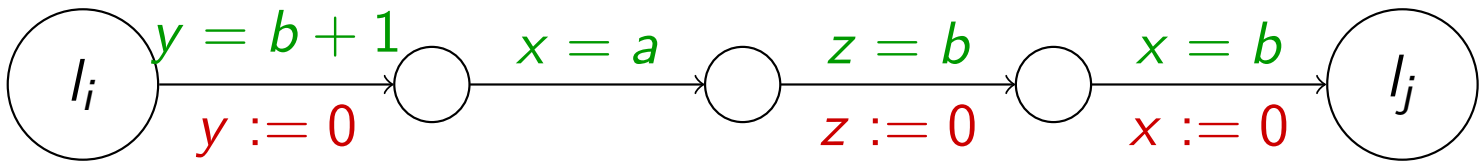

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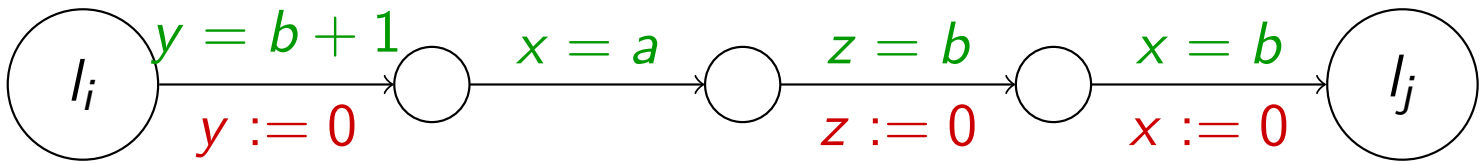

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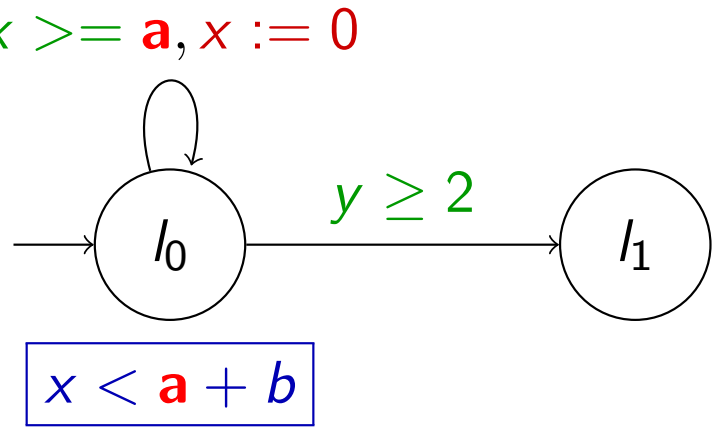
- If the machine does not halt, the PTA cannot reach l_{halt}: $F = \emptyset$;
- If the machine halts and c_1, c_2 are the max values of the counters then $F = \{a \geq c_1 \text{ and } b - a \geq c_2\} \neq \emptyset$.

L/U-automata [HRSV02]

- In L/U-automata, parameters are either **upper bounds** (e.g. $x \leq a$ or $x \geq 1 - a$) or **lower bounds** in the whole PTA, not both.

not LU



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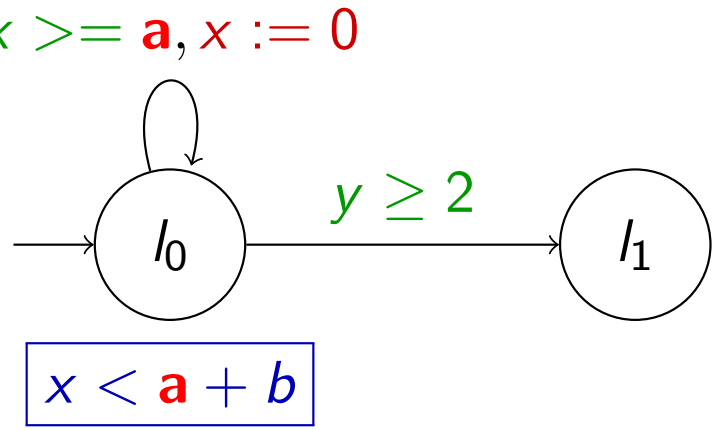

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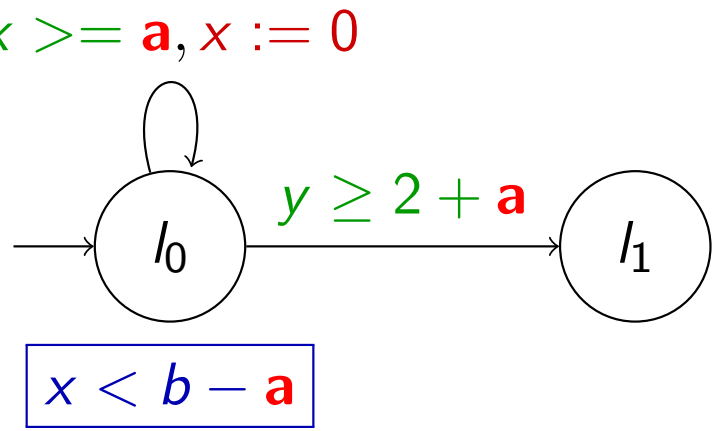

L/U-automata: Emptiness

- L/U automata have a **monotonicity** property:
  - decreasing lower bounds only **add** behaviors;
  - increasing upper bounds only **add** behaviors;
- They are "reachability-friendly": **all** the possible behaviors can be obtained by setting lower bounds to 0 and upper bounds to $+\infty$.

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Theorem ([BT09])

The "repeated reachability"-emptiness problem for L/U-automata is PSPACE-complete (also universality and finiteness).

L/U-automata: AF-Emptiness

- L/U automata are very well suited to **reachability** properties;
- But they fail for **unavoidability**:

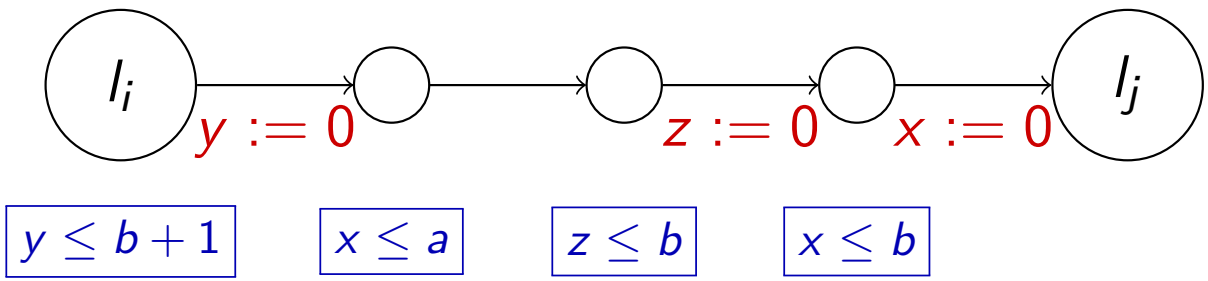
Theorem

*The AF-emptiness problem for **U**-automata is undecidable*

- We adapt the 2-counter machine reduction of [AHV93].

