

# Are Timed Automata Bad for a Specification Language?

*Language Inclusion Checking for Timed Automata*

# Contributors

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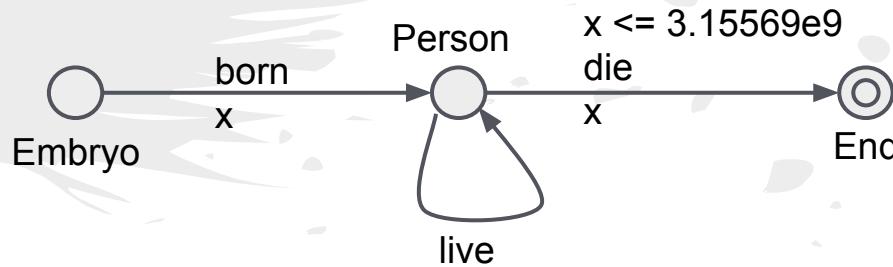
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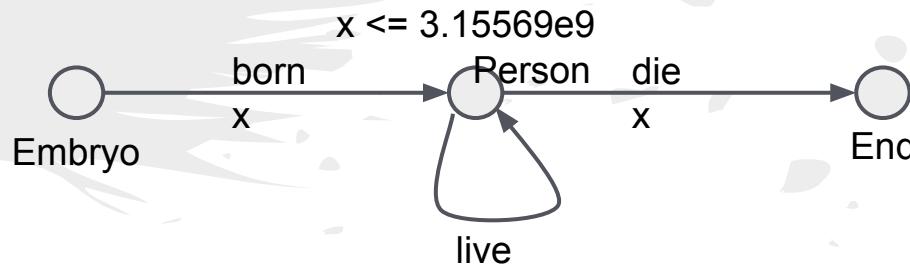
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# Timed Buchi Automata



\* “A Theory of Timed Automata”, 1994, Dill and Alur

# Timed Safety Automata



\* “Symbolic Model Checking for Real-Time Systems”, 1992, Hezinger et al.

\*\* Timed Automata means Timed Safety Automata hereafter

# Languages

A rooted run of the timed automaton:

*<Embryo, 50, Embryo, born, Person, live, Person, 1000, Person, live, Person, die, End>*

A word of the timed automaton:

*<(50,born),(0, live),(1000, live),(0, die)>*

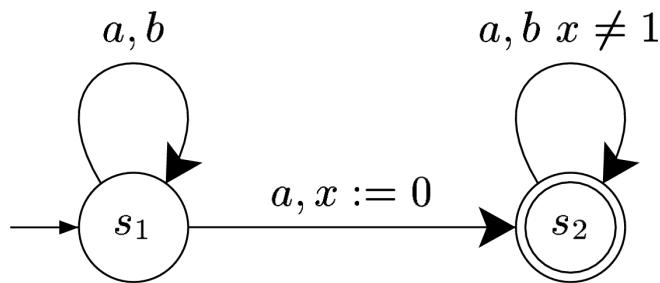
# The Problem

Let  $Impl$  be a timed automaton modeling an implementation;  $Spec$  be a timed automaton modeling a specification of the system.

Can we check  $Impl$  refines  $Spec$ , i.e., any word in  $Impl$  is in  $Spec$ ?

# The Problem is Undecidable

Timed automata are un-determinable\*.



\* “Decision Problems for Timed Automata: a Survey”, 1994, Dill and Madhusudan

# The Conclusion?

“... this result is an obstacle in using timed automata as a specification language ...”\*

Shall we look at event-clock timed automata,  
one lock timed automata, instead?

\* “A Theory of Timed Automata”, 1994, Dill and Alur

# This Work

We propose a semi-algorithm for checking whether an arbitrary timed automaton refines another.

We would argue that timed automata are not a bad specification language.

# The Result

Are timed automata good for specify commonly used timed properties?

Our semi-algorithm always terminates on commonly used timed properties.

# The Result

Does the Semi-Algorithm terminate often?

Highly likely (the answer is related to the transition density of the Spec).

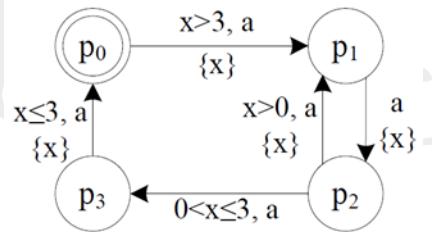
# The Result

Is the Semi-Algorithm Scalable in Practice?

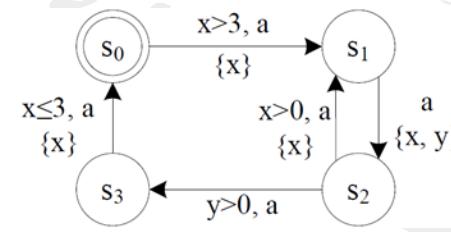
With the reduction techniques in place, it is perhaps as scalable as Uppaal is.

# The Approach

Here it goes ...

