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Hierarchical shape representation with geometric and topological guarantees

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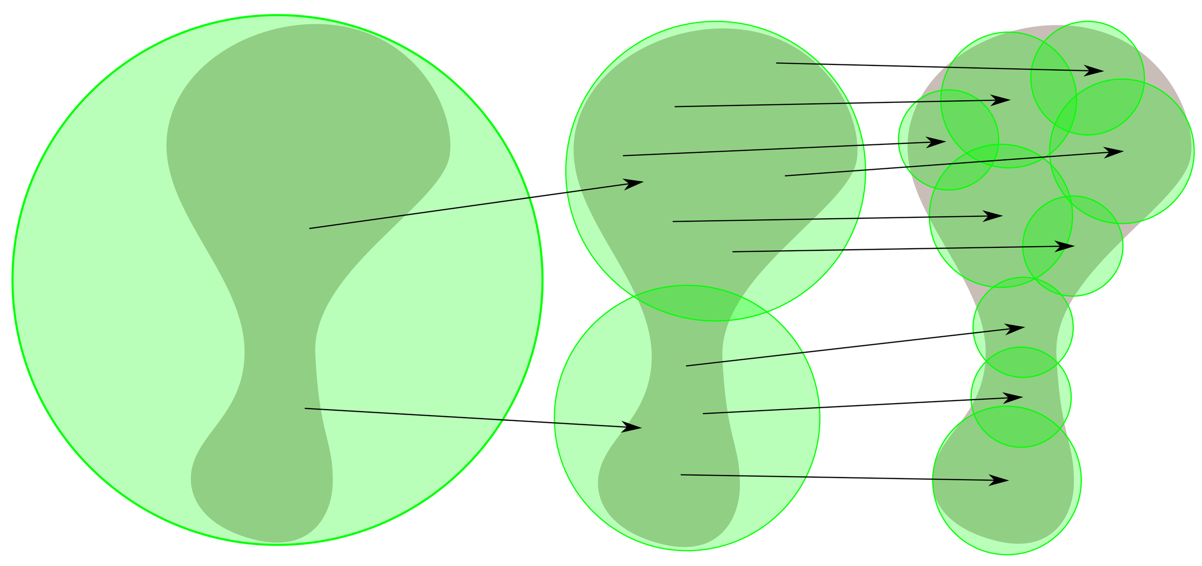
I. General Problem

Hierarchical representations provide a way to describe geometric shapes by a set of simple primitives at different scales. They are useful for collision detection [5]. To obtain a hierarchical representation from an input shape, different methods already exist [1, 7]. However these methods only consider the geometric error of the output and do not take into account topological aspects. Our aim is to define and compute a hierarchical representation with both **geometric** and **topological guarantees**.



II. Shape approximation at a given scale by union of balls

In order to generate a hierarchical representation, the shape first has to be approximated by the chosen primitives. For this, we formalize the problem with notions derived from mathematical morphology.

Let S be an open bounded set in an Euclidean space $\mathbb{E}$ and $\varepsilon > 0$. The erosion and dilation of S by parameter ε are respectively

