An applicative theory for the #P hierarchy FCH

— Work in progress —

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The class #P was introduced by Valiant [Val79] and consists of functions which count the number of accepting computations of non-deterministic Turing machines working in polynomial time. For this class, Wagner [Wag86] introduced a hierarchy FCH of counting functions.

In [DKO1x], Ugo Dal Lago and the first two authors of this work gave an implicit recursion-theoretic characterization of the classes #P and FCH by use of *tree recursion* instead of Wagner's sum operation.

The use of recursion in function-algebraic characterizations paves the way for relating them with formal theories which characterize the corresponding complexity classes, where the induction is tailored to match the recursion. In the context of *applicative theories* this was initiated by the third author [Str03, Str10] who treated, among other classes, FPtime and FPspace, the classes of functions computable in polynomial time and in polynomial space, respectively. The first two authors gave, for instance, a corresponding theory for the polynomial hierarchy and its levels [KO13].

In this work, we will present an applicative theory which characterizes the functions in FCH. The definition of the theory follows, to a certain degree, the examples of other applicative theories for classes of computational complexity. There are, however, two interesting technical problems which we like to discuss in more detail.

On the one hand, our theory will be intuitionistic, while most of the other applicative theories for complexity classes can be classical. The problem stems from a certain lack of (de)coding ability in FCH rather than from an intrinsic complexity reason.

On the other hand, we are not able to obtain theories for the levels $(\#\mathsf{P})_k$ of FCH as they have only restricted composition, which cannot be adequately mimicked in the applicative framework.

The aim of this talk is put up these problems for discussion and to look for possible solutions.

References

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