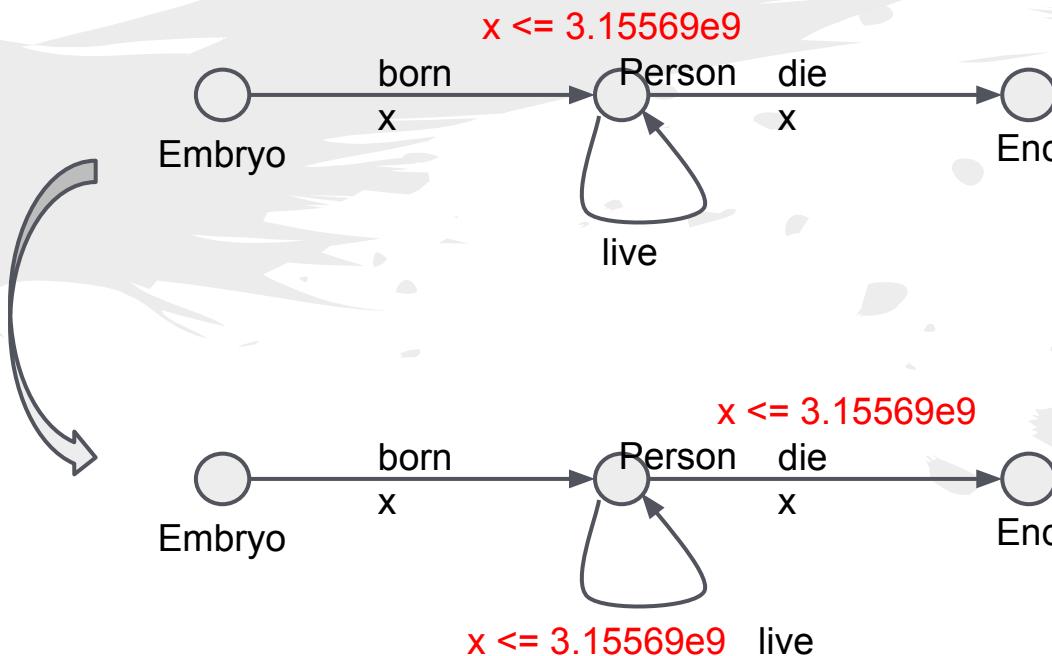


Impl

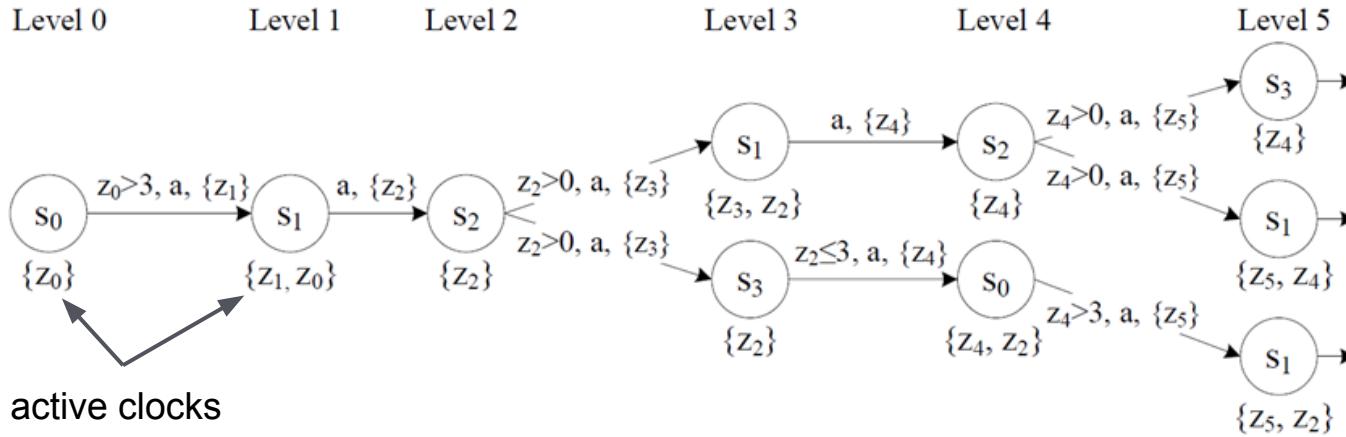
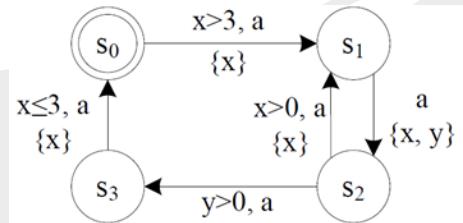


Spec

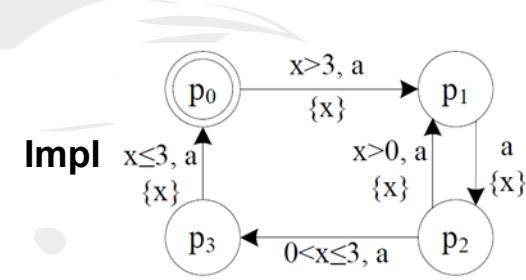
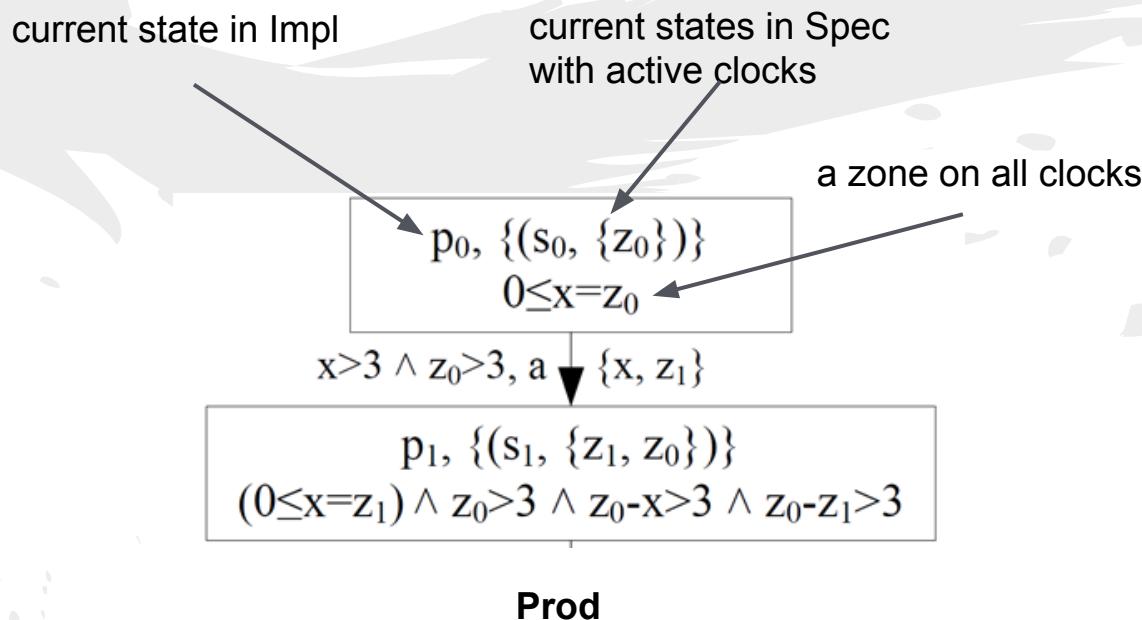
# Step 0: Remove Invariants