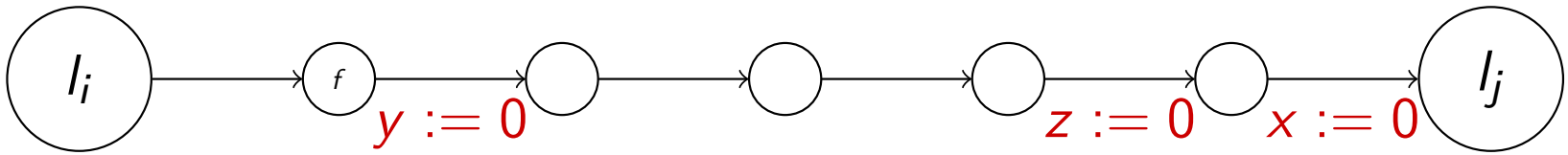
L/U-automata: AF-Emptiness

- All the runs go through the gadgets iff the **slowest run** ρ^* does:
  AF l_{halt} iff ρ^* reaches l_{halt}
- ρ^* simulates the machine;
- Increment C_1:



L/U-automata: AF-Emptiness

- Zero-testing C_1:



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The slowest run is **blocked** in f iff $c_1 \neq 0$

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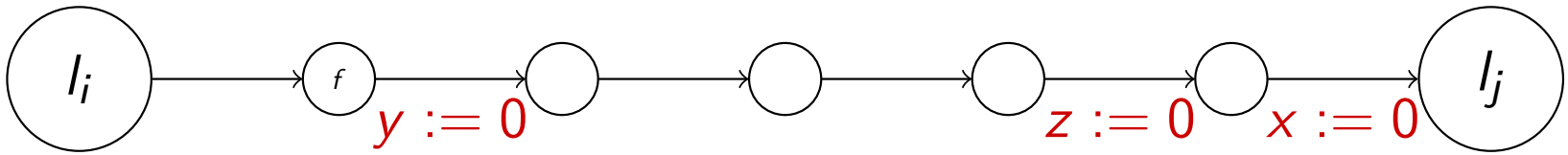

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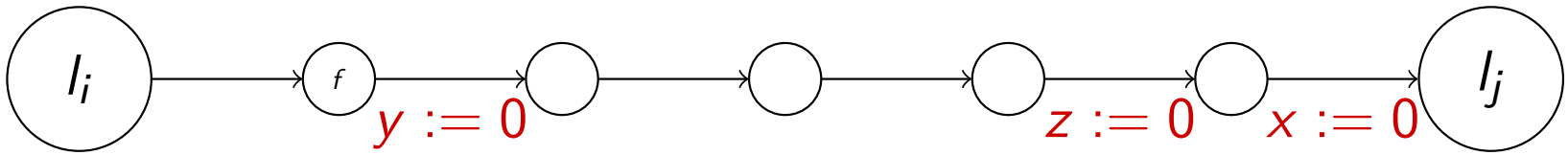

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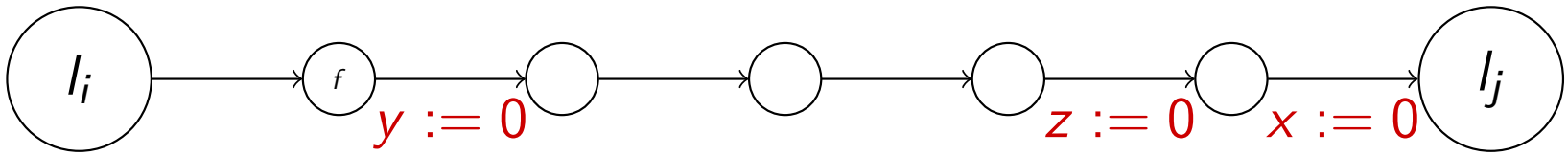

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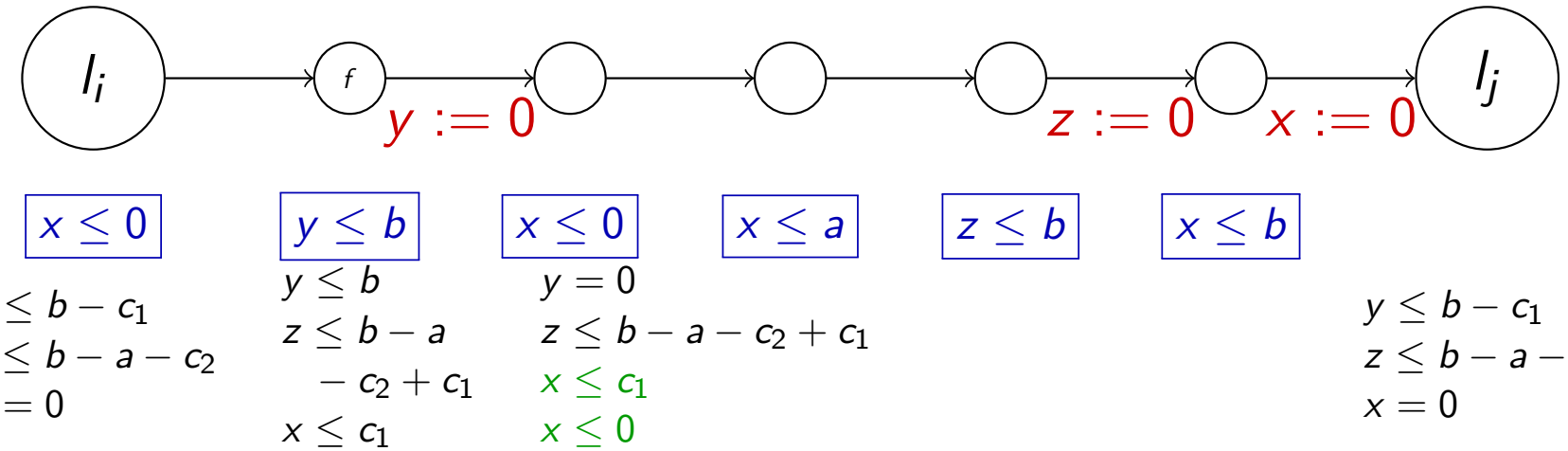

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  - otherwise the slowest run does not reach l_{halt} and again $F = \emptyset$.
- if the machine halts then no runs are blocked in f, and as before $F \neq \emptyset$.

L/U-automata: Synthesis

Theorem ([BT09])

The "repeated reachability"-synthesis problem can be explicitly solved for L- and U-automata (giving finite unions of convex polyhedra).

- In a PTA, if p_i is both a lower bound and an upper bound:
  - Replace p by a fresh parameter p_i^u when p is an upper-bound;
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Theorem ([BT09])

Constrained "repeated reachability"-emptiness is undecidable for L/U-automata (decidable if upper bounds are not compared to lower bounds in the initial constraints)

L/U-automata: Synthesis

- We can deduce something else from the previous reasoning:

Theorem

For L/U automata, the solution to the EF-synthesis problem cannot be represented using any formalism for which emptiness of the intersection with equality constraints is decidable.

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- These results can be trivially **extended** to "simple" L/U automata in which:
  - parametric constraints in the automaton are reduced to only one parameter;
  - each parameter is used only once in the whole automaton.