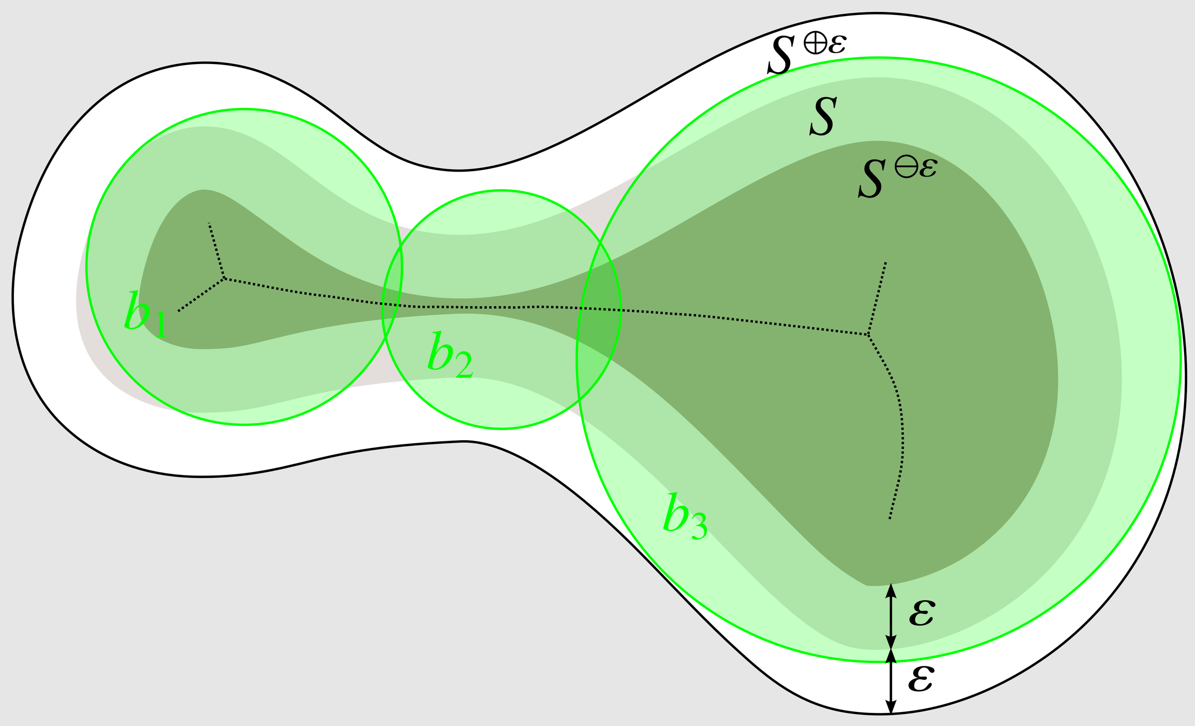
$$S^{\ominus\varepsilon} = \{e \in \mathbb{E} \mid b(e,\varepsilon) \subseteq S\}$$

and

$$S^{\oplus\varepsilon} = \{e \in \mathbb{E} \mid \exists s \in S,\ e \in b(s,\varepsilon)\} = \cup_{s\in S} b(s,\varepsilon)$$

where $b(e,r)$ denotes the open ball of center e and radius r.

Let $\mathscr{B}$ be a collection of balls of $\mathbb{E}$. We say that $\mathscr{B}$ is an ***ε*-covering** of S iff $S^{\ominus\varepsilon} \subseteq \bigcup\mathscr{B} \subseteq S^{\oplus\varepsilon}$, where $\bigcup\mathscr{B} = \cup_{b\in\mathscr{B}} b$.