# Step 1: Unfold Spec

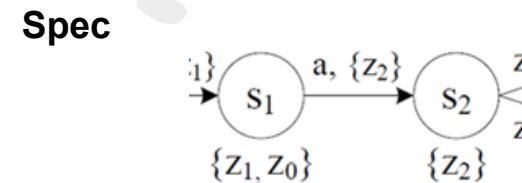
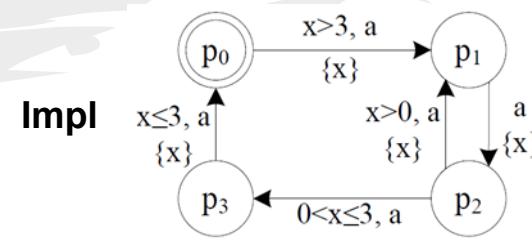
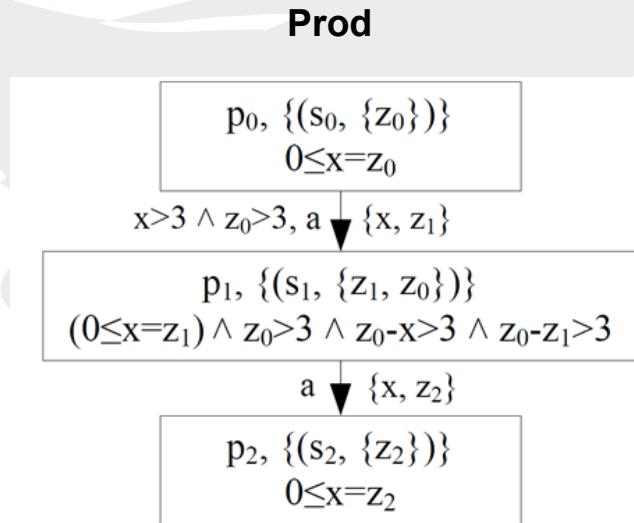


# Step 2: Compute the Product

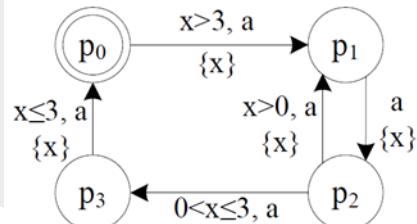


**Spec**

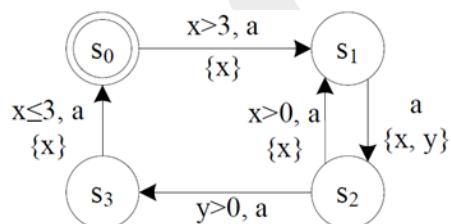
# Step 2: Compute the Product



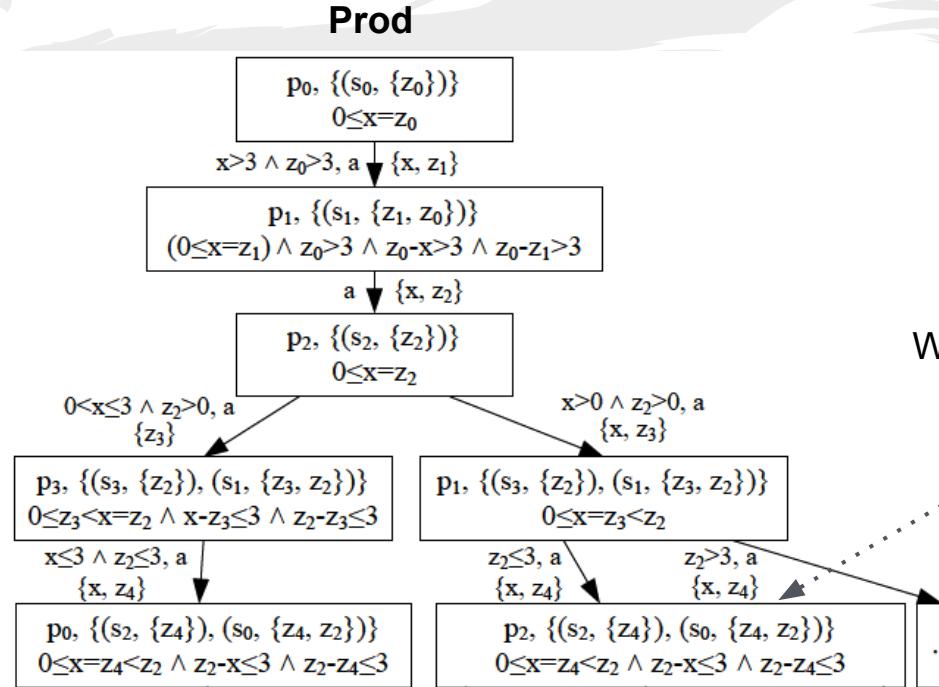
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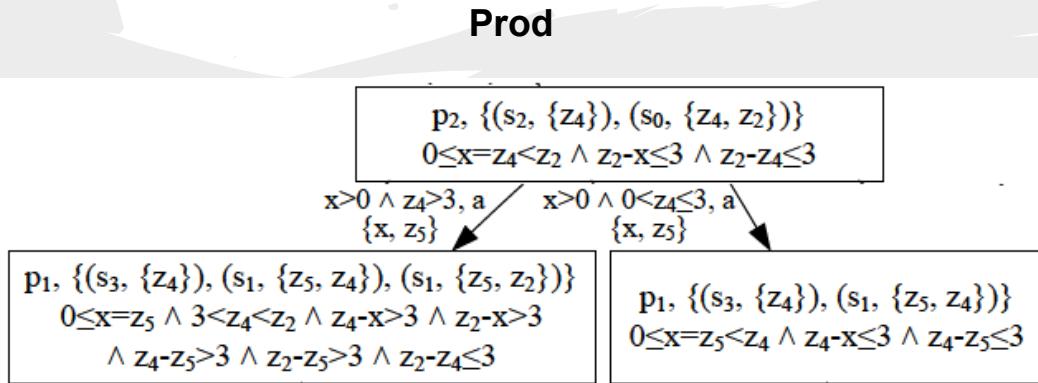
Impl



