Consistency in Parametric Interval Probabilistic Timed Automata

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Motivation

- Allow uncertainty in probabilistic timed models
  - On probabilities \( \Rightarrow \) Intervals
  - On timing constants \( \Rightarrow \) Parameters
  - Is it possible to concretize an abstract model containing uncertainties? \( \Rightarrow \) Consistency

- We want to compute the whole set of parameter values ensuring the desired properties.
Outline

Introduction

Probabilistic and Timed Specifications
  Timing Uncertainties
  Probabilistic Uncertainties
  Combining both approaches

The Consistency Problem
  Consistency in IMC/IMDP
  Consistency in Interval Probabilistic TA

Parameter Synthesis for PI\(^P\)TA Consistency
  Undecidability of Consistency for PI\(^P\)TA
  Semi-Algorithm

Conclusion
Outline

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Conclusion
Probabilistic Timed Automata (PTA)

- Clocks
- Discrete Probabilities

Restriction
- No invariants
Probabilistic and Timed Specifications

Timing Uncertainties

Parametric Probabilistic Timed Automata (PPTA)

\[\begin{align*}
\text{l}_0 & \xrightarrow{0.4} \text{l}_1 \\
\text{l}_1 & \xrightarrow{0.6} \text{l}_0 \\
\text{l}_2 & \xrightarrow{0.9} \text{l}_5 \\
\text{l}_5 & \xrightarrow{0.1} \text{l}_2
\end{align*}\]

\[x := 0, \quad x = 1 \land y \leq \alpha, \quad y := 0, \quad y < 2, \quad a\]

**Results**

- Reachability emptiness is *undecidable* [AHV93]
- Halting Problem of a 2-counter machine

**PPTA**

- Clocks compared to parameters
- Parameter Synthesis

Benoît Delahaye (Univ Nantes - LINA) Consistency in PPTA
Interval Probabilistic Timed Automata (\(I\text{PTA}\))

Symbolic Semantics: Classical LTS Semantics + probabilities

⇒ Interval Markov Decision Process (IMDP)
Implementation of $\mathbb{P}TA$

Satisfaction Relation

- Simulation-like relation
- **Structure not necessarily preserved**
- Clocks, Guards and Resets must be the same
Probabilistic and Timed Specifications Combining both approaches

Parametric Interval Probabilistic Timed Automata (PI\(\text{PTA}\))

Implementation: PI\(\text{PTA}\)

Same as for I\(\text{PTA}\):
- Simulation-like: Structure not preserved
- Same Clocks, Guards and Resets

+ Parameter Valuation fixed

Benoît Delahaye (Univ Nantes - LINA) Consistency in PI\(\text{PTA}\)
2016-10-18
The Consistency Problem

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Consistency

Does there exist a parameter valuation such that a given PI\textsuperscript{PTA} admits at least one implementation?
Consistency in IMC/IMDP

IMC/IMDP Consistency

- Decidable [DLLPW11]
- Polynomial Algorithm [D15]

Theorem [D15]: Structure can be conserved
The Consistency Problem Consistency in Interval Probabilistic TA

Theorem from [D15] does not hold for IP TA

- Consistent
- No implementation with same structure
Solution: Zone Graph

Theorem (Zone Graph Consistency)

An $\mathbb{IP}$TA is consistent iff its IMDP zone graph is consistent

Algorithm: Consistency of $\mathbb{IP}$TA

\begin{itemize}
  \item Build IMDP Zone Graph $\mathcal{IM}$
  \item Check Consistency of $\mathcal{IM}$
\end{itemize}

Constructive algorithm

$\Rightarrow$ Build $\mathbb{PTA}$ implementation from IMDP implementation
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Conclusion
More leverage than PPTA

We can use parameter values and probabilities to make inconsistent states unreachable.
Undecidability of Consistency for PI\(\mathbb{P}\)TA

**Theorem**

The consistency-emptyness for PI\(\mathbb{P}\)TA is undecidable

- Reduction from the halting problem of a 2-counter machine
- Halting-state is made inconsistent by adding an inconsistent transition
- 2-counter machine halts iff PI\(\mathbb{P}\)TA is inconsistent for all parameter valuations
Parameter Synthesis for PI\(^\text{PTA}\) Consistency