Bounded Integer Parametric Problems

- L/U automata:
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- **Syntactical** restriction of the constraints is not the only possibility;
- If we look for parameter values as **bounded integers** "everything" is decidable and computable!
- A finite number of possible values is somewhat trivial but:
  - EF-emptiness for **bounded rational** values is undecidable [Mil00];
  - EF-emptiness for possibly **unbounded integer** values is undecidable: in the proof of [AHV93], $b = c_1 + c_2$ and $a = c_1$ are in F when the machine halts, with c_1, c_2 max values of the counters on the run.

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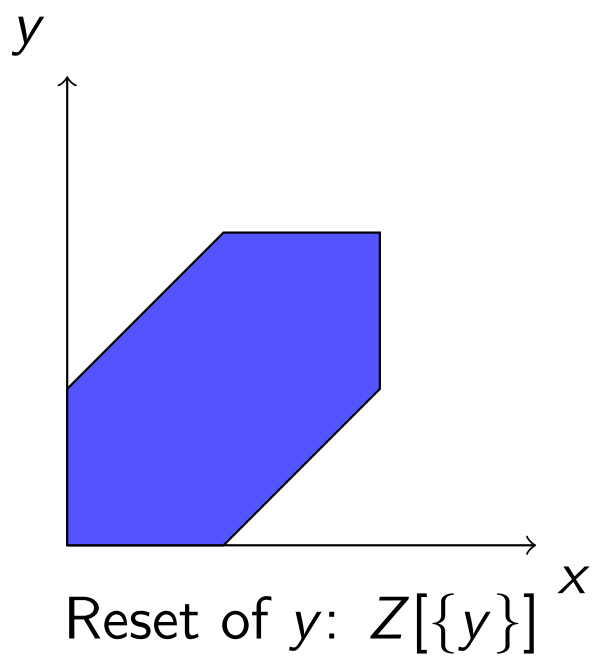

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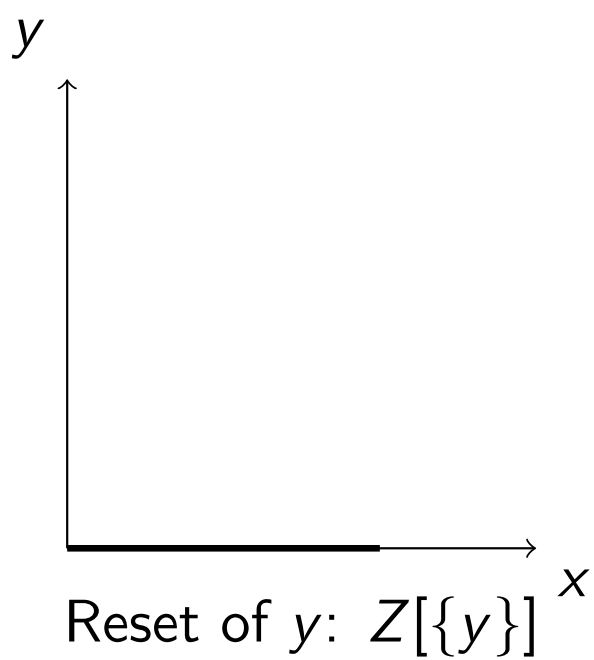

Symbolic States for PTA

- We do **not** want to **enumerate** all possible values of the parameters;
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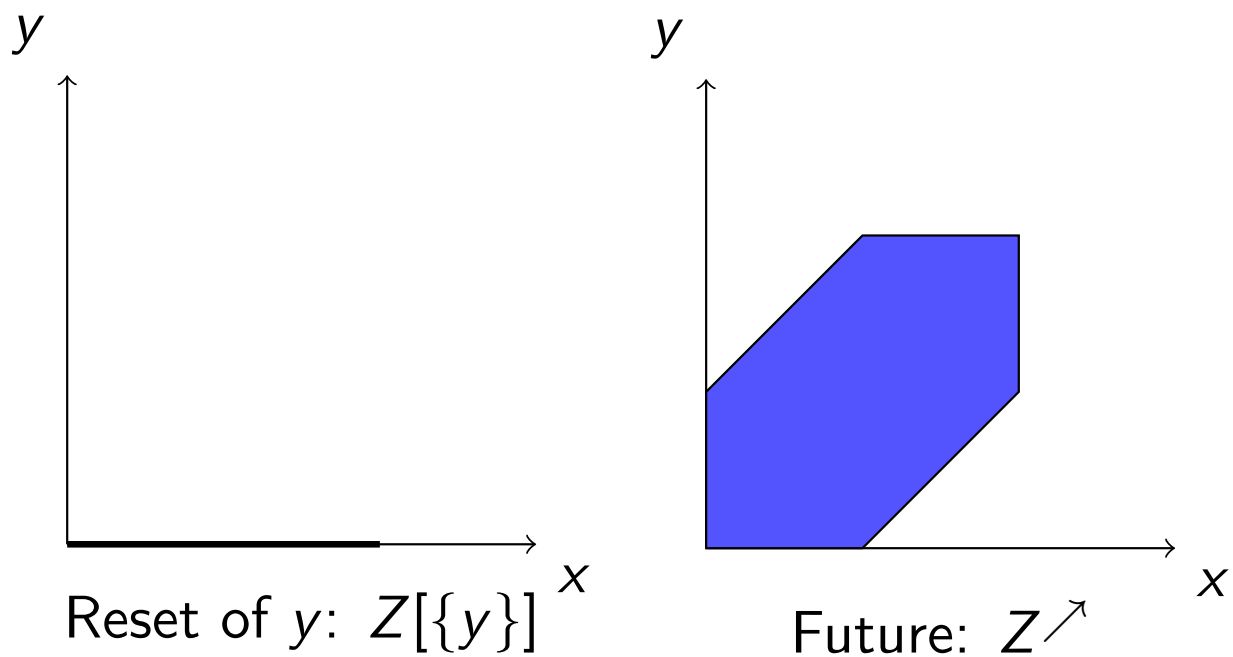

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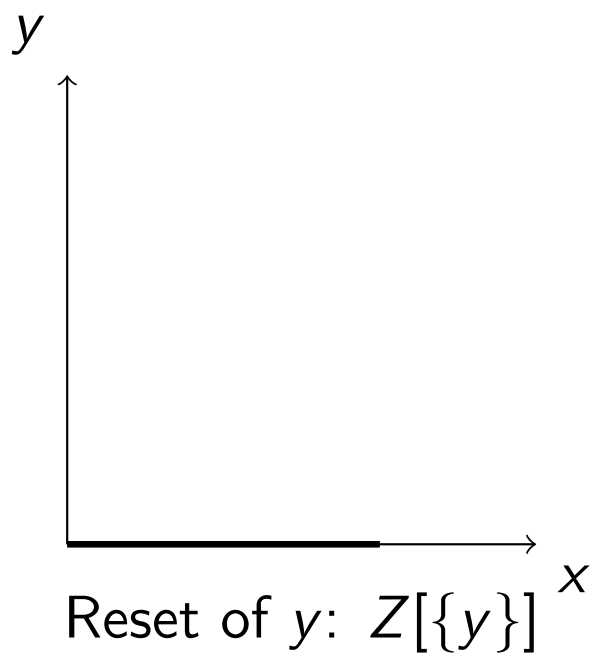
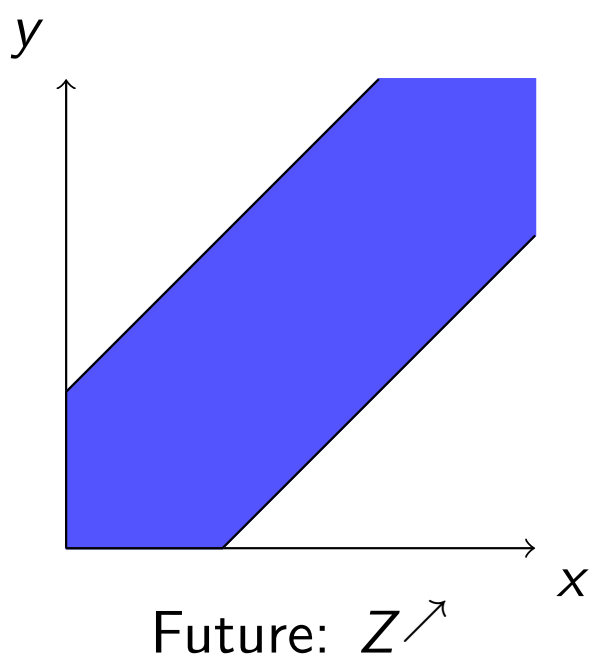