Spec

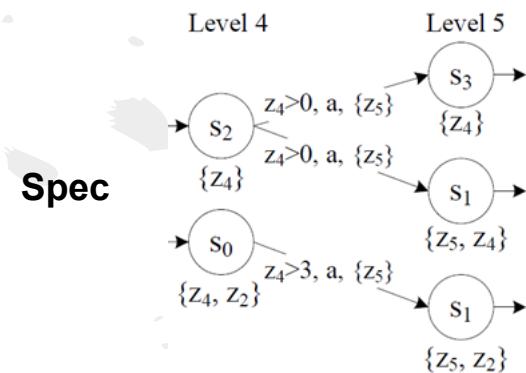
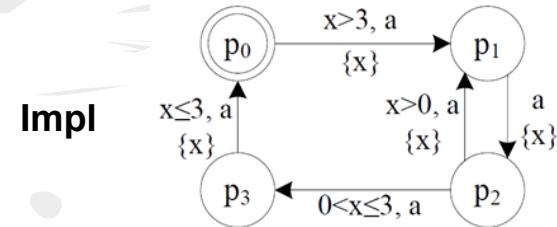


# Step 2: Compute the Product

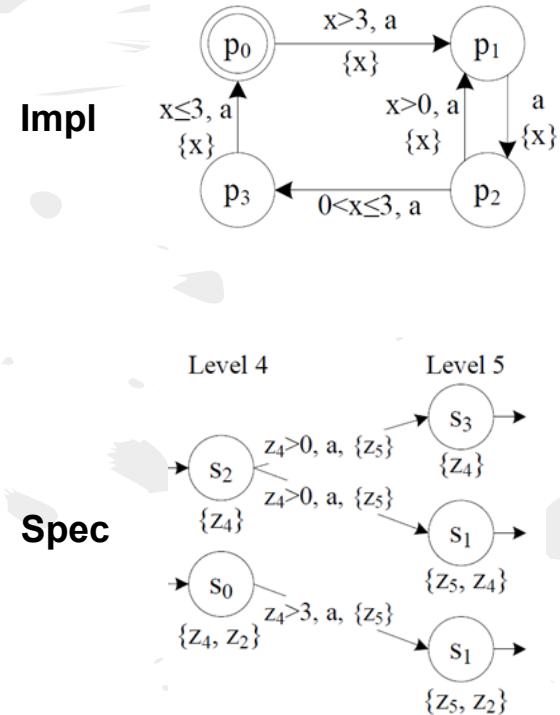
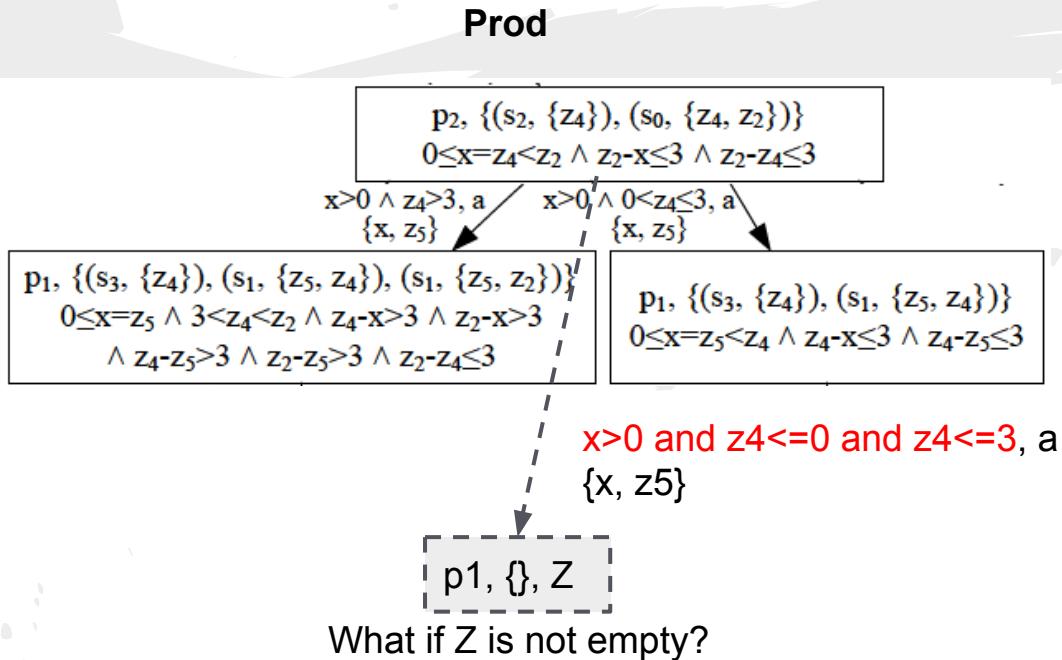


## Four combinations:

$x > 0$  and  $z_4 > 0$  and  $z_4 > 3$   
 $x > 0$  and  $z_4 > 0$  and  $z_4 \leq 3$   
 $x > 0$  and  $z_4 \leq 0$  and  $z_4 > 3$   
 $x > 0$  and  $z_4 \leq 0$  and  $z_4 \leq 3$



