The Diameter of Pancake Networks

1 Introduction

Given a stack of $n$ pancakes in arbitrary order, all of different size, the goal is to sort them so that the largest pancake is on the bottom and the smallest on top. The allowed sorting operation is a "spatula flip": a spatula is inserted beneath any pancake, and all pancakes above the spatula are lifted and replaced in reversed order. The problem is to find (or bound) $f(n)$, the minimum number of flips required in the worst case to sort a stack of $n$ pancakes.

Variant of the problem: consider the pancakes are two-sided (one side is "burnt"), and must be sorted into the size-ordered configuration in which each pancake has its burnt side down. Let $g(n)$ be the worst-case number of flips to sort $n$ "burnt pancakes". The problem is to find (or bound) $g(n)$.

2 The Problem

Let $U_n$ and $B_n$ be the unburnt and the burnt pancake networks, respectively. $U_n$ and $B_n$ are Cayley networks. The symmetric group $S_n$ is the group of $U_n$, while the group $P_n$ of $B_n$ consists of all "signed permutations" on $n$ elements. $U_n$ is generated by the set $\{u_1, \ldots, u_n\}$ and $B_n$ is generated by the set $\{b_1, \ldots, b_n\}$, where generators $u_j$ and $b_j$ correspond to a flip of the top $j$ unburnt (burnt) pancakes, respectively. $U_n$ has $n!$ vertices and is regular of degree $n - 1$. Similarly, $B_n$ has $2^n \cdot n!$ vertices and is regular of degree $n$.

Note that $u_1$ is the identity, whereas $b_j b_i b_j$ serves to flip the $j$-th burnt pancake in place.

Question: What are the values of the diameters $D(U_n) = f(n)$ and $D(B_n) = g(n)$?

3 Previous Partial Solutions

Each node of a network ($U_n$ or $B_n$) can be identified with a unique stack of $n$ pancakes (unburnt or burnt). A path from $A$ to $B$ through a network corresponds to a sequence of flips transforming stack $A$ into stack $B$. Let $g \in S_n$ (in $P_n$, resp.). We write $gA = B$ and we say that "$g$ sorts $A$" if $gA = I_n$ (the identity permutation).

With notations $D(U_n) = f(n)$ and $D(B_n) = g(n)$, the current results hold:

- Bounds for $f(n)$ of $17n/16$ and $5/3 (n + 1)$ where shown in [2].
- It was found in [1, 3] that $3n/2 \leq g(n) \leq 2n - 2$, where the upper bound holds for $n \geq 10$.
- Under the conjecture that $-I_n$ is the worst case among all stacks of $n$ burnt pancakes, it is shown in [1, 4] that $f(n)$, $g(n) \leq 47n/30 + c \approx 1.566 \ldots n + c$.

Références