Reset of y: $Z[\{y\}]$

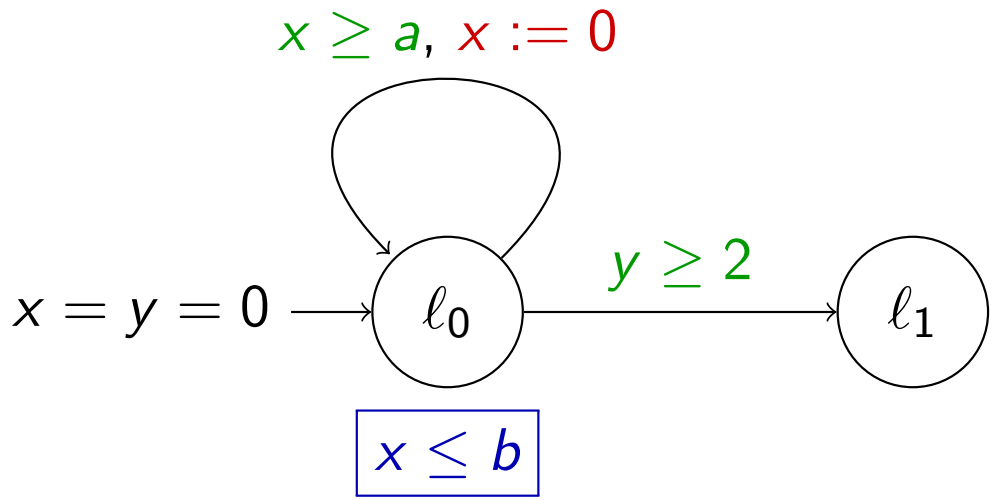


Future: $Z^{\nearrow}$

Symbolic States for PTA

- **initial** symbolic state of the PTA $\mathcal{A}$:
  $\mathsf{Init}(\mathcal{A}) = (l_0, \{v \in \mathbb{R}^{X \cup P} \mid v_{|X} \in \{\vec{0}_X\}^{\nearrow} \cap v_{|P}(\mathsf{Inv})\})$;
- **successor** by an edge $e = (l, a, \gamma, R, l')$:
  $\mathsf{Succ}((l, Z), e) = (l', (Z \cap \gamma)[R]^{\nearrow} \cap \mathsf{Inv}(l'))$

Example



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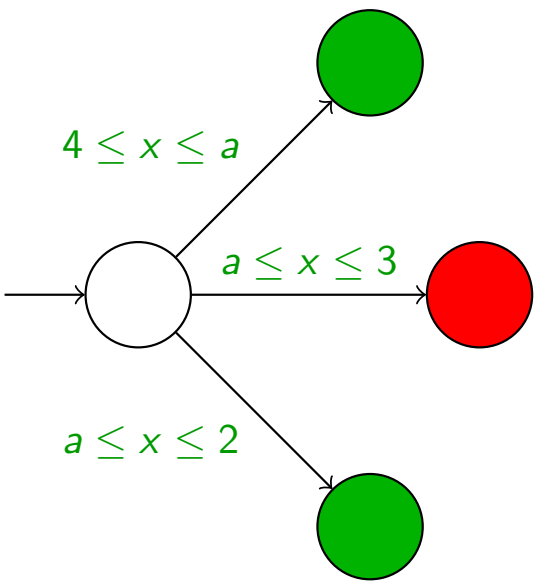
$$Z_n = \{0 \leq x \leq b, 0 \leq y, a \leq b, 0 \leq na \leq y - x \leq (n+1)b\}$$

EF semi-algorithm

- $S_{|P}$: with $S = (l, Z)$, projection of Z on parameters;
- M: passed list.

$$\mathrm{EF}_G(S, M) = \begin{cases} Z_{|P} & \text{if } S = (l, Z) \text{ and } l \in G \\ \emptyset & \text{if } S \in M \\ \bigcup_{\substack{e \in E \\ S' = \mathrm{Succ}(S,e)}} \mathrm{EF}_G\left(S', M \cup \{S\}\right) & \text{otherwise.} \end{cases}$$



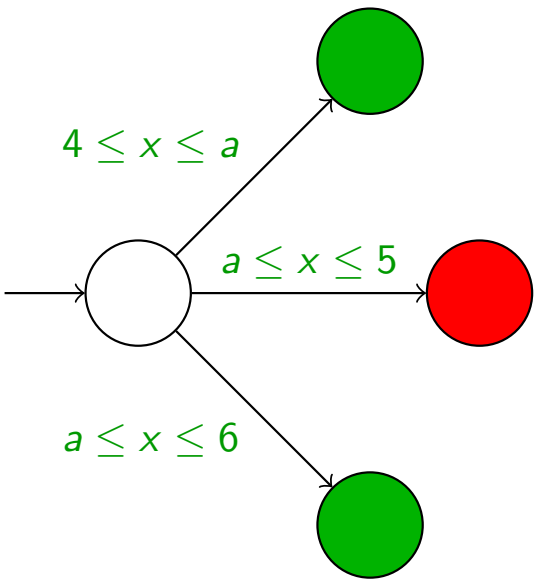
Collect the "good" parameter values:
$a \leq 2$ or $a \geq 4$

AF semi-algorithm

$$\mathsf{AF}_G(S,M) = \begin{cases} Z_{|P} & \text{if } S=(l,Z) \text{ and } l \in G \\ \emptyset & \text{if } S \in M \text{ or } S \text{ has no succ.} \\ \bigcap_{\substack{e\in E \\ S'=\mathsf{Succ}(S,e)}} \mathsf{AF}_G\big(S', M\cup\{S\}\big) \cup (\mathbb{R}^{\mathbf{P}} \setminus \mathbf{S'_{|P}}) \setminus \mathsf{dead}(S)_{|P} & \text{otherwise.} \end{cases}$$

with $\mathsf{dead}(S) = \left(\mathbb{R}^{X\cup P} \setminus \left(\bigcup_{(l,a,g,R,l')\in E} (g\cap S)^{\swarrow}\right)\right)$

States from which no outgoing transition can be taken anymore



Collect the "good" parameter values:

$a \ge 4$ and $a \le 6$

And remove the "bad" ones:

and $a > 5$

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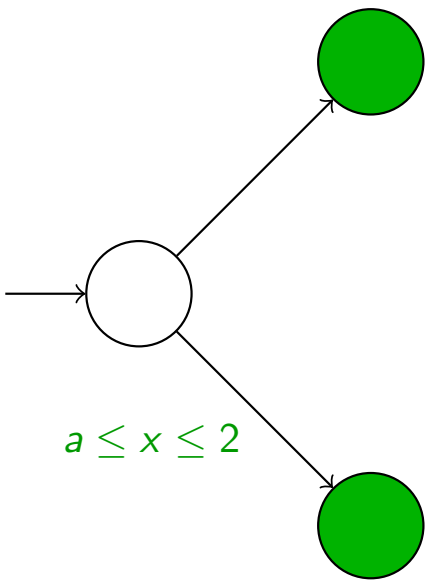

Cutting a "good" branch may be needed:

$a \geq 0$

AF semi-algorithm

$$\mathrm{AF}_G(S,M) = \begin{cases} Z_{\downarrow P} & \text{if } S=(l,Z) \text{ and } l \in G \\ \emptyset & \text{if } S \in M \text{ or } S \text{ has no succ.} \\ \bigcap_{\substack{e\in E \\ S'=\mathrm{Succ}(S,e)}} \mathrm{AF}_G\big(S', M\cup\{S\}\big) \cup (\mathbb{R}^{\mathbf{P}} \setminus \mathbf{S'_{\downarrow P}}) \setminus \mathrm{dead}(S)_{\downarrow P} & \text{otherwise.} \end{cases}$$

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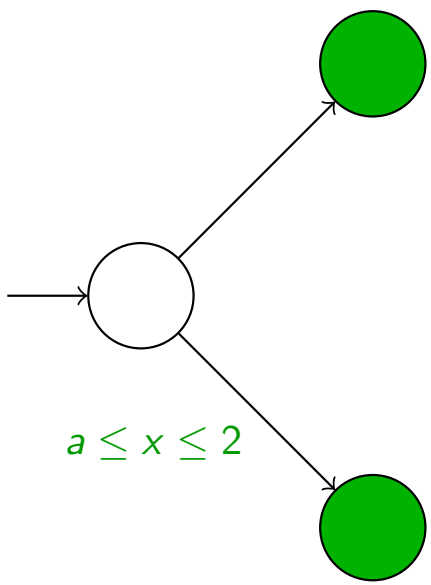

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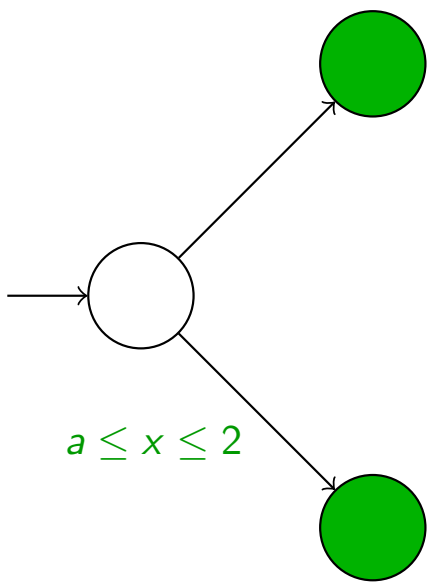

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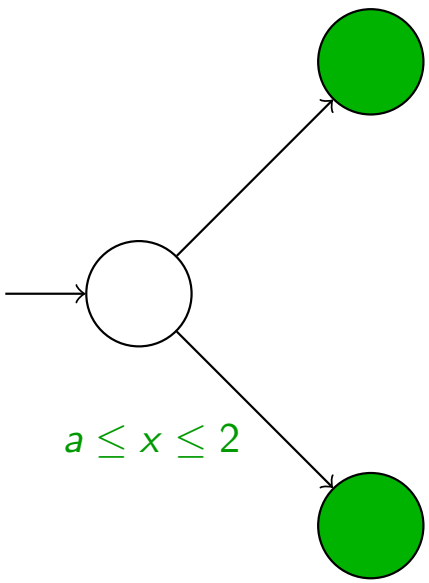

Cutting a "good" branch may be needed:

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EF and AF semi-algorithms

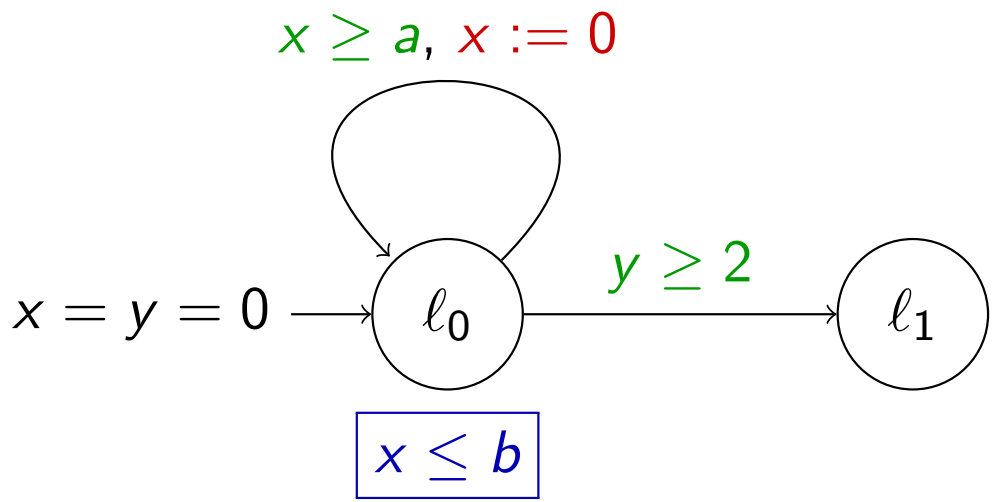
Theorem

For any PTA $\mathcal{A}$ and any subset of its locations G, upon termination, $\mathsf{EF}(\mathsf{Init}(\mathcal{A}), G, \emptyset)$ *is the solution the EF-synthesis problem for PTA $\mathcal{A}$ and set of locations to reach G.*

idem for AF

Termination is **not** guaranteed

Example



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$$Z_n = \{0 \leq x \leq b, 0 \leq y, a \leq b, 0 \leq na \leq y - x \leq (n+1)b\}$$

- $\forall n \neq m, Z_m \neq Z_n$ so the computations do not terminate.