# Step 2: Compute the Product



# Theorem

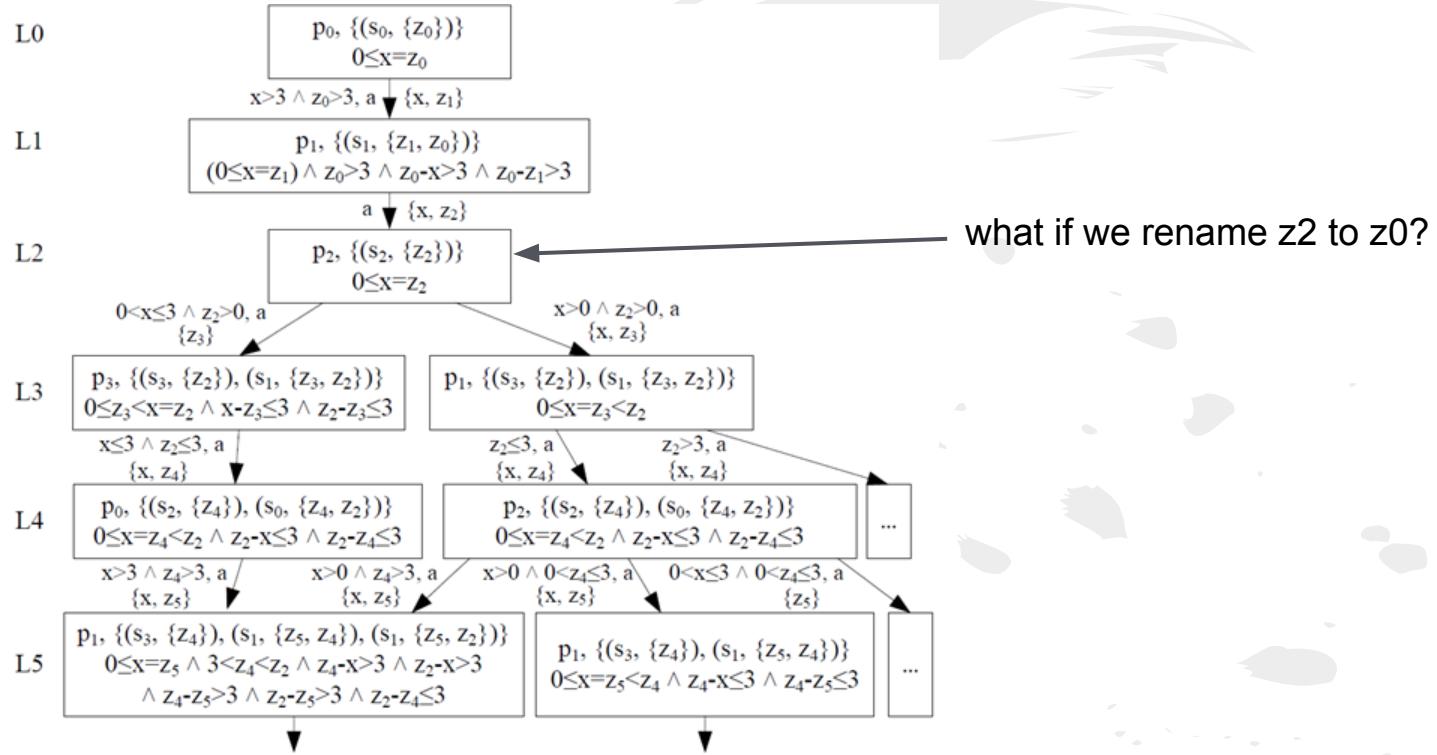
*Impl* refines *Spec* iff there is no reachable state  $(p, \{\}, Z)$  in *Prod*.

One minor problem: the product has infinitely many states.

# Reducing Prod

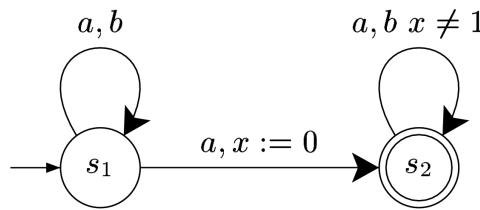
as much as we could ...

# Clock Renaming



# Infinite Clocks

There might be infinitely many active clocks at a state in Prod.



If #clocks are bounded, Prod is finite after clock renaming (with zone normalization).

# Simulation Reduction

If  $s$  simulates  $s'$  (w.r.t a set of accepting states),  
then if  $s'$  can be skipped if  $s$  has been explored.

Identifying the simulation relationship is  
expensive in general.

# LU-Simulation

Let  $(p_1, X_1, Z_1)$  and  $(p_2, X_2, Z_2)$  be two states in Prod.  $(p_2, X_2, Z_2)$  simulates  $(p_1, X_1, Z_1)$  iff

- $p_2 = p_1$  and  $X_2 = X_1$  and
- for all clock valuation  $v_1$  in  $Z_1$ , there exists  $v_2$  in  $Z_2$  such that  $v_1(x) = v_2(x)$ , or  $L(x) < v_2(x) < v_1(x)$ , or  $U(x) < v_1(x) < v_2(x)$  for all  $x$ .