Semi-Algorithm

Consistency Synthesis 1/2

Algorithm (Sketch)

Input: Labeled IMDP semantics (zone graph) of PI\(^\text{PTA}\)
Output: Constraint \(K\) on parameters ensuring consistency

- Identify locally inconsistent states \(\text{Inc}\)
- While \(\text{Inc} \neq \emptyset\)
  - Pick \(s \in \text{Inc}\)
  - Remove \(s\) from \(\text{Inc}\) and mark\(^1\) \(s\)
  - If possible, use probabilities to make \(s\) unreachable
  - Else mark\(^2\) predecessor states as inconsistent
- If \(s_0\) is not marked\(^2\) then return \(\top\)
- Remove unreachable states
- For all marked\(^2\) states \(s\)
  - \(K \leftarrow K \setminus C_s\)
- Remove all states \(s\) s.t. \(C_s \cap K = \emptyset\)
Consistency Synthesis Example

\[ y < 2 \quad \text{a} \]
\[ [0, 0.5] \quad x := 0 \]
\[ x := 0 \quad \text{b} \]
\[ 2 \leq x \leq \gamma \]
\[ y := 0 \]
\[ [0, 1] \]
\[ c \]
\[ [0, 0.3] \quad x, y := 0 \]
\[ [0, 0.2] \quad y := 0 \]
\[ d \]
\[ x = 5 \]
\[ 2 \leq x \leq \gamma \quad \text{e} \]
\[ \text{x := 0} \]
Consistency Synthesis Example

\[ y < 2 \quad \xrightarrow{a} \quad l_0 \]

\[ x := 0 \quad \xrightarrow{b} \quad l_1 \]

\[ 2 \leq x \leq \gamma \quad \xrightarrow{c} \quad l_2 \]

\[ x = 1 \land y \leq 2 \]

\[ y := 0 \quad \xrightarrow{d} \quad l_3 \]

\[ x, y := 0 \]

\[ x = 5 \quad \xrightarrow{e} \quad l_4 \]

\[ 2 \leq x \leq \gamma \]

\[ x := 0 \]
Consistency Synthesis Example

\[
\begin{align*}
0.1 & \quad x := 0 \\
0 & \quad x := 0 \\
0.8 & \quad x = 1 \land y \leq 2 \\
\end{align*}
\]

\[
\begin{align*}
\text{a} & \quad \text{y} < 2 \\
\text{b} & \quad \text{y} := 0, 2 \leq x \leq \gamma \\
\text{c} & \quad x, y := 0 \quad [0, 0.3] \\
\text{d} & \quad x := 5 \quad [0, 0.2] \\
\text{e} & \quad x := 0 \\
\end{align*}
\]
Consistency Synthesis Example

\[ y < 2 \]
\[ x := 0 \]
\[ x = 1 \land y \leq 2 \]
\[ c \]
\[ 2 \leq x \leq \gamma \]
\[ b \]
\[ K = (\gamma < 2) \]
\[ y := 0 \]
\[ 0 \leq x < 0.8 \]
\[ 0.1 \]
\[ x := 0 \]
\[ 0.8 \]
\[ 0 \leq x < 0.8 \]
\[ l_0 \]
\[ l_1 \]
\[ l_2 \]
\[ l_3 \]
\[ l_4 \]
\[ l_5 \]
**Consistency Synthesis Example**

Parameter $\gamma < 2$

**Result**

$\gamma < 2$
Consistency Synthesis 2/2

Theorem

A parameter valuation $v$ satisfies $K$ iff $v$ ensures that $\PiPTA$ is consistent

But...

- Semi Algorithm because IMDP semantics might be infinite
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Conclusion – Future work

- New formalism taking into account uncertainty on probabilities and timing constants
- Decidability of Consistency for $\mathbb{I} \mathbb{P} \mathbb{T} \mathbb{A}$
- Undecidability of Consistency for $\mathbb{P} \mathbb{I} \mathbb{P} \mathbb{T} \mathbb{A}$
- Semi-Algorithm

- Under-approximation that always terminates
- Subclasses for which exact synthesis can be achieved
- Parameters on probabilities