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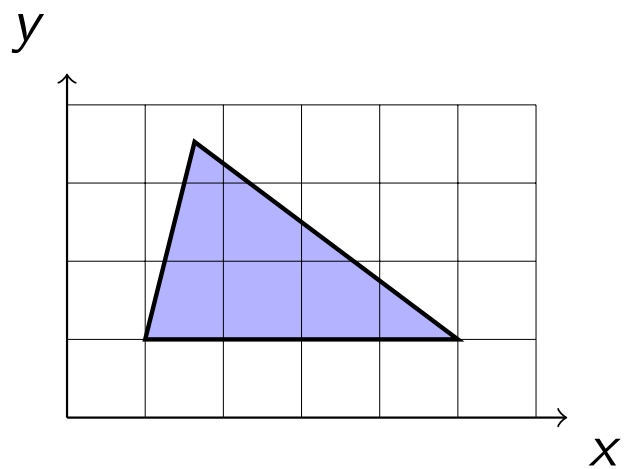
Computing the Integer Valuations: Integer Hulls

- We compute **integer hulls**: $\mathrm{IH}(Z)$



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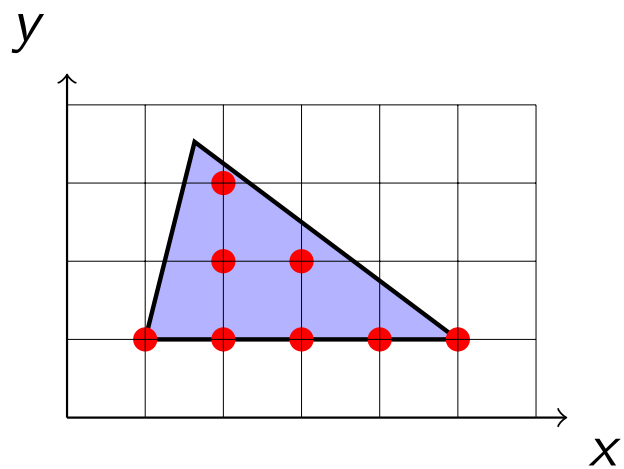

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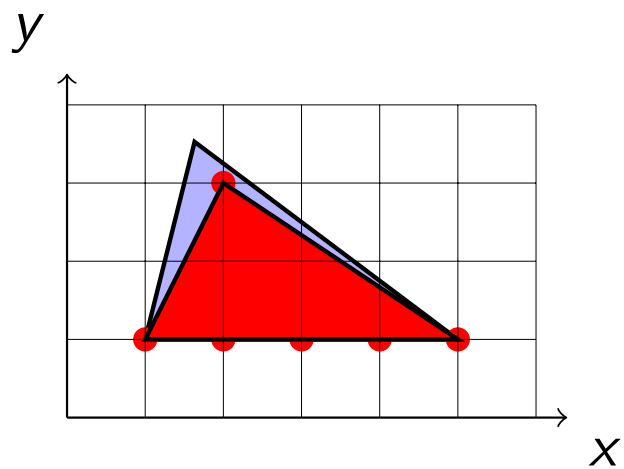

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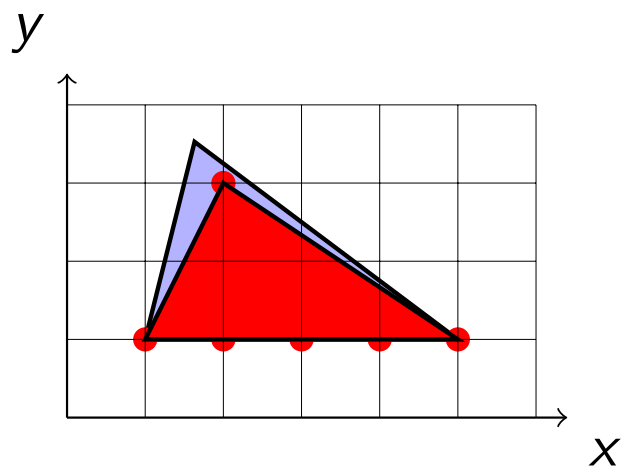

Lemma

For any symbolic state (l, Z) and any edge e:

$$\mathrm{IH}(\mathrm{Succ}(\mathrm{IH}((l, Z)), e)) = \mathrm{IH}(\mathrm{Succ}((l, Z), e))$$

Computing the Integer Valuations: Integer Hulls

- We note $\mathsf{ISucc}((l, Z), e) = \mathsf{IH}(\mathsf{Succ}((l, Z), e))$ and have:

Lemma

For a reachable symbolic states S, edge e, and **integer** *valuation v:*

$$v(\mathsf{ISucc}(S, e)) = \mathsf{Succ}(v(S), v(e))$$

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- We use ISucc instead of Succ in semi-algortihm EF to compute the **integer valuations**, giving semi-algorithm IEF.

Theorem

For any PTA $\mathcal{A}$ and any subset of its locations G, upon termination, $\mathsf{IEF}(\mathsf{Init}(\mathcal{A}), G, \emptyset)$ *is the solution the integer EF-synthesis problem for PTA $\mathcal{A}$ and set of locations to reach G.*

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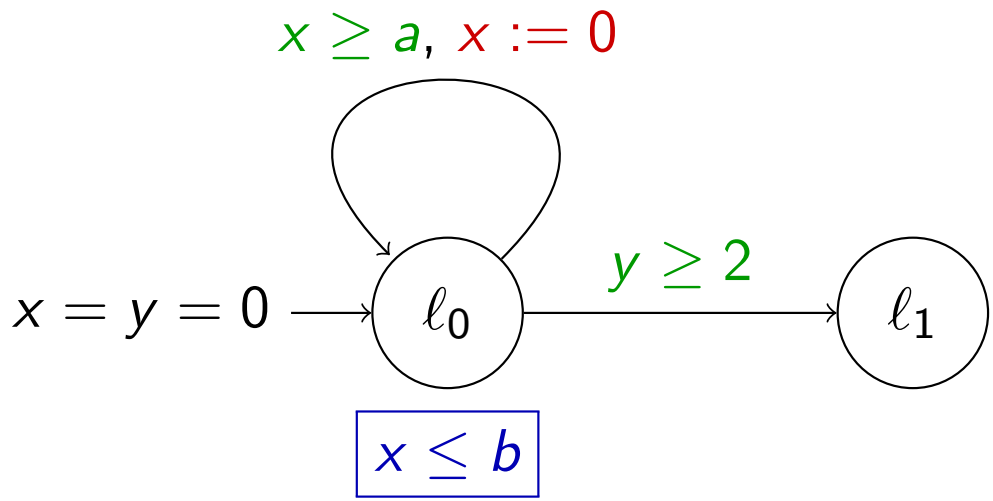
Example



- Initial symbolic state: (ℓ_0, Z_0) with $Z_0 = \{x = y, x \leq b\}$;
- After n loops: (ℓ_0, Z_n) with **still**

$$Z_n = \{0 \leq x \leq b, 0 \leq y, a \leq b, 0 \leq na \leq y - x \leq (n+1)b\}$$

- Actually, $Z_n = \mathsf{IH}(Z_n)$

Termination in the Bounded Case

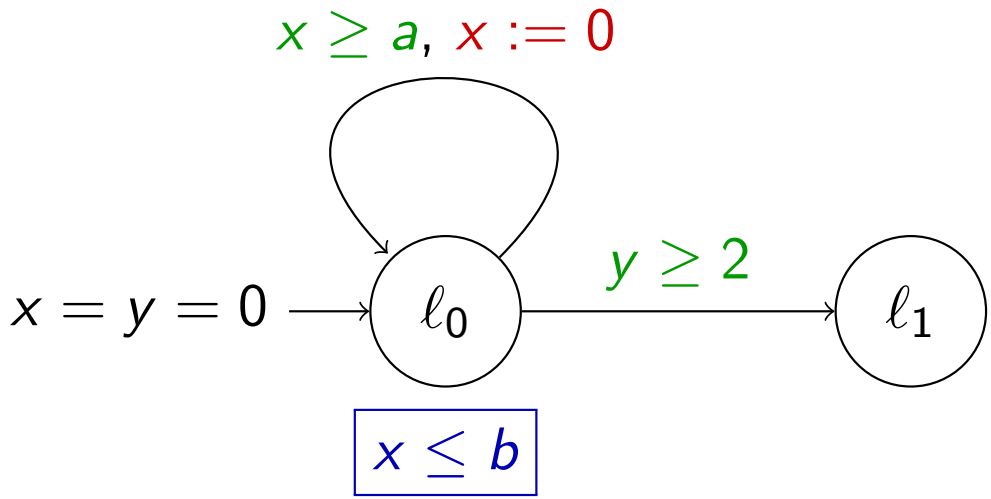
- When the possible parameter values are bounded, we can assume w.l.o.g. that all clocks are **bounded**;
- This ensures **termination** with the following result:

Lemma

For any symbolic state (l, Z) of the PTA $\mathcal{A}$:

$$\mathsf{IH}(Z) = \mathsf{Conv}\left(\bigcup_{v \in \mathsf{IntVects}(Z_{|P})} v(Z) \right)$$

Example



- Suppose all parameters are in $[0..3]$;
- Initial symbolic state: (ℓ_0, Z_0) with $Z_0 = \{x = y, x \leq b \leq 3\}$;
- After one loop: $Z_1' = Z_1 \cap \{y \leq a + 3, y \leq b + 2\}$,
- After two loops: $Z_2' = Z_2 \cap \{x \leq b - 2a + 2, y \leq a + 3, y - x \leq a + 2\}$,
- After three loops: $Z_3' = Z_3 \cap \{a \leq 1, y \leq a + 3, y \leq b + 3a\}$,
- After four loops: $Z_4' = Z_4 \cap \{y - x = 4a, x \leq 3 - 3a, x \leq b - a\}$,
- After $n > 4$ loops: $\mathbf{Z_n' = Z_{n+1}'} = \{a = 0, x = y, 0 \leq x \leq b, b \leq 3\}$.

Theoretical Complexity

- We can enumerate the parameter values in exponential time

Theorem

The ϕ-emptiness problem for PTA with bounded integer parameters is EXPTIME-complete for any class of problems ϕ that is EXPTIME-complete for TA.

- Better:

Theorem

The ϕ-emptiness problem for PTA with bounded integer parameters is PSPACE-complete for any class of problems ϕ that is PSPACE-complete for TA.

Proof by construction of a non-deterministic Turing machine that works in PSPACE

- TCTL model-checking for TA is PSPACE-complete [ACD93];

Properties Required from the Symbolic States

The proofs use (for any reachable symbolic state (l, Z)):

- Succ is non decreasing
- Z is convex
- $v(\text{Succ}((l, Z), e)) = \text{Succ}((l, v(Z)), v(e))$
- if v is an integer valuation of the parameters then $v(Z)$ has integer vertices
  $\Rightarrow$ does not work for, e.g., stopwatches

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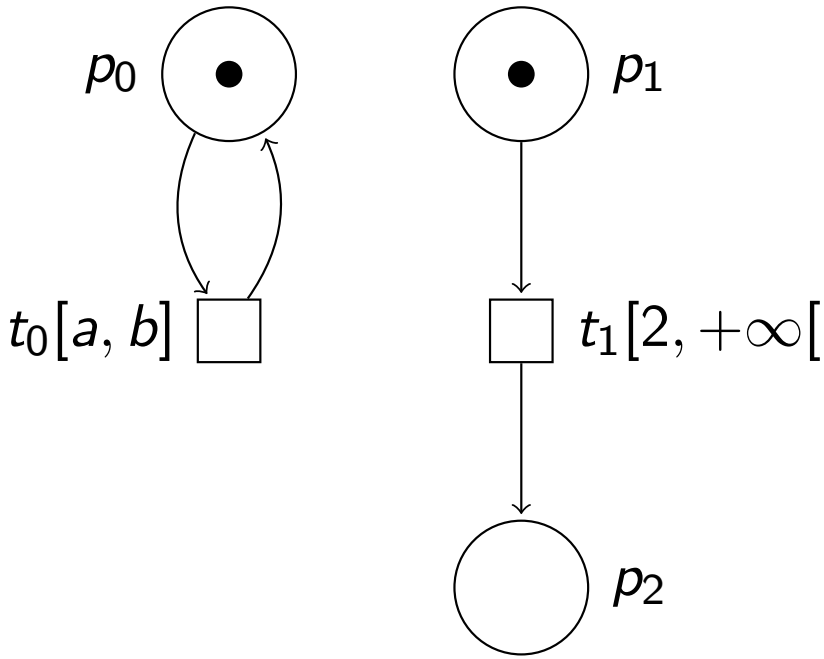
Parametric Time Petri Nets



Results for Parametric TPNs

- There are several time-bisimulation preserving **transformations** between TA and TPNs allowing to transfer (un)decidability results (e.g. [BCH$^+$05, CR06]);

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  - The historical approach is **state classes** [BD91]: they have **all** the good properties required before!
- In both cases, we can use the integer parameters approach.

Romeo: A Scheduling Problem

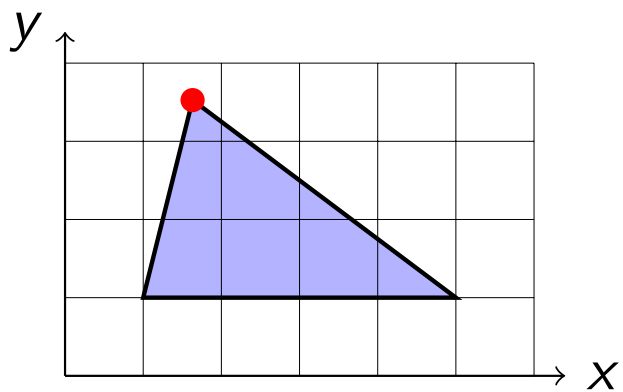
	$a \in [0, \infty)$ $b = 20$ $c = 18$ $d = 28$	$a \in [0, \infty)$ $b \in [10, \infty)$ $c = 18$ $d = 28$	$a \in [0, \infty)$ $b \in [10, \infty)$ $c \in [0, 28]$ $d = 28$	$a \in [0, \infty)$ $b \in [10, \infty)$ $c = 18$ $d \in [18, \infty)$	$a \in [0, \infty)$ $b \in [10, \infty)$ $c \geq 0$ $d \geq c$
IEF Time	1 s	2.8 s	27 s	840 s	DNF
Int. Hull	0.2 s (20%)	0.4 s (14%)	2.9 s (11%)	146 s (17%)	–
IEF Mem.	15 MB	35 MB	153 MB	1289 MB	–
EF Time	1.5 s	6.4 s	DNF	DNF	DNF
EF Mem.	19.6 MB	55 MB	–	–	–

Table : Scaling up the number of parameters in the schedulability problem

	$a = 50$	$a \in [40, 50]$	$a \in [0, 100]$	$a \in [0, 1000]$	$a \in [0, 10000]$
IEF Time	17 s	211 s	1079 s	1150 s	1178 s
Int. Hull	2 s (11.7%)	25.7 s (12.1%)	166 s (15.4%)	167 s (14.5%)	168 s (14.3%)
IEF Mem.	121 MB	425 MB	1598 MB	1667 MB	1667 MB