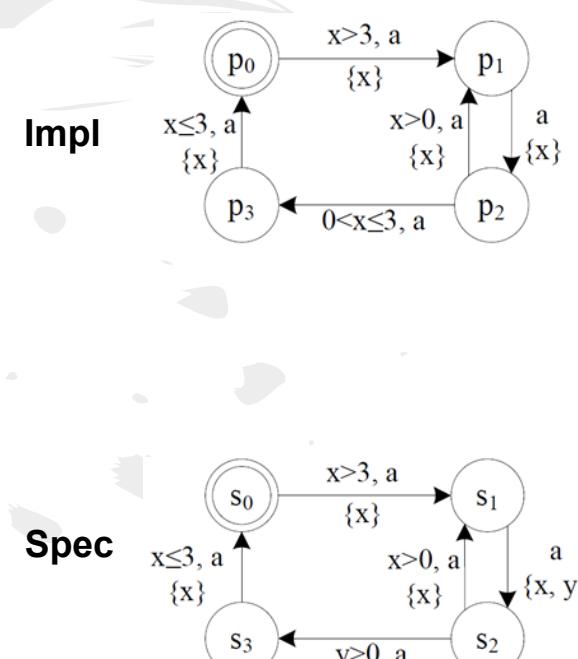
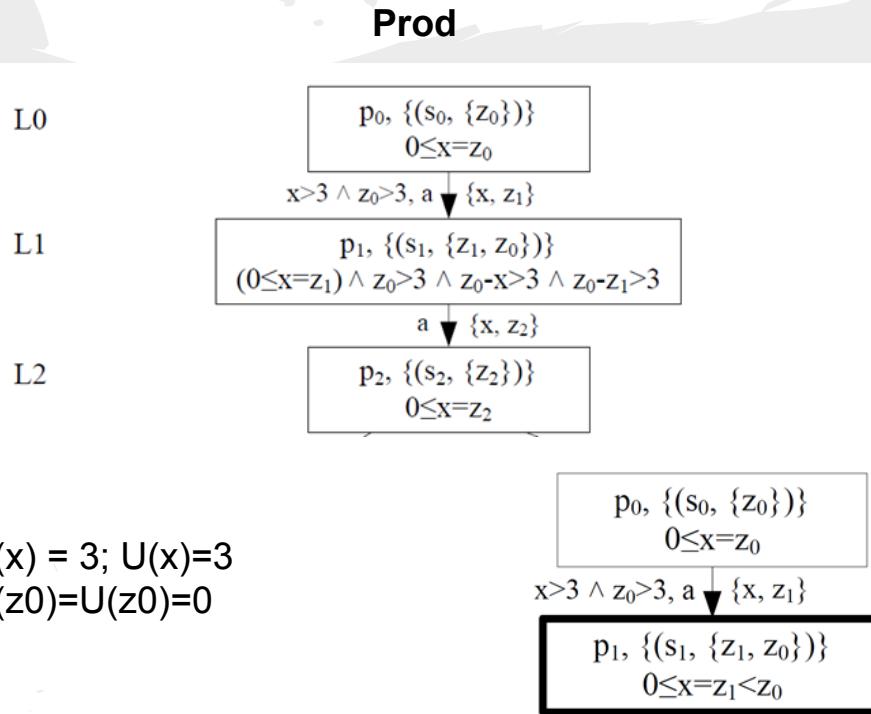
where  $L(x)$  is the maximal constant from a clock constraint of the form  $x > k$  or  $x >= k$ ;  $U(x)$  is the maximal constant from a clock constraint of the form  $x < k$  or  $x <= k$ .

# Zone Extrapolation

Given a state  $(p, X, Z)$ , enlarge  $Z$  s.t. it contains all clock valuation  $v_1$  s.t. there exists  $v_2$  in  $Z$  such that  $v_1(x) = v_2(x)$ , or  $L(x) < v_2(x) < v_1(x)$ , or  $U(x) < v_1(x) < v_2(x)$  for all  $x^*$ .

All clock valuations added to  $Z$  are simulated by an existing one.

# LU-Simulation: Example

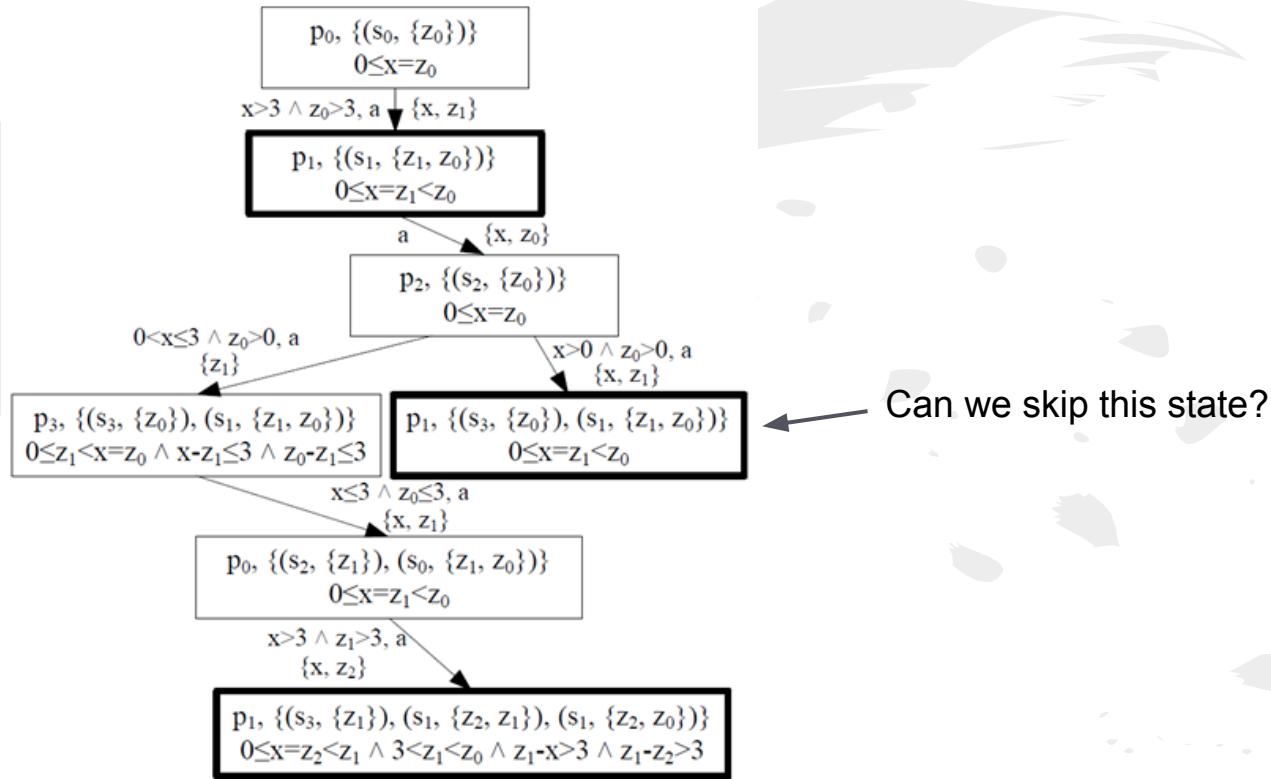


# LU Simulation Reduction

During exploration, a state  $(p, X, Z)$  can be skipped if a state  $(p, X, \text{extra}(Z'))$  where  $Z$  is a subset of  $\text{extra}(Z')$  has been explored.

\*  $\text{extra}(Z)$  is the enlarged zone based on  $Z$ .

