Hyper/Multi-Coherence Spaces  
and the Taylor Expansion of $\lambda$-terms

M1/M2 Internship proposal

Keywords  
Denotational Semantics, $\lambda$-Calculus, Sequential Computation,  
Linear Logic

Goal  
Characterizing the Taylor expansion of ordinary $\lambda$-terms via  
webbed denotational models.

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Brief description

$\lambda$-calculus is a mathematical model of sequential functional pro-
gramming. The basic notion is that of applying an argument to  
a function and to evaluate such an application by substituting all  
occurrences of the function parameter with the argument [0].

The process of substitution can erase or copy the argument as  
many times as needed. This fact has a mathematical appealing inter-
pretation through the so-called Taylor expansion of the $\lambda$-calculus  
into the resource calculus [1, 2]. The resource calculus has a bounded  
version of the $\lambda$-calculus application of the form $M(N^n)$, meaning  
that the argument $N$ is a resource that must be used by the func-
tion $M$ exactly $n$ times during the evaluation. The Taylor expansion  
then translates the unbounded $\lambda$-calculus application $M(N)$ into the
series \( \sum_n \frac{1}{n!} M(N^n) \) of resource applications, recalling the standard Taylor expansion of the exponential function.

Of course, not every sum of resource terms is in the Taylor expansion of an ordinary \( \lambda \)-term, and the question of characterizing the series converging to \( \lambda \)-terms wait to be settled. A partial answer has been achieved in [3] in the setting of propositional linear logic, with a combinatorial criterion characterizing the sums of differential nets converging to linear logic proof nets.

Here, we address the question in the framework of the \( \lambda \)-calculus and we look for a characterization based on denotational semantics. We plan to consider the enrichments of the category \( \text{REL} \) of sets and relations described in [4]. Any set \( S \) of resource terms can be interpreted as a relation \( [S] \) in \( \text{REL} \). We wonder whether there is a suitable enrichment \( C \) of \( \text{REL} \) such that \( [S] \) is a morphism in \( C \) if, and only if, the terms in \( S \) appears in the Taylor expansion of a \( \lambda \)-term.

Understanding the convergence which is at the base of the Taylor expansion of \( \lambda \)-terms might be the starting point of renewing our notion of sequential computation, represented here by the \( \lambda \)-calculus.

References