Table : Scaling up a's upper bound in the schedulability problem for $b \in [10, 100], c = 18$ and $d \in [18, 100]$

Computing the Integer Hull



We compute the **Gomory-Chvátal cuts** from integer linear programming (see, e.g. [Sch86])

- select the constraints defining a non-integer vertex:
  $y - 4x \leq -3$ and $3x + 4y \leq 19$

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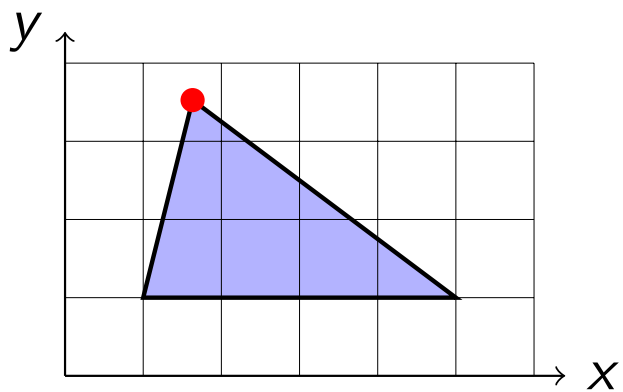

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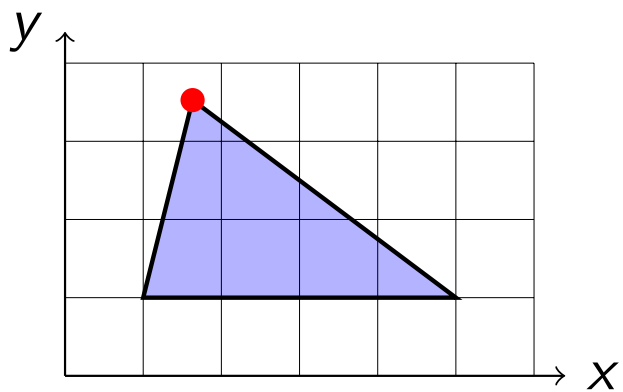

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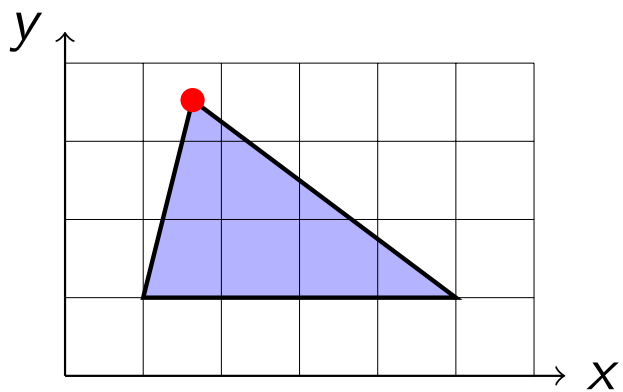

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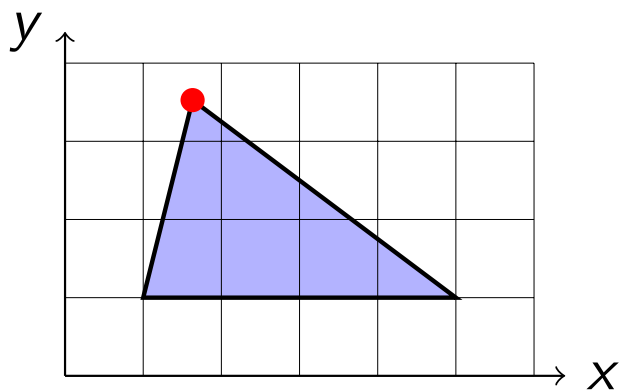
- slacks are non negative: $y + \lfloor \frac{3}{19} \rfloor s + \lfloor \frac{4}{19} \rfloor t \leq \frac{67}{19}$

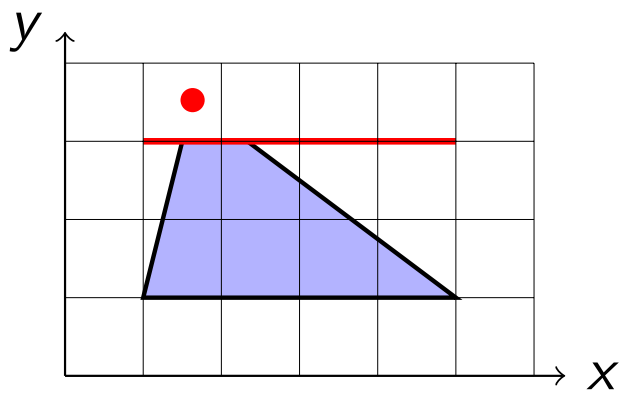
Computing the Integer Hull



We compute the **Gomory-Chvátal cuts** from integer linear programming (see, e.g. [Sch86])

- select the constraints defining a non-integer vertex:
  $y - 4x \leq -3$ and $3x + 4y \leq 19$
- add slack variables:
  $y - 4x + s = -3$ and $3x + 4y + t = 19$
- for a non-integer vertex coord. eliminate other non-slack variables:
  e.g. for y: $19y + 3s + 4t = 67$
- slacks are non negative: $y + \lfloor \frac{3}{19} \rfloor s + \lfloor \frac{4}{19} \rfloor t \leq \frac{67}{19}$
- y, s, t should be integers: $y + \lfloor \frac{3}{19} \rfloor s + \lfloor \frac{4}{19} \rfloor t \leq \lfloor \frac{67}{19} \rfloor$

Computing the Integer Hull



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- y, s, t should be integers: $y + \lfloor \frac{3}{19} \rfloor s + \lfloor \frac{4}{19} \rfloor t \leq \lfloor \frac{67}{19} \rfloor$
- Put the constraints in the above system and eliminate slacks: $y \leq 3$

Outline

Introduction

Parametric Timed Automata

(Un)decidability

Symbolic Semi-algorithms

Computing the Integer Valuations

Time Petri Nets

Conclusion

Performances Considerations

- **Termination** is guaranteed;
- We do **not** explicit **all** possible values;
- But we use **polyhedra** instead of **zones**;
- The computation of the integer hull is **not cheap** in practice
  Though most of the time is usually spent searching in the passed list

Practical Usefulness of Bounded Integer Values

- For (almost) all **practical** purposes, constants in TA or TPNs are chosen as **integers**;
- With adequate **scaling**, rationals can be transformed into integers
  This however sets the precision in advance
- The **symbolic** approach allows for **large** bounds;
- We need **no syntactical restriction** on the constraints in the PTA.

Conclusion and Perspectives

- Conclusion:
  - We have surveyed and completed a bit the theoretical results on L/U automata;
  - We offer an alternative with full PTA with **integer** parameters;
  - We argue this class has a great **practical** relevance;
  - We have shown how valuations can be computed **symbolically**;
  - The method also applies to **parametric TPNs**.

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  - We argue this class has a great **practical** relevance;
  - We have shown how valuations can be computed **symbolically**;
  - The method also applies to **parametric TPNs**.
- Perspectives:
  - Improve the implementation;
  - Investigate models with **stopwatches**.
  - Prove that the symbolic results of the algorithms are actually **dense**
    - Gives "big" underapproximations;
    - Useful, e.g., for robustness issues;
    - Straightforward for EF;
    - Seemingly also true for quite generic properties.

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