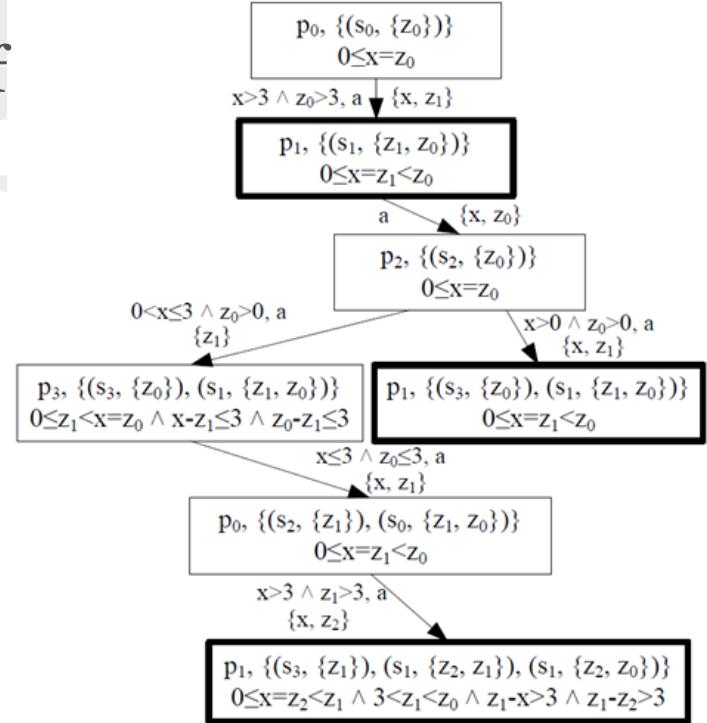
# Anti-Chain



# Anti-Chain

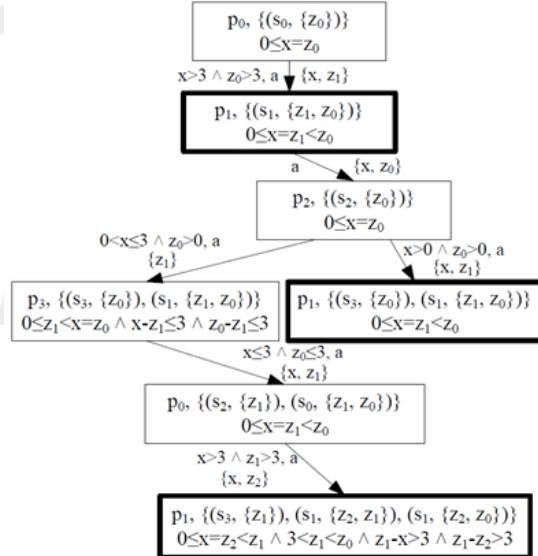
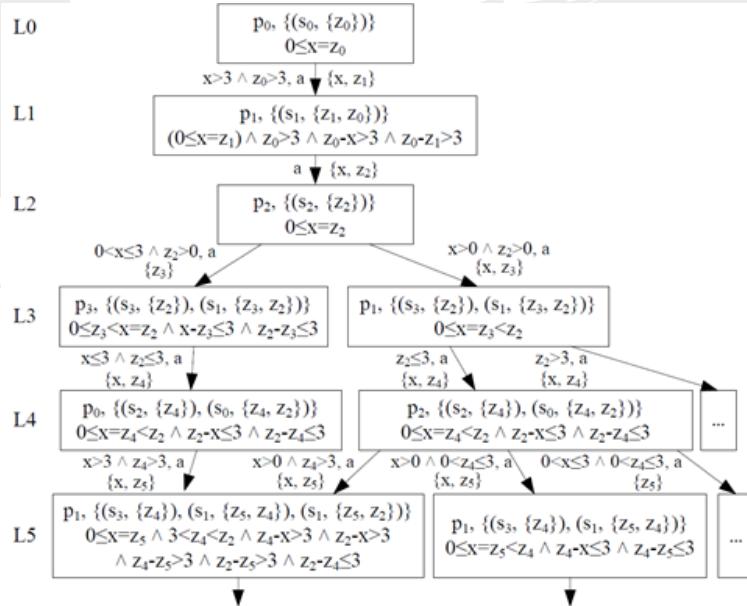
$(p_1, X_1, Z_1)$  simulates  $(p_2, X_2, Z_2)$  iff

- $p_1 = p_2$  and
- $X_1$  is a subset of  $X_2$  and
- $Z_2$  is a subset of  $Z_1^*$



\*with clock renaming

# The Reduction



# The Algorithm

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**Algorithm 1** Language inclusion checking

