Recovery of disrupted airline operations using k-Maximum Matching in graphs

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Assignment of slots for landing to aircrafts

Aircrafts arriving at some airport
Assignment of slots for landing to aircrafts

Each aircraft has a set of available and compatible slots depending on the tracks, the schedules, the companies...
Initially, one compatible slot is assigned to each aircraft.
Assignment of slots for landing to aircrafts

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Assignment of slots for landing to aircrafts

Imponderable problems may happen (no refund...)

Aréopart de Nice
Assignment of slots for landing to aircrafts

How to return to a normal situation?

i.e., maximize # of aircrafts having a slot for landing!!
A simple matching problem in bipartite graphs?

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Reminder on Matchings in Graphs

Let $G = (V, E)$ be a graph.

A matching $M \subseteq E$ is a set of pairwise disjoint edges.
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Exposed vertex: do not belong to the matching

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$\exists$-augmenting path: "alternating" with both ends exposed

Berge 1957: Let $G$ be a graph $M$ maximum matching ($|M| = \mu(G)$) iff no $M$-augmenting path.

⇒ the order in which the augmenting paths are augmented is not important
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Computing a maximum matching

**Maximum Matching in Bipartite Graphs** (flow problem)

“easy” [Hungarian method, Kuhn 1955]

**Maximum Matching** \( \mu(G) \)

Polynomial [Edmonds 1965]

finding an augmenting path in polynomial time + Berge’s theorem

Augment “greedily” paths of length \( \leq 2k - 3 \)

\((1 - \frac{1}{k})\)-Approximation for \( \mu(G) \) [Hopcroft, Kraft 1973]

Applications in wireless networks
Let $G$ be a graph, $M$ be a (partial) matching and $k \in \mathbb{N}$ odd.

Let $\mu_k(G, M)$ be the maximum size of a matching that can be obtained from $M$ by augmenting only paths of lengths $\leq k$. Here $k = 3$. 

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Recovery of disrupted airline operations
Our problem

Given a graph $G$, a matching $M$ and $k \in \mathbb{N}$ odd

Compute a matching of size $\mu_k(G, M)$ that can be obtained from $M$ by augmenting only paths of lengths $\leq k$.

**Goal:** algorithm that computes a sequence $(P_1, \cdots, P_r)$ such that:

- $\forall i \leq r$, $P_i$ a path of length $\leq k$ in $G$
- $\forall i \leq r$, after augmenting $P_1, \cdots, P_{i-1}$ starting from $M$

  $P_i$ is augmenting

- and $r$ is maximum (w.r.t. these constraints)

$$r_{\text{max}} + |M| = \mu_k(G, M)$$
Problem: Given a graph $G$, a matching $M$ and $k \in \mathbb{N}$ odd

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**Goal:** algorithm that computes a sequence $(P_1, \cdots, P_r)$ such that:

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- and $r$ is maximum (w.r.t. these constraints)

**Case** $M = \emptyset$. For any odd $k \geq 0$, $\mu_k(G, \emptyset) = \mu(G)$

Compute a maximum matching, augment its edges one by one.

**Case** $k = 1$. For any matching $M$, $\mu_1(G, M) = \mu(G \setminus V(M)) + |M|$.

The edges of $M$ cannot be “modified”
Compute a max. matching in $G \setminus V(M)$, augment these edges 1 by 1
Our contributions

$k = 3$

Let $G$ be a graph and $M$ be a matching

Computing a matching of size $\mu_3(G, M)$, obtained from $M$ by augmenting paths of length $\leq 3$, is in $P$.

$k \geq 5$

Let $G$ be a planar bipartite graph of max. degree 3 and $M$ be a matching

Computing a matching of size $\mu_k(G, M)$, obtained from $M$ by augmenting paths of length at most $k \geq 5$, is NP-complete.

$G = T$ is a tree.

Computing $\mu_k(T, M)$ ($k$ odd) can be done in polynomial-time in trees $T$

- with bounded max. degree $\Delta$ (dynamic prog., FPT in $k + \Delta$)
- or with vertices of degree $\geq 3$ pairwise at distance $> k$. 
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Why bounding the length of the augmenting paths makes the problem harder? ex: $k = 3$

(a) $\rightarrow$ (b) not optimal, but (a) $\rightarrow$ (c) $\rightarrow$ (d) optimal

$\Rightarrow$ The result is impacted by the order in which paths are augmented.
Why bounding the length of the augmenting paths makes the problem harder? ex: $k = 5$

⇒ The order in which paths are augmented impacts the creation of new augmenting paths that are necessary to reach the optimum.
Our contributions: ideas of the proofs

\( k = 3 \)

\( \mu_3(G, M) \) can be computed in polynomial time

- possible to focus only on 3-augmenting paths initially present;
- after “reducing” the graph, augmentations may be in any order.

\( k \geq 5 \)

\( \mu_k(G, M) \) is NP-hard even if \( G \) planar bipartite of max. degree 3.

Reduction from 3-SAT where “creating” new augmenting paths corresponds to a choice in the assignment.
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Reduction from 3-SAT where “creating” new augmenting paths corresponds to a choice in the assignment.
Question: Complexity of $\mu_k(T, M)$ in trees $T$?

What we know:

Computing $\mu_k(T, M)$ ($k$ odd) can be done in polynomial-time in trees $T$

- with bounded max. degree $\Delta$ (dynamic prog., FPT in $k + \Delta$)
- or with vertices of degree $\geq 3$ pairwise at distance $> k$.

Going further: $\Rightarrow$ a new problem

New Problem: Given a graph $G$, a matching $M$ and $k \in \mathbb{N}$ odd

Compute a matching of size $\mu_k(G, M)$ that can be obtained from $M$ by augmenting only paths of lengths $= k$.

Our results

- $\forall G$ and matching $M$, deciding if $\mu_3(G, M) \leq q$ is NP-complete
- given a tree $T$, $M$ a matching and $q, k \in \mathbb{N}$ as inputs, deciding if $\mu_k(T, M) \leq q$ is NP-complete
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Going further: $\Rightarrow$ a new problem

New Problem: Given a graph $G$, a matching $M$ and $k \in \mathbb{N}$ odd

Compute a matching of size $\mu_{-k}(G, M)$ that can be obtained from $M$ by augmenting only paths of lengths $= k$.

Our results

- $\forall G$ and matching $M$, deciding if $\mu_{=3}(G, M) \leq q$ is NP-complete
- given a tree $T$, $M$ a matching and $q, k \in \mathbb{N}$ as inputs, deciding if $\mu_{=k}(T, M) \leq q$ is NP-complete
Conclusion

Firstly: Learn how to communicate with companies.
After many months of discussion, hypotheses are changing...

Actual (?) constraints of the system/of Airline Operation Controllers:

- Switch alternating cycles of size 4 and augment paths of length 1.
- Two classes of slots: the ones of the company and the others.

Muchas Gracias / Muito Obrigado / Gràcies!

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- in other graph classes?

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