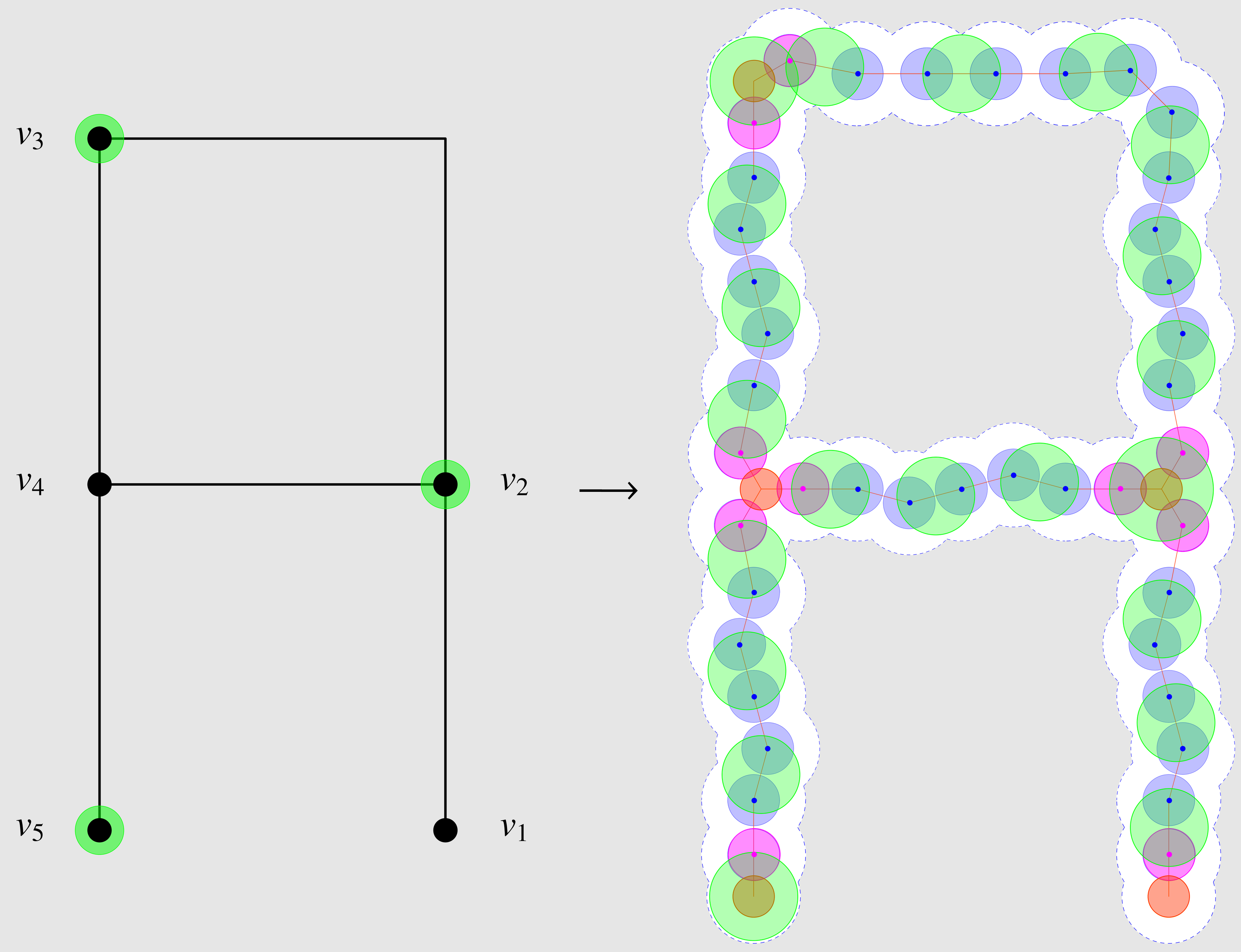
The collection $\{b_1,\ b_2,\ b_3\}$ is an ε-covering of S.

For ε sufficiently small, ε-coverings have good geometric and topological properties. Specifically, there is a universal constant C such that if $\varepsilon < C \times \mathrm{reach}(S)$, then the **Hausdorff distance** between S and $\bigcup\mathscr{B}$ satisfies $d_H(S, \bigcup\mathscr{B}) < \varepsilon$, and $\bigcup\mathscr{B}$ has the same **homotopy type** as S. We want to approximate S with ε-coverings.

III. ε-covering is NP-complete

If the set S is bounded, then it admits an ε-covering $\mathscr{B}$ of finite cardinal, i.e. $|\mathscr{B}| < +\infty$. In particular we are interested in **optimal** ε-coverings that achieve minimum cardinality.

When input shapes are restricted to finite unions of balls, computing an optimal ε-covering is an **NP-complete** problem. The NP property can be verified using arrangements of circles [4, 2]. The hardness property can be proved by reduction from the vertex cover problem.



Conversion of a graph into an adequate input shape

IV. Future works

- Develop an approximation algorithm to find good ε-coverings [3].
- Provide guarantees on the cardinal of the output with respect to the optimal.
- Generate a hierarchical representation from approximations at different scales.

References

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