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```
1: let working := Init;  
2: let done :=  $\emptyset$ ;  
3: while working  $\neq \emptyset$  do  
4:     remove  $ps = (s_p, X_s, \delta)$  from working;  
5:     add  $ps$  into done and remove all  $ps' \in done$  s.t.  $ps' \sqsubseteq ps$ ;  
6:     for all  $(s'_p, X'_s, \delta') \in post(ps, \mathcal{Z}_r^{LU})$  do  
7:         if  $X'_s = \emptyset$  then  
8:             return false;  
9:         end if  
10:        if  $\nexists ps' \in done$  such that  $(s'_p, X'_s, \delta') \sqsubseteq ps'$  then  
11:            put  $(s'_p, X'_s, \delta')$  into working;  
12:        end if  
13:    end for  
14: end while  
15: return true;
```

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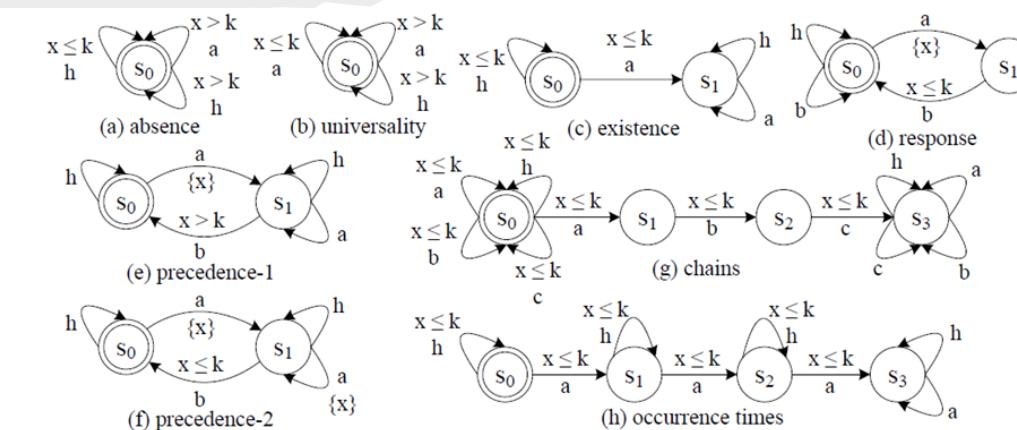
# Termination

Always terminates if active clocks are bounded (which includes SNZ, Event-clock timed automata, timed automata with integer resets).

Always terminates for one-clock timed automata.

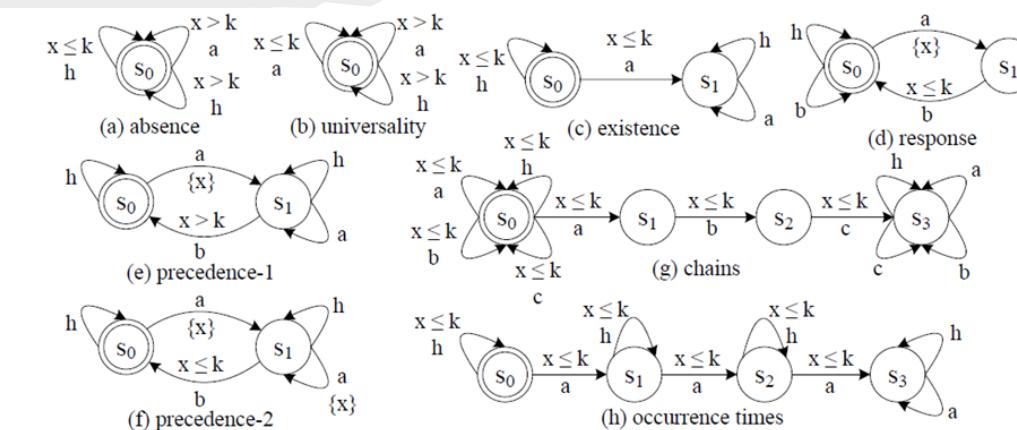
# Evaluation 0

Is the algo always terminates given a common timed property? Yes.



# Evaluation 0

Is the algo always terminates given a common timed property? Yes.



# Evaluation 1

Is the algo  
scalable?

**Table 1.** Experiments on Language Inclusion Checking for Timed Systems

System	$ C_s $	Det	$\sqsubseteq + LU$			$LU$			$\sqsubseteq$		
			stored	total	time	stored	total	time	stored	total	time
Fischer*8	1	Yes	91563	224208	28.3	138657	300384	516.7	-	-	-
Fischer*6	6	No	38603	78332	537.0	-	-	-	-	-	-
Fischer*6	2	No	27393	58531	6.8	36218	70348	30.3	-	-	-
Fischer*7	2	No	121782	271895	42.9	159631	326772	661.7	-	-	-
Railway*8	1	Yes	796154	1124950	142.1	-	-	-	-	-	-
Railway*6	6	No	23265	33427	7.2	27903	39638	20.4	-	-	-
Railway*7	7	No	180034	260199	66.7	222806	318698	1352.8	-	-	-
Lynch*5	1	Yes	3852	11725	0.6	16193	48165	6.0	45488	421582	377.2
Lynch*7	1	Yes	79531	400105	34.9	-	-	-	-	-	-
Lynch*5	2	No	8091	29686	2.4	63623	208607	151.3	56135	324899	290.1
Lynch*6	2	No	35407	162923	16.7	477930	1828668	5751.1	-	-	-
FDDI*7	7	Yes	1198	1590	7.4	8064	9592	36.4	8452	11836	125.5
CSMA*7	1	Yes	9840	36255	4.5	-	-	-	-	-	-

# Evaluation 2

Does it  
terminate?

**Table 2.** Experiments on Random Timed Automata

$ S $	$ C $	$Dt = 0.6$	$Dt = 0.8$	$Dt = 1.0$	$Dt = 1.1$	$Dt = 1.3$
4	1	1.00\0.99\0.98	0.99\0.93\0.74	0.99\0.82\0.59	0.99\0.63\0.39	0.89\0.18\0.09
4	2	0.99\0.98\0.94	0.98\0.87\0.68	0.94\0.72\0.51	0.85\0.49\0.33	0.45\0.12\0.06
4	3	0.99\0.98\0.93	0.95\0.82\0.65	0.89\0.67\0.52	0.75\0.42\0.28	0.31\0.10\0.06
6	1	1.00\0.99\0.98	0.99\0.97\0.90	0.99\0.61\0.41	0.97\0.43\0.29	0.83\0.13\0.08
6	2	0.99\0.99\0.98	0.99\0.96\0.88	0.88\0.49\0.32	0.79\0.34\0.22	0.44\0.09\0.05
6	3	0.99\0.99\0.98	0.99\0.94\0.85	0.78\0.44\0.29	0.69\0.31\0.21	0.34\0.11\0.07
8	1	1.00\0.99\0.99	0.99\0.92\0.83	0.96\0.53\0.40	0.94\0.37\0.31	0.55\0.08\0.07
8	2	0.99\0.99\0.99	0.99\0.91\0.84	0.84\0.48\0.37	0.73\0.32\0.25	0.25\0.10\0.09
8	3	0.99\0.99\0.99	0.98\0.91\0.83	0.78\0.47\0.38	0.70\0.40\0.32	0.20\0.08\0.07

$Dt = \#transitions/\#states$ ; a\b\c: percentage of termination (a: with reduction;  
b: without reduction; c: due to Spec being determinizable)

# Related Work

Zone abstraction

LU simulation reduction

Anti-chain simulation reduction

# Ongoing Work

How to extend the algorithm to deal with non-Zenoness?

What is the best way to verify timed automata with the assumption of non-Zenoness?



Q?